Revision Log for Uploads of
AN INTRODUCTION TO BRAILLE MATHEMATICS
Using UEB and the Nemeth Code
Provisional Online Edition 2017

Updated: 5-9-2020

This document contains information about revisions to the uploaded PDF and BRF files of *An Introduction to Braille Mathematics Using UEB and the Nemeth Code, Provisional Online Edition 2017.* Individuals who downloaded these files prior to the current upload dates should note this information about what has been changed or should download the most recent copy and discard the old one. Upload dates are reflected in each filename as well as in the footer of the print document and the title page of the braille document.

Changes were made to the following files for this update:

- FRONT MATTER
- LESSONS 1, 2, 3, 4, 5, 10, 11, 12, 13, 15, 17, and 18

Please report errors by sending an email to: transcribers@nfb.org.

FRONT MATTER
February 17, 2017: First upload
April 25, 2017: First revision
Added the revision date (footer in PDF; on title page in BRF); changed prerequisite title to Braille Formats 2016 (Pre-Publication Release); edited/added text to the first paragraph under the heading ABOUT THE PROVISIONAL ONLINE EDITION
BRF: Added a braille page number to even-numbered pages.
July 24, 2017: Second revision
BRF: title page: Lines 17-18 changed to say "Transcribed 2017 into Unified English Braille and Nemeth Braille by Lindy Walton"; inserted numeric indicator on line 24; Special Symbols page: added symbols according to Braille Formats Appendix G.
February 1, 2018: *Third revision*
"ABOUT" page: "completed" changed to "completely".
"PREREQUISITES" page: Corrected title of "Guidance"; deleted "(Pre-Publication Release)" from title of *Braille Formats*; deleted citation of "*Application of the Formats Guidelines 2011 to Nemeth Transcriptions*"; corrected date for Nemeth Code Updates to 2007-2015.
May 15, 2018: Fourth revision
Foreword: Sentence added regarding Appendix A; name added to thank yous.
Complete Table of Contents merged with Front Matter file.
June 25, 2018: Fifth revision
Table of Contents:
14.12.2 moved to become new "14.21 Margins for Embedded Expressions".
14.21-14.26 renumbered as 14.22-14.27
14.23-14.25: Heading changed to "Square Root Division"

17.8-17.11 updated.
18.5.3 title changed to "Context-Dependent Formats"

May 22, 2019: Sixth revision

Foreword: "Grafton Braille Service Center" added to list of helpers.
Repaired Roman page numbers

May 9, 2020: Seventh revision

Title Page: Changed book title to "AN INTRODUCTION TO BRAILLE MATHEMATICS USING NEMETH CODE WITHIN UEB CONTEXTS" and added the subtitle "A Course for Transcribers"
Table of Contents: "4.1.1 Capitalization" added.
BRF: Added a running head.

ALL BRAILLE FILES

August 28, 2018

Changes made to reflect new guidelines:

Special Symbols page: Nemeth Code symbols included in the main symbols list, with (Nemeth Code symbol) added to the end of the description
Transcriber's Notes page: Changes made to the statement about the Nemeth Codebook and the Guidance document.

May 9, 2020

Title page: Book title changed to "AN INTRODUCTION TO BRAILLE MATHEMATICS USING NEMETH CODE WITHIN UEB CONTEXTS: A Course for Transcribers"
Braille page 1: Book title changed to "AN INTRODUCTION TO BRAILLE MATHEMATICS USING NEMETH CODE WITHIN UEB CONTEXTS A Course for Transcribers"

LESSON 1

February 17, 2017: First upload

April 25, 2017: First revision

1.10: Clarified statement about Special Symbols page.
See also "Shared Note" #1 on the last page of this document.

July 24, 2017: Second revision

PDF: Brief contents: "General Rules" changed to "General Principles"; 1.7: "(blank lines)" inserted to clarify the term "line spacing".
BRF: 1.7: "(blank lines)" inserted to clarify the term "line spacing".
See also "Shared Note" #2 on the last page of this document.

February 1, 2018: *Third revision*

1.3.1, 1.4.1, 1.5, 1.6: Language clarified.
Example 1.3-1: Line numbers added to print copy.
Instructions before Practice 1A: First paragraph: "he" changed to "the"; second paragraph revised.
Practice 1B: "0.37" corrected in answer section.
See also "Shared Note" #3 on the last page of this document.

May 15, 2018: Fourth revision

BRF: page t3, Deleted all three switch indicators on Special Symbols page.
BRF: Practice 1B: Added transcriber's note referring to notes at end of lesson.
See also "Shared Note" #4 on the last page of this document.

August 28, 2018: Fifth revision

1.10: "is" changed to "are" in the BANA decision box.

May 22, 2019: Sixth revision

1.10: BANA box and text: The text regarding the Special Symbols page has been deleted.

May 9, 2020: Seventh revision

1.3.1, last paragraph: "plural numbers" added to what is considered to be an unmodified number. New example 1.3-4

LESSON 2

February 17, 2017: First upload

April 25, 2017: First revision

Practice 2E: Braille corrected to use a 40-cell line.

See also "Shared Note" #1 on the last page of this document.

July 24, 2017: Second revision

PDF: 2.14: Deleted the word "All" in "All paragraphs".

BRF: Example 2.7-7: Dot 6 deleted from minus sign in commentary; 2.14: deleted the word "All" in "All paragraphs"; Answer section: lesson number deleted from combined page numbers.

See also "Shared Note" #2 on the last page of this document.

February 1, 2018: *Third revision*

2.1: Language clarified regarding the term "switch".

2.3: Two additional paragraphs and examples added (Examples 2.3-5 and 2.3-6).

Example 2.4-1: Citation added to comment.

Example 2.5-5: Changed "12s" to "twelves".

Examples in 2.5.3 (print copy only): Added line numbers.

Practice 2C: Second paragraph changed.

Practice 2D: Added a period at the end of the second paragraph.

Example 2.7-2: Clarification added to the comment.

Example 2.7-3: Comment added.

Example 2.7-5: "Dan" changed to "Danny".

Text preceding Example 2.10-2: "UEB dash" changed to "UEB underscore".

2.14: Word added to note.

See also "Shared Note" #3 on the last page of this document.

May 15, 2018: Fourth revision

BRF: page t3: Deleted single-word switch indicator on Special Symbols page.

2.1, note: "See Appendix A" changed to "See Lesson 18".

BRF: Practice 2E, F: Added transcriber's note referring to notes at end of lesson.

See also "Shared Note" #4 and #5 on the last page of this document.

August 28, 2018: Fifth revision

Example 2.7-5 (braille and simbraille): letter “p” changed to capitalized letter “P” in the word “Phew!”

May 22, 2019: Sixth revision

2.1: Sentence added to second paragraph: "UEB symbols are not used inside of the Nemeth Code switches."

July 8, 2019: Seventh revision

Answers to PRACTICE 2C (print copy only): The first word "How" indented to begin in cell 3.

May 9, 2020: Eighth revision

2.1: Added "and indicators" to second sentence after box.

LESSON 3

February 17, 2017: First upload

April 25, 2017: First revision

3.5.3 Clock Time: Entire section rewritten; Example 3.18-2: replaced; Example 3.19-1: shadow dots added to PDF copy.

See also "Shared Note" #1 on the last page of this document.

July 24, 2017: Second revision

3.14: "math term" changed to "math expression".

BRF: 3.14: Two displayed chevron items moved to cell 3 (from cell 5); answer section: lesson number deleted from combined page numbers.

See also "Shared Note" #2 on the last page of this document.

February 1, 2018: *Third revision*

Example 3.5-1: Sentence added to comment.

Example 3.5-3: Sentence added to comment.

Practice 3B: Second sentence of instructions changed.

Example 3.8-1: Comma inserted after "14.6" in the (simulated) braille.

Example 3.8-4: Text added; period added at end of sentence.

Sample braille before Example 3.8-5: corrected the second cell of the equals symbol.

Example 3.8-5: Text added.

3.10: Corrected narrative in second sentence: "immediate" changed to "immediately".

3.13: Narrative changed to explain recommended practice; further clarification added before the second isolated example.

Example 3.13-2: Comment deleted.

3.13.1: New subsection added, with new Example 13.3-3.

3.14: Removed "it makes more sense to" and changed ellipsis to period; text after Example 3.14-2 changed.

Example 3.18-2: Comment removed.

Example 3.20-1: Comment added.

See also "Shared Note" #3 on the last page of this document.

May 15, 2018: Fourth revision

BRF: page t3: Deleted single-word switch indicator on Special Symbols page.

See also "Shared Note" #4 and #5 on the last page of this document.

May 9, 2020: Fifth revision

3.4: "Inner quotation marks" deleted from the table. BANA box inserted regarding future rule about "single" quotation marks in Nemeth context.

Example 3-17-1: "122" changed to "144"

LESSON 4

February 17, 2017: First upload

April 25, 2017: First revision

Many changes made in the first three sections and the following sections are renumbered: paragraph after Practice 4B: "in a later lesson" changed to "in Lesson 5"; answers to Practice 4C corrected to use a 40-cell braille line; FORMAT SUMMARY #2, last item: removed "or letter".

See also "Shared Note" #1 on the last page of this document.

July 24, 2017: Second revision

BRF: A displayed chevron item moved to cell 3 (from cell 5); inserted a blank line before Example 4.6-4; inserted a blank line before section 4.6.4; removed the box lines around the full-page Practice 4E; FORMAT SUMMARY #2, last item: removed "or letter".

See also "Shared Note" #2 on the last page of this document.

February 1, 2018: *Third revision*

Example 4.2-1 (line 2): Contracted "sh" in "Shape".

Example 4.2-5 Removed; 4.2-6 renumbered.

4.3: First sentence clarified.

Example 4.3-2: Remark reworded.

Practice 4A:

--Sentence "B" (braille): period deleted from the end.

--Sentence "C" (braille): the word "named" moved to fall at the end the first line.
--Sentence "D" added.
4.4: Text edited before Example 4.4-3; isolated example removed.
4.4.2: Heading changed to "Punctuation with Abbreviations"; first sentence clarified.
4.4.2.a: Language clarified.
4.5: Text added.
Practice 4B, item 2 (braille): The grave accent in "Gruyère" corrected.
Practice 4C, braille edition: Nemeth Code terminator corrected in second sentence.
4.9: New Example 4.9-2 with accompanying text; old Example 4.9-2 renumbered as 4.9-3; exception moved.
4.11.3: First sentence changed to "In mapping notation, the symbol : is brailled as an unspaced colon."
See also "Shared Note" #3 on the last page of this document.

May 15, 2018: Fourth revision

BRF: Practice 4A, B, D, F: Added transcriber's note referring to notes at end of lesson.
Practice 4A, Item C: Opening switch indicator moved to the beginning of the next line.
4.6: "See Appendix A" changed to "See Lesson 18".
Example 4.16-1 changed.
Practice 4F (braille) single-word switch indicator inserted before word "does".
See also "Shared Note" #4 and #5 on the last page of this document.

May 22, 2019: Fifth revision

Practice 4A: Added a fifth item.
4.6, after the box: Statement added regarding nonuse of the single-word switch with symbols or indicators.
4.6.3: Sentence inserted "A hyphenated word is considered to be more than one word." New Example 4.6-7 added; the next example renumbered to 4.6-8.
Example 4.11-2: Letters changed to m and W
After Example 4.12-1: Text edited; a mapping illustration added.
Example 4.15-2 deleted.
Practice 4F, Item E: Punctuation indicator inserted before closing quote in rjc”

July 8, 2019: Sixth revision

4.6 and 4.6.3: *Changes implemented in fifth revision reversed*. New Example 4.6-2.

May 9, 2020: Seventh revision

4.1.1: New subsection and example added regarding capitalized words in technical context.
4.6: Sentence added: " UEB typeform may be applied to the word, as appropriate." Clarification "single-word switch" added to last sentence.
4.6.1: "two" deleted in narrative.
Example 4.6–6: Comma removed; "and" underlined. Comment inserted regarding use of typeform indicators with the single-word switch indicator.
4.6.4: Deleted "(The correct transcription is shown.)"
Example 4.6–8: added commentary

LESSON 5

April 25, 2017: First upload

July 24, 2017: First revision

PDF: Added new section 5.1.1; old 5.1.1 renumbered as 5.1.2.
BRF: Added new section 5.1.1; old 5.1.1 renumbered as 5.1.2; several displayed chevron items moved to cell 3 (from cell 5); Greek Alphabet Table: information added to the transcriber's note regarding blank entries, unused column headings deleted from the third page of the table; pagination readjustments for a couple of pages after the Greek Alphabet Table.
See also "Shared Note" #2 on the last page of this document.

February 1, 2018: *Second revision*

Practice 5A, braille edition: Item #1, the opening Nemeth Code indicator moved to begin the runover line.

Example 5.3-1, first line of braille: "Formulas" contracted and "to" moved up to that line.

5.3.1: Last sentence deleted.

5.3.2: Removed redundancy; changed "is" to "are"; last sentence deleted.

Example 5.3-2: Sentence rearranged to include a comma; comment reworded.

5.5: Clarification added regarding required switch to Nemeth Code; cross-reference added.

Example 5.5-1: Wording changed; comment added.

Example 5.5-2: Comment added.

Example 5.9-4, Braille edition: Transcriber's Note indicators removed.

Practice 5F:

--Third sentence: opening quotation mark moved; note added regarding placement of the opening quotation mark before the switch indicator.

--Fourth sentence: a second sentence has been added to the note.

--Last sentence: comment rewritten.

Example 5.15-6: P(3 and 7) changed to P(3 AND 7); explanation of logical operators added to the comment.

New example 5.15-15 and accompanying text.

Practice 5G (braille), 16th item removed; opening parenthesis inserted before "San Antonio"; runover margins corrected.

Example 5.16-3: P(A and B) changed to P(A AND B).

Practice 5H:

--Item C: comma inserted after "nth".

--Item E: not and or changed to NOT and OR.

Example 5.23-1: Clarification added to the comment.

5.24: Entirely revised, including new subsections 5.24.1 and 5.24.2.

Practice 5I: Deleted first two sentences in the instructions.

See also "Shared Note" #3 on the last page of this document.

May 15, 2018: Third revision

BRF: Practice 5F: Added transcriber's note referring to notes at end of lesson.

See also "Shared Note" #4 and #5 on the last page of this document.

June 25, 2018: Fourth revision

Practice 5C: Letter a changed to letter A in the second sentence.

August 28, 2018: Fifth revision

Example 5.9-5: (Nemeth Code) changed to (Nemeth Code symbol)

May 22, 2019: Sixth revision

5.2, second sentence: word "cues" changed to "clues"

5.10, second box: third letter name changed to "Fao"

5.12, second box: third letter name changed to "Shah"

Example 5.21-4, commentary “5.1.1” changed to “5.1.2”

May 9, 2020: Seventh revision

5.16, third paragraph, second sentence: deleted the word "be".

LESSON 6

April 25, 2017: First upload

July 24, 2017: First revision

6.7: Wording change (clarification).

See also "Shared Note" #2 on the last page of this document.

February 1, 2018: *Second revision*

6.2: New paragraph added to the end.

6.3: Language clarified (after the table).
6.3.1-6.3.6: Each symbol is restated before the examples.
6.4.1-6.4.12: Each symbol is restated before the examples.
6.5: Text clarified; print example inserted before commentary; another line added to sample; commentary adjusted accordingly. BRF: ellipsis corrected to Nemeth Code symbol.
Example 6.7-2: Deleted a space within the word “Relational” in the braille.
6.10: Added "Negated Perpendicular To" symbol; added to the second example.
Practice 6F: Instructions added (to treat the headings as cell-5 headings) and in the braille copy the headings were adjusted.
See also "Shared Note" #3 on the last page of this document.

May 15, 2018: Third revision

6.5: Symbols box added to show column separation line and guide dots.
6.5 (print): Commentary moved to follow the braille example.
BRF: Practice 6C: Added transcriber's note referring to notes at end of lesson.
Practice 6F: The first three item numbers 1. 2. 3. (braille) changed to (1) (2) (3) to match print.
See also "Shared Note" #4 and #5 on the last page of this document.

May 22, 2019: Fourth revision

Example 6.7-6, line 1: English letter indicator inserted before the first letter "x"
Example 6.11-1: Line 10 - Nemeth code terminator deleted; Line 13 - Single-word switch indicator inserted before “Multiply.”; opening Nemeth code indicator deleted following the word "Multiply."

LESSON 7

July 24, 2017: First upload

February 1, 2018: *First revision*

7.10: Changed fifth sentence slightly.
Example 7.11-3, note: The specific section number in the *Braille Formats* citation was removed.
Example 7.18-1: Nemeth Code terminator deleted from the end of item b; single-word switch indicator applied to the word "If" at the beginning of item c; punctuation indicators added before the two periods which now fall inside the switches.
Practice 7C: Nemeth Code terminator removed from the end of item 2 as Nemeth Code continues in item 3; punctuation indicators adjusted and identifier "3" changed to Nemeth Code.
See also "Shared Note" #3 on the last page of this document.

May 15, 2018: Second revision

Insignificant capitalization changes.
Practice 7A: Reorganized for clarity. One more row added to the beginning. Commas deleted.
Example 7.5-7: Math moved to begin on the second line and continue on the third.
7.6: Comment added.
BRF: Practice 7D: Added transcriber's note referring to notes at end of lesson.
See also "Shared Note" #4 and #5 on the last page of this document.

June 25, 2018: Third revision

Practice 7B (print): Period deleted from end of sentence ii.

May 22, 2019: Fourth revision

Text before Example 7.10-2: "Sha" changed to "Shah"
Example 7.18-1, subitem b (print copy only): period inserted at end of equation.
Example 7.18-7, item 3c (print copy only): "485" changed to "458".
Example 7.18-7 (BRF): spaces deleted before and after hyphen in Example label
Example 7.18-8, item 3c (print copy only): "485" changed to "458".
Practice 7C: Inserted a blank line before item #1, after the transcriber's note.
Practice 7F: "ar" in "n-ary" changed to be uncontracted (Nemeth context).

LESSON 8

July 24, 2017: First upload

February 1, 2018: *First revision*

Practice 8B, item 4 (print): Quotation mark deleted before the word "is".

8.6.1: Third mini-example removed.

Example 8.7-3: Comment added.

8.7.3: (print copy): Apostrophes changed to look like punctuation, not like prime signs.

8.10.4: Sentence moved from "is not used" paragraph to "must be used" paragraph; order of paragraphs switched.

Example 8.11-1 (braille): The word “and” added.

Example 8.11-2 (braille): Corrected spelling of the word “shown”.

8.11.2, second mini-example (braille): Corrected spelling of the word "obtuse".

8.11.3 (print): Code switch indicators deleted from the two isolated examples.

Example 8.16-1 (braille): Opening Nemeth Code indicator added following the word “anion”.

Practice 8D, item 6 (braille): Single-word switch indicator added to the word “or”.

Practice 8I (braille): Opening Nemeth Code indicators moved to follow the last word in each cell-5 heading; several extraneous punctuation marks and one extraneous code switch indicator deleted.

See also "Shared Note" #3 on the last page of this document.

May 15, 2018: Second revision

Example 8.3-7 Changed; comment removed.

See also "Shared Note" #4 and #5 on the last page of this document.

January 31, 2019: Third revision

Practice 8A, second paragraph (braille and simbraille): Dot 6 added to the sanserif indicator in the second and third instances.

Practice 8G, #4 (print): The two vector letters "v" changed to bold type.

Example 8.16-1 (braille and simbraille): Superscript corrected to 127.

May 22, 2019: Fourth revision

8.16, second sentence: cross-reference 8.9 changed to 8.10.

Example 8.24-1: Period inserted before closing TN symbol; space removed.

Example 8.25-4 (print copy only): Word "to" changed to "do"

Practice 8J, item 1, last line: Dot 2 in front of the numeral 6 changed to dot 5 (baseline indicator).

LESSON 9

February 1, 2018: *First upload*

May 15, 2018: First revision

Practice 9A: Note added to "Instructions" regarding end-of-sentence punctuation.

BRF: Practice 9B: Added transcriber's note referring to notes at end of lesson.

9.7.1: Description of last mini-example clarified.

See also "Shared Note" #4 and #5 on the last page of this document.

May 22, 2019: Second revision

Practice 9C: Directive inserted: "Treat "FRACTION REVIEW" as a centered heading."

Example 9.9-3 (print copy only): Period added to end the sentence.

Practice 9I: Directive inserted: "Treat "MULTIPLYING MIXED NUMBERS" as a centered heading."

Practice 9B, item B: Extra blank cell deleted after the word “Energy”.

Practice 9C: "FRACTION REVIEW" changed to a centered heading, followed by a blank line.

LESSON 10

February 1, 2018: *First upload*

May 15, 2018: First revision

BRF: Practices 10A, C: Added transcriber's note referring to notes at end of lesson.

10.2: Second sentence deleted.
BRF: 10.5: Opening switch indicator corrected. Examples changed to two columns to match print copy. Pagination changes throughout.
10.15: Second sentence repaired.
Third mini-example before 10.29: Stroke direction changed to be upper left to lower right.
BRF: Transcriber's note added to explain the three print cancellation strokes.
Example 10.29-3: Misplaced carried numbers corrected in the print example.
10.34: Question box inserted regarding BANA ruling about opening switch indicator.
Example 10.37-1: Punctuation indicator inserted before period in identifier 2.
See also "Shared Note" #4 and #5 on the last page of this document.

May 22, 2019: Second revision

Example 10.9-1: ELI inserted before letter "f"; space deleted following colon. Commentary corrected.
Examples 10.26-5 and 10.26-6: Inserted a plus sign on second line of spatial addition problem.

May 9, 2020: Third revision

10.34: Inserted "itemized" in the first sentence to read"…. an itemized spatial problem ..."
Example 10.27-2: "vertically" added to end of commentary.

LESSON 11

May 15, 2018: First upload

August 28, 2018: First revision

11.23.1: "as a square shape" changed to "as square shapes".
Practice 11G, Instructions: Wording of standard statement changed to "Mathematical content is transcribed according to *The Nemeth Braille Code for Mathematics and Science Notation, 1972 Revision,* 2007-2015 Updates including the *Guidance for Transcription Using the Nemeth Code within UEB Contexts."*
Practice 11G, Braille: same as above.

January 31, 2019: Second revision

Practice 11A (print): Several corrupted images corrected.
Practice 11E (print): Corrupted image of pencil icons corrected.
Practice 11E (braille and simbraille): Opening Nemeth Code indicator inserted before first keystroke indicator; Nemeth Code terminator inserted at the end of the third line of keystrokes; Second line of keystrokes: symbol for multiplication dot corrected.

May 9, 2020: Third revision

11.30: "general omission indicator " changed to " general omission symbol".

LESSON 12

May 15, 2018: First upload

August 28, 2018: First revision

Practice 12I, Instructions: Wording of standard statement changed to "Mathematical content is transcribed according to *The Nemeth Braille Code for Mathematics and Science Notation, 1972 Revision,* 2007-2015 Updates including the *Guidance for Transcription Using the Nemeth Code within UEB Contexts."*
Practice 12I, Braille: same as above.

May 9, 2020: Second revision

12.17.3: "Step-Number Format" narrative changed to reflect the possibility of print showing no numbers or column headings.
Example 12.17-1: Last item in commentary changed to fit on one print line.
Pagination issues corrected; some images improved.

LESSON 13

May 15, 2018: First upload

August 28, 2018: First revision

13.1.2 Deleted the sentence "List Nemeth Code symbols under a separate cell-5 heading." Sample Special Symbols page changed to comply with new guidelines.

May 22, 2019: Second revision

13.7, sixth sentence: “identity” changed to “identify”.

13.7.1: Added: the colon as a mathematical symbol

13.7.2: Added: two forms of the "colon"

May 9, 2020: Third revision

Practice 13G, braille answer: Margins of the six spatial division problems changed to be displayed mathematical material, starting in cell 3, and showing three blank cells between symbols of neighboring arrangements.

LESSON 14

June 25, 2018: First upload

May 22, 2019: First revision

Example 14.13-5, line 1: Corrected the word “To” (should not be contracted).

LESSON 15

June 25, 2018: First upload

May 22, 2019: First revision

Challenge Problem (after Practice 15C): Inserted a second paragraph to describe the format. In the braille, inserted a UEB ellipsis before the opening Nemeth Code indicator; indented the entire arrangement two cells to the right, using proper displayed margins.

Example 15.10-1, Practice 15C, and Practice 15D: Inserted a UEB ellipsis before the opening Nemeth Code indicator; indented the entire arrangement two cells to the right, using proper displayed margins.

May 9, 2020: Second revision

Example 15.11-3, line 8: Opening Nemeth Code indicator moved to the runover cell of the preceding text (cell 13). Added explanation of this placement to the commentary.

LESSON 16

July 17, 2018: First upload

May 22, 2019: First revision

Example 16.5-1 line 4: Baseline indicator inserted before right enlarged brace.

Challenge Problem #1: Text and provisional transcription changed.

16.16.4: Wording changed in this paragraph.

LESSON 17

July 17, 2018: First upload

May 22, 2019: First revision

Example 17.9-2 (print copy only): “Triangle 3” moved left one cell to begin in cell 7.

17.12.3: Several improvements made to the text. An additional example given.

Example 17.12-1, first column heading: Grade one indicator removed from letter "A" in "Town A".

Example 17.12-2, bulleted list: Margins adjusted to 1-3.

May 9, 2020: Second revision

Example 17.6-4: Nemeth Code terminator moved to a new line; bottom box line changed to a full line of dots 1245. Commentary adjusted.

Example 17.6-5: Opening Nemeth Code indicator moved to the line preceding the box; top box line changed to a full line of dots 2356. Commentary adjusted.

Practice 17C: Nemeth Code terminator moved to a new line; bottom box line changed to a full line of dots 1245. Commentary adjusted.

LESSON 18
July 17, 2018: *First upload*
August 28, 2018: First revision
18.4.1 Special Symbols Page: Revised to comply with newest guidelines.
18.4.2 Transcriber's Notes Page: Revised to comply with newest guidelines.
Practice 18A: Special symbols page and transcriber's notes page revised to comply with newest guidelines.
May 9, 2020: Second revision
18.4.1, second paragraph: "switching" changed to "switch"

APPENDICES

APPENDIX A
May 15, 2018: First upload (partial)
June 25, 2018: Second upload (complete) –
Corrections made to readings 3, 5, 8, and 11 (print) and readings 8, 9, 11, 12, and 13 (braille).

APPENDIX B
May 15, 2018: First upload

APPENDIX C
July 24, 2017: First upload (through Lesson 8)
February 1, 2018: *Second upload (through Lesson 10)*
May 15, 2018: Final upload (all lessons)
May 22, 2019 revision
Switch Indicators ... with Spatial Arrangements: Fifth sentence changed to say "If there is not room on the line with the preceding text, the opening Nemeth Code indicator is placed in cell 1 on a line by itself." Sixth sentence deleted.
July 8, 2019 revision
Switch Indicators ... with Spatial Arrangements: Fifth sentence changed again to make a distinction between itemized and unitemized formats.

SHARED NOTES

Note #1 (Lessons 1-4) *April 25, 2017*

Added the revision date (footer in PDF; on title page in BRF).

BRF: Added a braille page number to even-numbered pages; added a running head; deleted "brief contents".

Note #2 (Lessons 1-6) *July 24, 2017*

BRF: title page: Lines 17-18 changed to say "Transcribed 2017 into Unified English Braille and Nemeth Braille by Lindy Walton"; inserted numeric indicator on line 24; Special Symbols page: added symbols according to Braille Formats Appendix G; Transcriber's Notes page: inserted capitals indicator for letter S in TRANSCRIBER'S; added a paragraph about the dash being omitted from page numbers; page 1: inserted full book title on lines 1-4; page 1: added statement to recheck the NFB website periodically for updated file.

PDF: page 1: Added statement to recheck the NFB website periodically for updated file; brief contents: moved FORMAT list to be a right column (if applicable).

Note #3 (Lessons 1-8) *February 1, 2018*

Braille examples changed to a 3-1 paragraph style in order to avoid misunderstanding. Sometimes this required slight changes in wording.

PDF: "Answers to Practice Material" section forced to start on a page front.

Note #4 (Lessons 1-10) *May 15, 2018*

Throughout: The name of the BANA Mathematics Committee changed to "Nemeth Code Technical Committee".

Throughout: The name of the BANA "Guidance" document changed to "Guidance for Transcription Using the Nemeth Code within UEB Contexts".

Note #5 (Lessons 2-10) *May 15, 2018*

BRF: page t4 (Transcriber's Notes page): Added paragraph about use of the Nemeth Code; changed the last paragraph to have a paragraph heading.

AN INTRODUCTION TO BRAILLE MATHEMATICS USING NEMETH CODE WITHIN UEB CONTEXTS

A Course for Transcribers

Provisional Online Edition 2017

Revised by Lindy B. Walton
Barbara Taffet, advisor

www.loc.gov/nls

National Library Service for the Blind and Physically Handicapped

The Library of Congress



I believe that I could not have reached my potential in mathematics without the Nemeth Code. With it, I am able to read and write mathematics, as well as other sciences, at all levels, limited only by my talent and my ambition.

—Dr. Abraham Nemeth, creator of the braille code for mathematics and science notation

DEDICATION

I credit my interest in the continued training of braille transcribers in the Nemeth Code to my friend and mentor, Helen Hay, whose fascination and enthusiasm about this braille code was contagious. —Lindy Walton

THANKS

I offer my gratitude to the original authors of this lesson manual, Helen Roberts, Bernard M. Krebs, and Barbara Taffet, for their insight into the learning process and for their eye for detail. Many of the excellent examples from the original book are preserved in this edition. I also wish to thank my supervisors and colleagues in the Madison Metropolitan School District for realizing the importance of the development of this curriculum. —Lindy Walton

ABOUT THE PROVISIONAL ONLINE EDITION

Due to the need to make this training manual available to transcribers as soon as possible, we are offering a provisional version of the lessons in downloadable electronic format before being completely field tested and while BANA is revising the Nemeth Code. Exercises at the end of each lesson will not be available until a path toward certification is established. You may proceed through the course as each lesson is released with the understanding that certain details are still under construction and that some rules may change. A certification exam will not be available until the revised Nemeth Code is released. To apply for the exam, you must have turned in passing transcriptions of each of the yet-to-be-released lesson exercises.

We encourage you to contact us to report errors or to comment on topics that are unclear. As a result of user feedback, you can expect changes to appear in the online edition. All changes will be documented. A running list will be maintained and will be posted on www.nfb.org/transcribers.

Once a final version is approved, this lesson book will also be available in printed form.

To contact us by e-mail, send your message to transcribers@nfb.org.

FOREWORD TO THE 2017 EDITION

The first edition of the *Introduction to Braille Mathematics* was published in 1978 and was written by the late Helen Roberts and Bernard M. Krebs. It was my privilege to complete the text with Mr. Krebs after Helen passed away. Since that time, numerous corrections and updates have been made both to the Nemeth Code itself and to this manual. Now, however, a major change has necessitated a complete re-writing of the lessons. 2016 was the implementation year in the United States for new transcriptions to be produced using the Unified English Braille Code. Because Nemeth Code works *within* UEB, many of the rules of Nemeth Code must be modified.

After the first lesson most examples, practices, and exercises are shown in a text-like context. In this way, the student can see how the Nemeth Code works in a real setting such as found in texts of many grade levels and complexities.

The practices within each lesson are available for self-checking by the student. Answers to the practices are given at the end of each lesson. Braille reading practice will soon be offered in Appendix A. In the future, each lesson will conclude with an exercise which will be graded and evaluated by your teacher or by your NFB-assigned grader.

The braille examples are written on a 38-cell line in the lesson manual to allow for a reasonable margin on the given page size. However, the student should use the standard 40-cell line when transcribing the practices and exercises.

The student should understand that the Nemeth Code itself is the authoritative source for all mathematics transcriptions. The student should also be thoroughly familiar with the sourcebooks listed in the PREREQUISITES which follow this Foreword.

It has long been my hope that this manual could be brought into the present era. Lindy Walton, an experienced transcriber who works with the NLS Nemeth certification program, led the writing of this Second Edition. Once again, it is my honor to work with an exceptional member of the braille transcriber community.

Both Lindy and I thank the following for their support and help: Mary Denault, Peggy Jackson, Bill Jackson, Kyle DeJute, Julie Sumwalt, Lynnette Taylor, the members of the BANA Nemeth Code Technical Committee, and the Grafton Braille Service Center. We would also like to thank the National Federation of the Blind which has lent support to the development and publication of this comprehensive manual.

Barbara Taffet

PREREQUISITES

A prerequisite to the study of the Nemeth Code within UEB context is certification in Unified English Braille, adequate experience in literary braille transcription, and confidence in your production method. Before beginning this course of study the student should also be thoroughly familiar with current methods for transcribing a textbook. Rules and guidelines are found in the following sourcebooks, all of which are available from the Braille Authority of North America (BANA) at www.brailleauthority.org. Dates shown below are the editions used as a resource in this lesson manual.

The Rules of Unified English Braille, Second Edition 2013
Guidance for Transcription Using the Nemeth Code within UEB Contexts, 2016
Braille Formats: Principles of Print-to-Braille Transcription, 2016
The Nemeth Braille Code for Mathematics and Science Notation, 1972 Revision, 2007-2015 Updates
Guidelines and Standards for Tactile Graphics, 2010
Braille Code for Chemical Notation 1997

STUDY TIPS

HOW TO BECOME AN EXCELLENT NEMETH BRAILLE TRANSCRIBER

Don't race through the lesson material.

- Read carefully and deliberately as the narrative is compact and the language is exact.
- Study the examples and understand the point being made with each one but do not rely on the examples alone for an understanding of the rules. Braille the examples to reinforce the rule.
- Do the practice drills. Proofread them before checking the answers. See more tips below.
- Try back-translating the braille examples and practices without looking at the print.
- Take special note of rules regarding spacing, punctuation, abbreviations, and format.
- Make lists to help you remember differences between Nemeth and UEB rules.
- Don't be afraid to underline, highlight, or write notes in the margins of your lesson manual.

If the braille or the print doesn't make sense to you ...

- Compare new information to similar topics learned in previous lessons.
- Some of the lesson material is grouped in "use of" and "nonuse of." Compare them and look closely at the braille examples.

THE PRACTICE MATERIAL

- Slow down. By using 6-key entry instead of a translator you will better understand the braille from the reader's point of view.
- Compare your braille to the answers to the practice material found at the end of each lesson. Read each cell closely.
- At the end of each line, look at the braille cell in the line above and in the line below and compare it to the answer key. Any misalignment indicates an error on that line.
- When you identify your errors, return to the lesson to review the applicable rule.

PREPARING THE EXERCISE FOR GRADING

- Don't try to copy braille examples that look like the exercise material – understand and apply the rule.
- Don't guess. Don't rely on the proofreader's report to find your mistakes.
- Proofread carefully before turning in for grading. Your knowledge and understanding of the Nemeth Code will improve dramatically if you proofread from an embossed copy or from a simulated braille (print) copy, without looking at the print.
- Make note of items you are unsure of. If your transcription is correct, look these items over again after receiving your report to reinforce the rule.

RESEARCH/REVIEW

- Analyze the mistakes found in your exercise and make sure you understand your errors before moving ahead to the next lesson. Ask questions until you are sure of the rule.
- Return to earlier lessons. Topics will make more sense to you in retrospect.
- Read the index. Terminology used there will help you understand the language of Nemeth braille.
- Review format rules learned in earlier lessons. Study the examples.
- Go back to an earlier lesson exercise and back-translate the practices or your braille exercise by writing in longhand. Don't look at the print copy until you are finished. Giving yourself some distance from the lesson material is a good review strategy.
- In later lessons, research the topic in the Nemeth Code in addition to studying the lesson book. Not only will this enrich your understanding of the current subject, you will also review material already learned in a new context.

PROOFREADING TIPS

Accuracy is crucially important in technical work. Your proofreading skills will be challenged.

- Is your lighting adequate?
- Use a magnifier when print is questionable.
- Use a straightedge when levels are in question.
- Take breaks when your concentration wanes. Then go back a few pages when resuming proofreading.
- Read the braille dots. Compare often to the print copy.
- Vary your reading medium -- don't always proofread from the screen or from simulated braille or from embossed braille.

BRAILLE TRANSLATION SOFTWARE

Many students of the Nemeth Code have been brailling for years and have thousands of pages of braille to their credit. They also have been taking advantage of the many electronic input and proofreading aids available to transcribers and are quite adept at turning out high quality work. We expect you are one of those transcribers.

You are undertaking a serious study of one of the technical braille codes, and we would like you to consider stepping back a bit and learning the old fashioned way, using 6-key entry in your braille software program. It is our experience that the best braille transcribers are those that can read and write braille as the 6-dot code that it is, not solely reading a back translation or a source file and not using another input code to 'type' math problems. Using proofreading and production aids for more accurate and faster work is certainly something you will continue to use – it is important that you understand how your particular software and translation tools work in Nemeth mode – but we are convinced you will understand the Code better if you take the 6-key approach while learning.

Best of luck to you!

CONTENTS

About the Provisional Online Edition
Foreword to the 2017 Edition
Prerequisites
Study Tips

Lesson 1

1.1 Philosophy
1.2 Non-technical and Technical Texts
1.2.1 Non-technical Texts
1.2.2 Technical Texts

INTRODUCTION TO NUMERALS AND THE NUMERIC INDICATOR

1.3 Representation of Arabic Numerals
1.3.1 English Braille Numerals
1.3.2 Nemeth Code Digits
1.4 Numeric Indicator
1.4.1 *Special Case:* Segmented Numbers

THE PRACTICE MATERIAL *Practice 1A*

THE MATHEMATICAL COMMA AND DECIMAL POINT

1.5 Mathematical Comma
1.6 Mathematical Decimal Point
1.6.1 Spacing of the Decimal Point
1.6.2 The Decimal Point and the Numeric Indicator
1.7 *Format:* General Principles *Practice 1B*

INTRODUCTION TO SIGNS OF OPERATION

1.8 Signs of Operation
1.8.1 Spacing with Signs of Operation
1.8.2 Positive and Negative Numbers *Practice 1C*

INTRODUCTION TO SIGNS OF COMPARISON

1.9 Signs of Comparison
1.9.1 Spacing with Signs of Comparison *Practice 1D*

MONETARY, PERCENT, AND PRIME SIGNS

1.10 Monetary Signs
1.10.1 Spacing with Monetary Signs
1.11 Percent and Per Mille Signs
1.11.1 Spacing with Percent and Per Mille Signs
1.12 Prime Sign *Practice 1E*

CONTINENTAL SYMBOLS

1.13 The Continental Comma
1.14 The Continental Decimal Point

Answers to Practice Material

Lesson 2

INTRODUCTION TO CODE SWITCHING

2.1 A Complete Transcription
2.2 Use of the Switch Indicators *Practice 2A*
2.3 Which Code?
2.4 Placement of Literary Punctuation *Practice 2B*
2.5 *Format: Keep Together*—General Principle Regarding Mathematical Expressions

Placement of the Switch Indicators

2.5.1 Keep Together If Possible
2.5.2 A Switch Indicator May Stand Alone on a Line if Unavoidable
2.5.3 Switch Indicators at Page Turns *Practice 2C*
2.6 Consistency with Mathematical Symbols *Practice 2D*

THE HYPHEN AND THE DASH

2.7 The Hyphen and the Dash As Punctuation
2.7.1 A Hyphen May Divide a Long Numeral
2.7.2 A Hyphen May Connect Numerals
2.7.3 A Dash Separates
2.7.4 Hyphen, Dash, or Minus Sign? *Practice 2E*

SIGNS OF OMISSION

2.8 General Use of Signs of Omission
2.9 Ellipsis
2.10 Long Dash
2.11 General Omission Symbol
2.12 Other Omission Symbols
2.13 Spacing of the Ellipsis and Long Dash
2.14 : Paragraph Margins for Narrative Portions of Text (3-1) *Practice 2F*

INTRODUCTION TO IDENTIFIERS

2.15 Terminology
2.16 *Format:* Margins for Itemized Material with No Subdivisions (1-3) *Practice 2G*

FORMAT SUMMARY #1

Answers to Practice Material

Lesson 3

MORE ABOUT PUNCTUATION

3.1 Punctuation Mode
3.2 Spacing of UEB Punctuation and Code Switch Indicators
3.2.1 Spacing of the UEB Dash
3.3 Nemeth Braille Punctuation *Practice 3A*

PUNCTUATION IN NEMETH CODE

3.4 Background

The Punctuation Indicator

3.5 Role of the Punctuation Indicator

3.5.1 Comparison Sign in Quotes

3.5.2 Number in Quotes

3.5.3 Clock Time

3.6 Punctuation with Omission Signs

3.6.1 SPACING EXCEPTION —The Hyphen

3.7 Punctuation and Spacing of Plural or Possessive Endings *Practice 3B*

3.8 SUMMARY—Situations That Do Not Require a Puncuation Indicator

INTRODUCTION TO SIGNS OF GROUPING

3.9 Definition

3.10 Signs of Grouping with Numerals

3.11 Punctuation with Grouping Symbols

3.12 Nested Grouping Symbols

Code-Switching Considerations

3.13 "Enclosed" Technical Material

3.13.1 Punctuation Following a Sign of Grouping

3.14 The Bigger Picture

Spacing with Signs of Grouping

3.15 Spacing Inside of the Grouping Signs

3.16 Spacing Outside of the Grouping Signs *Practice 3C*

IDENTIFIERS, cont.

3.17 Code Switching and Identifiers—Maintaining Alignment

3.18 Context Decisions and Code Switching *Practice 3D*

3.19 *Format: Keep Together*—Hyphenated Expressions

3.20 *Format:* Margins for Itemized Material with No Subdivisions—Side-by-Side Layout

Answers to Practice Material

Lesson 4

WORDS

4.1 Words in Mathematical Context

4.1.1 Capitalization

4.2 Words in Narrative

4.3 Punctuation With Words *Practice 4A*

Introduction to Abbreviations

4.4 Abbreviations

4.4.1 *Format:* Keep Together

4.4.2 Punctuation with Abbreviations
4.4.2.a Abbreviations with a Related Period
4.4.3 Spacing with Abbreviations
4.5 Numbers with Ordinal Endings *Practice 4B*
Single-Word Switch Indicator
4.6 The Single-Word Switch Indicator
4.6.1 Consider the Context
4.6.2 Lower Wordsigns
4.6.3 Two or More Words
4.6.4 Switch Considerations
Switch Indicators at Page Turns, cont.
4.7 Review of "Keep Together" Considerations *Practice 4C*
4.8 New Print Page
LETTERS
4.9 Single English Letters in Narrative
4.10 Single English Letters in Nemeth Code
4.10.1 Nemeth Code Definition of "Single Letter"
Introduction to the English Letter Indicator
4.11 Use of the English Letter Indicator with a "Single Letter"
4.11.1 Capitalization of "Single Letters"
4.11.2 Punctuation of "Single Letters"
4.11.3 Mapping Notation *Practice 4D*
4.12 Nonuse of the English Letter Indicator with a "Single Letter"
4.12.1 Comparison Sign
4.12.2 Enclosed Within Grouping Symbols
4.12.3 Unspaced Sequence of Terms
4.12.3.a Probability Notation
4.13 Letters as Identifiers *Practice 4E*
Mathematical Letter Combinations
4.14 Mathematical Letter Sequence
4.15 Capitalized Letter Sequence
4.16 Shortform Letter Combinations
4.16.1 Use of the English Letter Indicator with a "Shortform Letter Combination"
4.16.2 Nonuse of the English Letter Indicator with a "Shortform Letter Combination" *Practice 4F*
FORMAT SUMMARY #2
Answers to Practice Material

Lesson 5

MORE ABOUT LETTERS

Variables

5.1 Mathematical Variables

5.1.1 *Format:* Keep Together

5.1.2 Abbreviation or Variable? *Practice 5A*

Roman Numerals

5.2 Letter or Numeral?

5.3 Transcribing Roman Numerals in Mathematical Context

5.3.1 Roman Numerals Consisting of Lowercase Letters

5.3.2 Roman Numerals Consisting of One Capital Letter

5.3.3 Roman Numerals Consisting of Two or More Capital Letters

5.3.4 Punctuation with Roman Numerals

5.3.5 Nonuse of the English Letter Indicator with Roman Numerals

5.3.6 Possessive Endings

5.3.7 Large Roman Numerals

5.4 Roman Numerals Used as Identifiers

5.5 Mathematical Letter Combinations Similar to Roman Numerals *Practice 5B*

Non-Decimal Bases

5.6 Letters Used to Represent Numerals in Non-Decimal Bases

5.6.1 Transcriber's Note Required

5.7 Non-Alphabetic Symbols Used to Represent Numerals *Practice 5C*

OTHER ALPHABETS

5.8 Alphabetic Indicators

5.8.1 Code Switching and Use of Letter Indicators

5.8.2 Capitalization and Punctuation

5.9 Greek Alphabet

Greek Alphabet Table

5.9.1 Code Switching with Greek Letters

5.9.2 Alternate Form of Greek Letters *Practice 5D*

5.10 German Alphabet

5.11 Hebrew Alphabet

5.12 Russian Alphabet *Practice 5E*

5.13 A Sequence of Unspaced Letters

5.13.1 Derivatives

5.14 Mathematical Constant *Practice 5F*

ENCLOSED LISTS

5.15 *Special Case:* Definition of "Enclosed List"

5.15.1 Nonuse of the Numeric Indicator in an " Enclosed List"

5.15.2 Nonuse of the English Letter Indicator in an " Enclosed List"

5.15.3 *Format:* Keep Together

5.15.3.a Division Between Lines *Practice 5G*

MORE ABOUT ENGLISH LETTERS

5.16 English Letters and Grouping Symbols

5.17 English Letters and Endings *Practice 5H*

MORE ABOUT ABBREVIATIONS

5.18 More Spacing Rules

5.18.1 Spacing of Abbreviations With Operation Signs

5.18.2 Spacing of Omission Symbols

5.19 Single-Letter Abbreviations

5.20 Abbreviations Whose Letters Correspond to a Shortform

5.21 Context Clues

5.22 Fully Capitalized Acronyms

5.23 UEB vs. Nemeth Code

CODE SWITCHING, cont.

5.24 Initiating Nemeth Code Before Itemized Material, Following a Heading

5.24.1 Centered Heading

5.24.2 Cell-5 and Cell-7 Heading *Practice 5I*

Answers to Practice Material

Lesson 6

SIGNS OF OPERATION, cont.

6.1 Review of Signs of Operation

6.2 Signs of Operation Using Plus and Minus

6.3 Signs of Operation That Look Like Literary Symbols

6.3.1–6.3.6 Ampersand; Asterisk; Crosshatch; Dagger and Double Dagger; Paragraph Mark; Section Mark *Practice 6A*

6.4 Signs of Operation Unique to Mathematics

6.4.1–6.4.12 Slash; Back Slash; Dot; Hollow Dot; Intersection; Logical Product; Logical Sum; Minus With Dot Over; Tilde; Union; Vertical Bar; Negated Vertical Bar

Format: Simple Tables

6.5 Introduction to Table Format *Practice 6B*

SIGNS OF COMPARISON, cont.

6.6 Review of Signs of Comparison

6.7 More Signs of Comparison

6.7.1–6.7.16 Greater Than With Curved Sides; Less Than With Curved Sides; Arc, Concave Upward; Arc, Concave Downward; Equivalence; Identity (Triple Bar); Inclusion; Membership; Parallel To; Perpendicular To;

Relation; Reverse Inclusion; Reverse Membership;
Tilde as a Sign of Comparison; Variation; Vertical Bar
6.7.16.a Colon Meaning "Such That" *Practice 6C*
6.8 Signs of Comparison Compounded Vertically
6.8.1–6.8.9 Greater Than Or Equal To; Inclusion; Intersection;
Less Than Or Equal To; Logical Product; Logical Sum;
Reverse Inclusion; Tilde; Union *Practice 6D*
6.9 Signs of Comparison Compounded Horizontally
6.9.1–6.9.2 Greater Than; Less Than
6.10 Negated Signs of Comparison *Practice 6E*
Format: INSTRUCTIONS
6.11 Margins for Instructions Preceding Itemized Material (5-3)
6.11.1 Code Switching and Instructions
6.12 Not "Instructions"
6.12.1 Margins for Instructions Preceding Unitemized Material (3-1)
6.12.2 Itemized Instructions
6.12.3 Margins for Narrative Preceding Itemized Material (3-1) *Practice 6F*

Answers to Practice Material

Lesson 7

TYPEFORMS
7.1 Introduction to Typeforms
LETTERS AND NUMERALS
7.2 Determining Significance of a Variant Typeform
THE FOUR MATHEMATICAL TYPEFORM INDICATORS
7.3 Mathematical Typeforms and Their Indicators
7.4 Boldface, Italic, Script, and Sanserif Type for Letters and Numerals
7.5 Typeform Indicators with One Letter
7.5.1 Recognition of Script Type in Other Alphabets *Practice 7A*
7.5.2 Script Letter of Special Interest
7.5.3 Boldface Letters of Special Interest—Vectors
7.5.4 Boldface Letters of Special Interest—German Letters
7.6 Typeform Indicators with One Numeral
7.6.1 Boldface Zero
7.7 Nonregular Typeform next to Signs of Grouping
7.7.1 Nonregular Typeform in an Enclosed List
7.7.2 Nonregular Typeform in Contact with a Grouping Symbol *Practice 7B*
Typeform Indicators with More Than One Letter or Numeral
7.8 More Than One Letter
7.9 More Than One Numeral

Other Details
7.10 Underlining and Other Typeforms
7.11 Termination of a UEB Typeform Passage
7.11.1 Capitalization
7.12 Context Clues *Practice 7C*
BOLDFACED MATHEMATICAL SYMBOLS
7.13 Dots 456
7.13.1 Signs of Operation in Boldface Type
7.13.2 Equals Sign in Boldface Type
7.13.3 Grouping Signs in Boldface Type *Practice 7D*
COMPOUND EXPRESSIONS
7.14 Definition of "Compound Expression"
7.14.1 Extent of Capitalization
7.14.2 Hyphen Followed by a Number
7.15 Typeform Indicators in Compound Expressions *Practice 7E*
Typeform Summary
UNSPACED NUMBER/LETTER COMBINATIONS
7.16 Number/Letter Combinations Without Hyphens
REVISITING THE NUMERIC INDICATOR
7.17 Summary of the Numeric Indicator *Practice 7F*
7.18 *Format:* Margins for Itemized Material with Subdivisions (1-5; 3-5)
7.18.1 Paragraphs Within Itemized Material with Subdivisions (7-5)
7.18.2 Side-by-Side Option *Practice 7G*
7.18.3 Tabular Form
7.18.3.a When to Retain Column Format
7.18.3.b When Not to Retain Column Format
7.19 Margins for Exercise Sets

Answers to Practice Material

Lesson 8

LEVEL INDICATORS
8.1 Definition
Superscripts
8.2 Superscript Level Indicator
8.2.1 Abbreviations with Superscripts *Practice 8A*
8.3 Returning to the Baseline Level
8.4 Raised Hollow Dot *Practice 8B*
Introduction to the Baseline Indicator
8.5 Function of the Baseline Indicator
8.5.1 Abbreviations with Superscripts *Practice 8C*
8.6 Higher Levels of Writing

8.6.1 Combinations
8.7 Certain Raised Signs
8.7.1 Raised Ordinal
8.7.2 Prime Sign
8.7.3 Apostrophe-s *Practice 8D*

Subscripts

8.8 Subscript Level Indicators *Practice 8E*
8.9 Returning to the Baseline Level *Practice 8F*
8.10 *Special Case:* Nonuse of the Subscript Level Indicator
8.10.1 Further Conditions
8.10.2 Restrictions
8.10.3 Rewind
8.10.4 Summary *Practice 8G*
8.11 Spaces Within Superscripts and Subscripts
8.11.1 Comma
8.11.2 Words
8.11.3 Comparison Signs
8.11.4 Ellipsis or Long Dash in Superscripts and Subscripts
8.11.5 Ellipsis or Long Dash on the Baseline of Writing
8.11.6 Segmented Numbers *Practice 8H*

More about Superscripts and Subscripts

8.12 Superscript and Subscript Combinations *Practice 8I*
8.13 Left Subscripts and Superscripts
8.13.1 Raised Negative Sign
8.13.2 More About the Temperature Abbreviations F (Fahrenheit) and C (Centigrade)
8.14 Further Combinations
8.15 Consecutive Superscripts and Consecutive Subscripts
8.16 Simultaneous Superscripts and Subscripts
8.17 Nonsimultaneous Superscripts and Subscripts
8.18 Detached Superscripts and Subscripts
8.19 Literary Symbols and Level Indicators

Summary *Practice 8J*

GROUPING SIGNS, cont.

8.20 Review of Rules
8.21 Grouping Symbols and Level Indicators
8.22 Grouping Symbols with Super/Subscripts
8.23 More Signs of Grouping
8.24 Transcriber-Devised Grouping Symbols *Practice 8K*

INTRODUCTION TO DISPLAYED MATHEMATICAL EXPRESSIONS

8.25 Displayed Mathematical Expressions
8.25.1 Placement of Code Switch Indicators
8.25.2 Look for Context Clues

8.25.3 Instructions with Displayed Material
8.26 Displayed Narrative Material *Practice 8L*

Answers to Practice Material

Lesson 9

INTRODUCTION TO FRACTIONS
9.1 Recognition and Layout
Simple Fractions
9.2 Definition of Simple Fraction
9.3 Simple Fraction Indicators
and the Horizontal Simple Fraction Line *Practice 9A*
9.4 The Diagonal Simple Fraction Line
9.4.1 Use of simple fraction indicators with the diagonal simple fraction line
9.4.2 Nonuse of simple fraction indicators with the diagonal simple fraction line
9.4.3 Reminder About the UEB Slash
9.5 Additional Considerations
9.5.1 Fractions and Abbreviations
9.5.2 Code Switch Reminders *Practice 9B*
Mixed Numbers
9.6 Definition of Mixed Number
9.6.1 Use of Mixed Number Fraction Indicators
9.6.2 Mixed Numbers and Omissions
9.6.3 Nonuse of Mixed Number Fraction Indicators *Practice 9C*
Complex Fractions
9.7 Definition of Complex Fraction
9.7.1 Use of Complex Fraction Indicators and Complex Fraction Lines *Practice 9D*
More Fraction Rules
9.8 Fractions and the Baseline Indicator
9.9 Further Observations Regarding Spacing
9.10 Fractions and the Ellipsis and Long Dash
9.11 Enclosed Lists *Practice 9E*
RADICAL EXPRESSIONS
9.12 Terminology
9.13 The Termination Indicator
9.14 Spacing
9.15 Index of Radical *Practice 9F*
9.16 Nested Radical Expressions
9.16.1 Nested Radical Expression with an Index

9.17 Radical Expressions and the Baseline Indicator
9.18 Radical Expressions and the Ellipsis and Long Dash
9.19 Radical Expressions and Abbreviations
9.20 Enclosed Lists with Radical Expressions *Practice 9G*

LINKED EXPRESSIONS

9.21 Definition of Linked Expression
9.22 Division of Linked Expressions
9.22.1 Restrictions
9.22.2 Other Considerations *Practice 9H*
9.23 *Special Case:* Certain Displayed Linked Expressions

Format: Margin Requirements for a Linked Expression Requiring Special Margins

9.23.1 In Narrative
9.23.2 In Itemized Text Without Subdivisions
9.23.3 In Itemized Text With Subdivisions *Practice 9I*

Answers to Practice Material

Lesson 10

ARROWS

10.1 Arrows Used in Mathematics

Construction of Braille Arrows

10.2 Introduction to the Shape Indicator
10.3 Horizontal Arrow Shafts
10.4 Arrowheads
10.4.1 Barbed Arrowheads
10.5 Arrows With Barbed Ends
10.6 Spacing and Punctuation with Arrows
10.7 The Contracted Form of the Right-Pointing Arrow
10.7.1 Nonuse of the Contracted Form of the Right-Pointing Arrow *Practice 10A*
10.8 Arrows With Dotted Ends
10.9 Arrows With Other Types of Arrowheads *Practice 10B*

Vertical, Slanted, and Curved Arrow Shafts

10.10 Arrow Direction Indicators
10.10.1 Vertical Arrow Directions
10.10.2 Slanted Arrow Directions
10.11 Curved Arrows *Practice 10C*

Boldface and Compounded Arrows

10.12 Boldface Arrows
10.13 Arrows Used as Signs of Comparison Compounded Vertically

10.14 Arrows Used as Signs of Comparison Compounded Horizontally *Practice 10D*
10.15 Other Arrows
INTRODUCTION TO SPATIAL ARRANGEMENTS
10.16 Background
Spatial Arrangements with Addition and Subtraction
10.17 Numeric Indicator
10.18 Separation Line
10.19 Alignment with Addition and Subtraction
10.20 Placement of Symbols
10.20.1 Operation Symbols
10.20.2 Currency Symbols
10.21 Side-by-Side Layout
10.20.1 Page Number Restriction
10.22 Blank Lines *Practice 10E*
10.23 Omissions in Work Arranged Spatially for Computation
10.24 Polynomials
10.25 Abbreviations
10.26 Fractions
10.27 Placement of Identifiers
10.27.1 Side-by-Side Arrangement
10.27.2 Page Number Restriction *Practice 10F*
10.28 Carried Numbers with Addition
10.28.1 Placement of Identifiers with Carried Numbers
Introduction to Cancellation
10.29 Cancellation in Subtraction Problems
10.29.1 Placement of Identifiers with Spatial Subtraction *Practice 10G*
Arrangement on the Page
10.30 Blank Lines and the Page Change Indicator
10.31 Pagination and Blank Lines
10.31.1 Starting a Braille Page with a Spatial Arrangement
10.31.2 Ending a Braille Page with a Spatial Arrangement
10.32 Wide Arrangements
10.33 Itemized Spatial Problems with Subdivisions
Placement of Code Switch Indicators
10.34 Opening Nemeth Code Indicator
10.35 Nemeth Code Terminator
10.36 Braille Page Turn
10.37 Instructions Preceding Itemized Spatial Arrangements
10.38 Displayed Spatial Arrangements

Answers to Practice Material

Lesson 11

SIGNS OF SHAPE

11.1 Definition

Basic Shapes

11.2 Basic Signs of Shape Represented by Numbers—Regular Polygons

11.2.1 Unlisted Regular Polygons

11.3 Basic Signs of Shape Represented by Letters—Irregular Polygons

11.3.1 Unlisted Irregular Polygons

11.4 Other Basic Signs of Shape Represented by Letters

11.4.1 Other Unlisted Basic Shapes

11.5 Basic Signs of Shape Represented by Other Dot Combinations

11.6 Filled-In and Shaded Shapes *Practice 11A*

Shapes with Structural Modification

11.7 Definition and Construction

11.8 Structurally Modified Triangles

11.9 Structurally Modified Angles

11.10 Unlisted Shapes with Structural Modification *Practice 11B*

Shapes with Interior Modification

11.11 Definition and Construction

11.12 Circles with Interior Modification

11.13 Angles with Interior Modification

11.14 Rectangles and Squares with Interior Modification

11.15 Words Enclosed in Shapes

11.16 Two or More Vertically Arranged Modifiers

11.17 Two or More Horizontally Arranged Modifiers

11.18 Unlisted Shapes with Interior Modification *Practice 11C*

Other Details

11.19 Spacing with Signs of Shape

11.20 Punctuation with Signs of Shape

11.21 Plurals/Possessives

11.22 Further Considerations Regarding Transcriber-Devised Shapes

11.22.1 Usage Rules Regarding Interior Numerals and Arrows

11.22.2 Shapes Represented by Drawing *Practice 11D*

Calculators and Keyboards

11.23 The Keystroke Indicator

11.23.1 Shape in Print

11.24 Other Details Concerning Keystrokes

11.24.1 The Label

11.24.2 Spacing

11.25 Long Keystroke Constructions

Icons

11.26 Consistency in Representation of Icons

Shapes Used as Signs of Omission
11.27 Spacing
11.28 The English Letter Indicator and Comparison Signs
11.29 Use of the Multipurpose Indicator
11.30 Omissions in Spatially-Arranged Problems *Practice 11E*
Identified Signs of Shape
11.31 Spacing
11.31.1 Keep Together
11.31.2 Surrounding Symbols
11.32 A Shape Within a Superscript or a Subscript
11.33 A Shape Which Carries a Superscript or a Subscript
11.34 The English Letter Indicator
11.34.1 The Letter "m"
11.35 Use of the Numeric Indicator in an Enclosed List *Practice 11F*
LABELED FORMATS, cont.
Displayed Material with Labels
11.36 Recognition
11.36.1 Braille Layout
11.36.2 Transcriber's Note Required
11.37 Page Number Citation *Practice 11G*
TYPEFORMS, cont.
Labeled Mathematical Statements
11.38 Recognition of a Labeled Mathematical Statement
11.38.1 The Label
11.38.2 The Statement
11.38.3 Spacing and Margins
11.39 Significant Typeface *Practice 11H*
Typeform Indicators for Mathematical Words and Phrases
11.40 Italic and Boldface Typeform Indicators
11.40.1 One Word in Italics or Boldface
11.40.2 A Phrase in Italics or Boldface
11.41 Code Switching Within an Emphasized Passage
11.42 Revisiting Typeform in Labeled Statements *Practice 11I*
Answers to Practice Material

Lesson 12

MODIFIERS AND MODIFIED EXPRESSIONS
12.1 Definition
12.2 Construction of Simple Modified Expressions – The Five-Step Rule
Common Modifiers
12.3 Arrows as Modifiers

Special Situations Involving Arrows
12.3.1 When to Omit Arrows
12.3.2 When the Arrow is Being Modified
12.3.3 When Other Rules Apply *Practice 12A*
12.4 Carets as Modifiers
12.5 Horizontal Bar as a Modifier
12.5.1 The Contracted Form of Bar Over/Bar Under
12.5.1.a Bar Above
12.5.1.b Bar Below *Practice 12B*
12.6 Other Symbols as Modifiers
12.6.1 Dot
12.6.2 Arc
12.6.3 Tilde
12.6.4 Question Mark
12.7 Expressions as Modifiers
12.7.1 Binomial Coefficient *Practice 12C*
12.8 Spacing with Modified Expressions *Practice 12D*
Modified Expressions and Superscripts/Subscripts
12.9 Modified Expression on the Baseline
12.9.1 Superscript/Subscript After the Modification
12.9.2 Superscript/Subscript Within the Modification
12.9.3 Binomial Coefficient *Practice 12E*
12.9.4 Modified Expression on the Baseline That Follows a Superscript or a Subscript
12.10 Modified Expression Within a Superscript or Subscript *Practice 12F*
12.11 Horizontal Grouping Signs as Modifiers
Modified Signs of Comparison
12.12 Definition
12.13 Transcription *Practice 12G*
Expressions with More Than One Modifier
12.14 Modifiers of Higher Order
12.14.1 Parallel Horizontal Bars
12.15 Individual Modifiers
12.16 Simultaneous Modifiers *Practice 12H*
Format: FORMAL PROOF
12.17 Definition
12.17.1 Spacing and Margins
12.17.2 Auxiliary Captions
12.17.3 Step-Number Format *Practice 12I*
Practice 12J
SPATIAL ARRANGEMENT WITH MULTIPLICATION
12.18 Alignment
12.19 Placement of Multiplication Symbol

12.20 Separation Line
Alignment of Partial Products
12.21 Partial Products
12.21.1 Spacing *Practice 12K*
12.22 Omissions
12.23 Fractions and Mixed Numbers
12.24 Polynomials
12.25 Subscripts Denoting Nondecimal Bases
12.26 Carried Numbers with Multiplication
12.27 Placement of Identifiers with Spatial Multiplication *Practice 12L*

Answers to Practice Material

Lesson 13

MISCELLANEOUS SYMBOLS
Unspaced Miscellaneous Symbols
13.1 Spacing Rules for Unspaced Symbols
13.1.1 Caret
13.1.1.a Use of the UEB Caret
13.1.2 Crossed Letters
13.1.3 Del
13.1.4 Derivative and Partial Derivative (round d)
13.1.5 Empty Set (null set, void set)
13.1.6 Factorial Sign
13.1.7 Infinity
13.1.8 Integral
13.1.8.a Upper and Lower Integral Signs
13.1.9 Quantifiers *Practice 13A*
Spaced Miscellaneous Symbols
13.2 Spacing Rules for Spaced Symbols
13.2.1 "At" Sign in Mathematical Context
13.2.2 Check Mark
13.2.3 Ditto Marks
13.2.4 Since (because)
13.2.5 Therefore
13.2.6 Boldface Vertical Bar (end of proof) *Practice 13B*
Spacing with the Angstrom Unit and Tally Marks
13.3 Angstrom Unit
13.3.1 Punctuation
13.4 Tally Mark
13.4.1 Grouping
13.4.2 Spacing
13.4.3 Punctuation *Practice 13C*

SUPERPOSED SIGNS

13.5 Definition and Analysis

13.6 Transcription of Superposed Signs

13.6.1 Integral Modified by Superposition

13.6.2 Signs of Operation Modified by Superposition

13.6.3 Horizontal and Vertical Bars Modified by Superposition

13.6.4 Signs of Shape Modified by Superposition

13.6.5 Signs of Comparison Modified by Superposition

13.6.6 Symbols That Are Not Superposed Signs *Practice 13D*

AMBIGUOUS SIGNS

13.7 Context

13.7.1 Vertical Bar and Colon

13.7.2 Spacing

13.7.3 Uppercase Greek Letters

13.7.4 Chemical Notation *Practice 13E*

MULTIPURPOSE INDICATOR

13.8 Review

13.9 Additional Uses of the Multipurpose Indicator

13.9.1 Letter Followed by a Decimal Point and a Numeral

13.9.2 Numeric Subscript Followed by a Numeral

13.9.3 Decimal Point Followed by a Nonnumeric Character

13.9.4 Side-by-Side Vertical Bars *Practice 13F*

DIVISION PROBLEMS

13.10 Linear Representation *Practice 13G*

SPATIAL ARRANGEMENTS WITH DIVISION

13.11 Notation Devices

13.11.1 Length of the Separation Line

13.12 When a Spatial Arrangement is Required

13.12.1 More Than Just Numerals

13.12.1.a Spacing with Abbreviations

13.12.2 A Quotient is Present

13.12.2.a A Quotient with a Remainder

13.12.3 Long Division *Practice 13H*

13.13 Blank Lines Required

13.14 Omissions

13.15 Carried Numbers in Long Division

13.16 Cancellation in Long Division

13.17 Placement of Identifiers with Spatial Division *Practice 13I*

Answers to Practice Material

Lesson 14

FUNCTION NAMES AND THEIR ABBREVIATIONS

14.1 Functions
14.1.1 Code Switching
14.2 Spacing of Function Names
14.2.1 Spacing with Signs of Operation
14.2.2 Spacing with Indicators *Practice 14A*
14.3 Nonuse of the English Letter Indicator
14.4 Unabbreviated Function Names in Mathematical Context
14.5 Consecutive Function Names
14.5.1 "Arc" in Context
14.5.1.a Do Not Confuse
14.6 Punctuation
14.7 Keep Together
14.8 *Clarification*—Function Names in an Enclosed List *Practice 14B*
14.9 Function Names and Superscripts/Subscripts
14.9.1 Use/Nonuse of the Subscript Indicator
14.9.2 Function Names Within a Superscript or a Subscript *Practice 14C*
14.10 Modifiers with Function Names
14.10.1 *Special Case:* Upper Limit and Lower Limit *Practice 14D*

Format: Division of Mathematical Expressions Between Braille Lines

14.11 Review
14.12 The Concept of Logical Mathematical Units
14.12.1 The Priority List
14.13 Priority #1—Before a Sign of Comparison
14.13.1 Identifier May Stand Alone
14.13.2 Logical Mathematical Units
14.14 Priority #2—Before a Sign of Operation
14.14.1 Logical Mathematical Units *Practice 14E*
14.15 Priority #3—Before a Fraction Line
14.15.1 Logical Mathematical Units
14.16 Priority #4—Before a Baseline Indicator
14.16.1 Logical Mathematical Units *Practice 14F*
14.17 Priority #5—Before a Level Indicator
14.18 Priority #6—Between Grouped Factors
14.18.1 Logical Mathematical Units
14.19 Priority #7—After a Termination Indicator *Practice 14G*
14.20 Application of this Rule to Special Linked Expressions
14.20.1 Long Anchor
14.20.2 Long Link *Practice 14H*
14.21 Margins for Embedded Expressions
14.22 Items That Must Not Be Divided

14.22.1 Symbols to Keep Together
14.22.2 Expressions to Keep Together
Spatial Arrangements, cont.
SQUARE ROOT DIVISION
14.23 Review of Terminology
14.24 Spatial Arrangement for Square Root Problems
14.25 Placement of Identifiers with Spatial Radical Expressions *Practice 14I*
OTHER PRINT LAYOUTS SHOWING DIVISION
14.26 Partial Quotients
14.27 Synthetic Division
14.27.1 Alignment and Spacing
14.27.2 Vertical Line
14.27.3 Another Print Style—Divisor on the Right
14.27.4 Another Print Style—Boxed Divisor
14.27.5 Placement of Identifiers with Synthetic Division *Practice 14J*
Answers to Practice Material

Lesson 15

SPATIAL ARRANGEMENTS OF FRACTIONS
15.1 Spatial Fraction Line
15.2 Numerator and Denominator
15.3 Placement of Identifiers with Spatially Arranged Fractions
Situations Requiring Spatial Presentation
15.4 Simple Fractions Arranged Spatially for Illustration *Practice 15A*
15.5 Cancellation Within Fractions
15.5.1 Extent of Cancellation
15.5.2 Cancellation and Level Indicators
15.5.3 Canceled Abbreviations
15.6 Chemistry Exception *Practice 15B*
HYPERCOMPLEX FRACTIONS
15.7 Definition and Recognition
15.8 Hypercomplex Fraction Indicators
15.9 Higher Orders of Complexity *Practice 15C*
CONTINUED FRACTIONS
15.10 Definition and Recognition *Practice 15D*
Format: REMARKS AND COMMENTS
15.11 Guidelines
15.11.1 OPTION #1—Continue the Commentary on the Same Line
15.11.2 OPTION #2—Indent the Commentary on the Next Line *Practice 15E*
Format: STEM-AND-LEAF PLOTS
15.12 Definition and Recognition

15.13 The Table
15.14 The Key
15.15 Data Consisting of More Than One Character; Punctuation Between Entries
15.16 Alphabetic Data
15.16.1 Alphabetic Key
15.17 Blank Entries
15.18 Runovers Within the Table
15.19 Back-To-Back Plot *Practice 15F*
Answers to Practice Material

Lesson 16

SYSTEMS OF EQUATIONS
16.1 Definition and Recognition
16.2 Transcription Rules for Systems of Equations *Practice 16A*
16.3 Unified System of Equations and Enlarged Grouping Signs
Enlarged Signs of Grouping
16.4 Transcription Rules for Enlarged Signs of Grouping
16.4.1 Enlarged Left Brace
16.4.2 Enlarged Right Brace
16.5 Embedded Vertical Groupings *Practice 16B*
16.6 Enlarged Parentheses
16.7 Placement of Symbols
16.8 Placement of Identifiers and Punctuation
16.9 Grouping Symbols Shown Taller in Print *Practice 16C*
16.10 Remarks Printed Next to Spatial Arrangements
16.10.1 Remarks Brailled Beside Unified Expressions
16.10.2 Remarks Brailled Below the Arrangement *Practice 16D*
16.11 More Enlarged Signs of Grouping *Practice 16E*
DETERMINANTS AND MATRICES
16.12 Definition and Recognition
16.13 Transcription Rules for Determinants and Matrices
16.13.1 Blank Lines
16.13.2 Grouping Symbols
16.13.3 Placement of Items
16.13.4 Numeric and Letter Indicators
16.13.5 Placement of Identifiers, Symbols, and Punctuation *Practice 16F*
Further Considerations with Determinants and Matrices
16.14 Multiplying Arrays
16.15 Omission Dots
16.15.1 No Dots Are Printed Between Columns
16.15.2 Dots Are Printed Between Columns or Some Entries are Blank *Practice 16G*

16.16 Space-Saving Techniques
16.16.1 Runovers With Indentation
16.16.2 Runovers Without Indentation
16.16.3 Fractions in Arrays
16.16.4 Keying *Practice 16H*
16.17 Row Matrix
16.18 Embedded Arrays *Practice 16I*
16.19 Use of Tactile Graphics for Enlarged or Horizontal Grouping Signs

Answers to Practice Material

Lesson 17

TABLES
17.1 Structure of Tables
17.2 Table Label and Title
17.3 Column Headings
17.4 Table Entries
17.4.1 Tables in UEB
17.4.2 Tables in Nemeth Code
17.4.3 Code Switching Decisions *Practice 17A*
17.5 When Row Headings are Words *Practice 17B*
Boxed Tables
17.6 Code Switching and Box Lines
17.6.1 Switching Within the Table
17.6.2 Switching Within the Box Lines
17.6.3 Technical Material Before or After a Box
17.6.4 Placement of Transcriber's Note *Practice 17C*
Table Rules Specific to the Nemeth Code
17.7 Table of Numbers *Practice 17D*
FIGURES AND DIAGRAMS
17.8 Which Code?
17.8.1 Letters Used as Diagram Labels
17.9 Switch Indicators and Tactile Graphics
17.10 Graphic Number Lines
17.11 Diagrams in Exercise Material *Practice 17E*
KEYING TECHNIQUE
17.12 Keying
17.12.1 Alphabetic Key
17.12.2 Numeric Key
17.12.3 The Key List *Practice 17F*

CHEMISTRY

17.13 Two BANA Publications

Answers to Practice Material

Lesson 18

18.1 Preparing for the Certification Exam

18.2 The Nemeth Codebook

18.3 Beyond the Nemeth Code

Structuring a Textbook

18.4 Transcriber-Generated Pages and Front Matter

18.4.1 Special Symbols Page

18.4.1.a Braille Order

18.4.2 Transcriber's Notes Page *Practice 18A*

18.5 Body of Text

18.5.1 Follow Nemeth Code Formatting Rules

18.5.2 Follow *Braille Formats* Formatting Guidelines

18.5.3 Context-Dependent Formats

Four Practices *Practice 18B*

Practice 18C

Practice 18D

Practice 18E

Answers to Practice Material

Appendices

Appendix A Reading Practice

Appendix B Glossary Of Terms

Appendix C Nemeth Code Format Summaries

Appendix D Sample Page

LESSON 1

Read about this PROVISIONAL EDITION in the front matter to this book.
Check the NFB website periodically for updates to this lesson.

- *INTRODUCTION TO NUMERALS AND THE NUMERIC INDICATOR*
- *THE MATHEMATICAL COMMA AND DECIMAL POINT*
- *INTRODUCTION TO SIGNS OF OPERATION*
- *INTRODUCTION TO SIGNS OF COMPARISON*
- *MONETARY, PERCENT, AND PRIME SIGNS*
- *CONTINENTAL SYMBOLS*

FORMAT

- *General Principles*

1.1 Philosophy: The braille code for mathematics is especially designed for the representation and transcription of mathematical and scientific notation. Its purpose is to convey, as accurately as possible, a clear conception of the printed text to the braille reader. Using braille indicators in conjunction with the 63 braille characters, this code is capable of providing equivalent symbols for the hundreds of mathematical and scientific print signs now in use and yet to be devised. The one-to-one correspondence between braille and print symbols makes it possible to produce an accurate transference from print to braille or from braille to print.

1.2 Non-technical and Technical Texts

1.2.1 Non-technical Texts: As defined in the Nemeth Code, the designation "non-technical" implies only the absence of mathematical or scientific notation. A work in law or medicine may be quite technical in its field but is regarded as non-technical in the sense just mentioned.

The BANA Nemeth Code Technical Committee is discussing details regarding the term "non-technical text." This section will be completed after decisions are made.

1.2.2 Technical Texts: A technical text is any work in which mathematical or scientific notation appears. Narrative is transcribed in accordance with the rules of Unified English Braille ("UEB"). The mathematical or scientific notation is transcribed in accordance with the rules of the Nemeth Code. Technical text is referred to as "mathematical context" in this document.

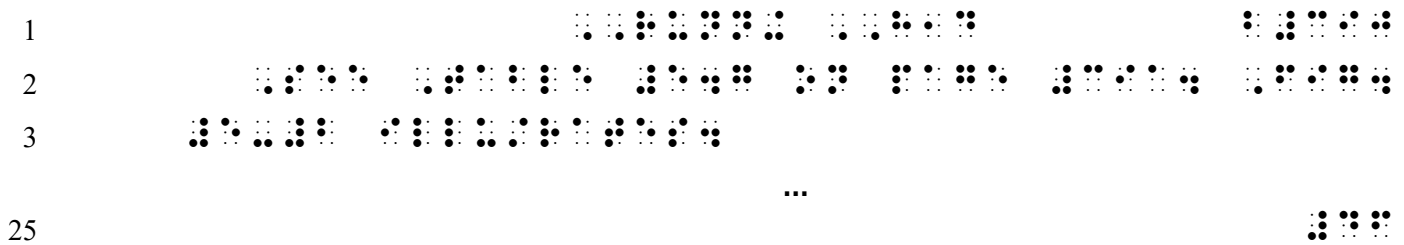
INTRODUCTION TO NUMERALS AND THE NUMERIC INDICATOR

Note: This section does not cover all of the rules regarding the use/nonuse of the numeric indicator.

1.3 Representation of Arabic Numerals: In the transcription of a technical text, digits are represented in two ways:

1.3.1 English Braille Numerals: Numbers used to label figures, tables, sections, etc. as well as any associated punctuation are transcribed in UEB. Print page numbers, braille page numbers, and page numbers referred to within the text are transcribed in UEB.

Example 1.3-1 See Table 5.7 on page 391. Fig. 5-2 illustrates.



The number at the end of line 1 is print page number b390. The last number represents braille page 46 on line 25 of the braille page.

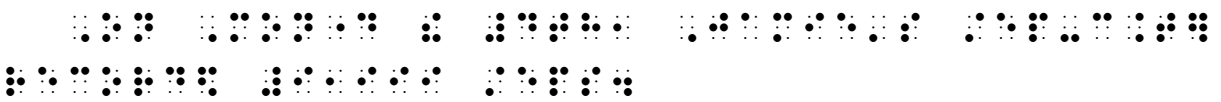
Numbers that have mathematical meaning within narrative context may be transcribed in UEB as long as the number is freestanding and is unmodified. ("Unmodified" in this context means there is no symbol associated with the number, such as a monetary symbol or a percent sign, for example.)

Example 1.3-2 9 inches and 15 inches are 2 feet.



"Freestanding and unmodified" includes numbers with an internal comma, ordinals, and plurals. Also note that a number that touches literary punctuation is considered to be freestanding.

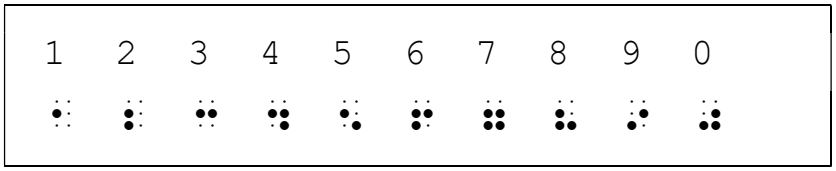
Example 1.3-3 On Monday the 4th, Jamie's step-counter recorded 9,999 steps.



Example 1.3-4 Which is greater: two 3's or three 2's?



1.3.2 Nemeth Code Digits: Nemeth Code digits are used to represent Arabic numerals which occur in mathematical context, as well as a freestanding mathematical number in narrative context which is combined with anything other than an internal comma (e.g., a minus sign or a decimal point). The ten Arabic digits are represented by the letters "a" through "j" dropped to the lower part of the braille cell.



Although unmodified, freestanding mathematical numbers can be brailled in UEB, assume mathematical context in the isolated examples presented throughout the remainder of this lesson.

1.4 Numeric Indicator: Unless otherwise stated, the numeric indicator is required before a numeral that <u>follows a space</u> or before a numeral that <u>begins a braille line</u>.

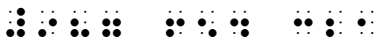


Example 1.4-1 5 10 15 20



1.4.1 SPECIAL CASE—Segmented Numbers: The numeric indicator is <u>not</u> used following a space that partitions a number into segments. *The numeric space indicator of UEB is not used in a technical transcription. Segmented numbers must be transcribed in Nemeth Code .*

Example 1.4-2 987 654 321



Note: A particular book may show large numbers in this manner rather than using commas to delineate place value. Check for context clues to be sure this represents 987 million 654 thousand 321 and not three separate 3-digit numbers.

THE PRACTICE MATERIAL

By transcribing the practice material you will gain first-hand experience with the topics presented in each lesson and you will be better prepared to braille the exercise for grading. Many of the points discussed in the lesson are illustrated only in the practice material. The Study Tips on pages i-ii offer more ways to get the most out of these activities.

Use a 40-cell line when transcribing the practice material. Check your work by comparing your transcription to the simulated braille located at the end of each lesson.

PRACTICE 1A

Instructions: Do not braille a heading for any of the practices in this lesson. Using a 40-cell braille line, transcribe the following numbers using the lower-cell digits of the Nemeth Code. Begin in cell 1. Leave one blank cell between each number. Begin a new line in cell 1 when you do not have room on a line to complete a number. None of these numbers are partitioned into segments—each is a new number. Check your accuracy by comparing your transcription to the practice answers at the end of this lesson.

123 456 7890 295 431 61 507 3196 15837 808 46 282 2802

61640 74 9559 404 75134 13579

THE MATHEMATICAL COMMA AND DECIMAL POINT

1.5 Mathematical Comma: The mathematical comma is used for a comma occurring in a long numeral. It is also used for a comma which follows a numeral or other mathematical expression.

Mathematical Comma	⠂

Although numbers with commas can be brailled in UEB, assume mathematical context in the isolated examples presented below.

Example 1.5-1 987,654,321

⠼⠊⠓⠋⠂⠑⠙⠙⠂⠛⠉⠁

This represents the number 987 million, 654 thousand, 321.

Example 1.5-2 997, 998, 999, 1,000

⠼⠊⠓⠋⠂ ⠼⠊⠓⠊⠂ ⠼⠊⠓⠊⠊⠂ ⠼⠁⠂⠚⠚⠚

These are four individual numbers, separated by a comma and a space. The last number contains an internal comma.

*SYMBOL RECOGNITION See **1.14** for a discussion of the Continental decimal point, which is depicted by the same print symbol as a mathematical comma.*

1.6 Mathematical Decimal Point

Mathematical Decimal Point	⠲

1.6.1 Spacing of the Decimal Point: In a numeral, no space is left between the decimal point and the digits to which it applies.

Example 1.6-1 3.14159

⠼⠉⠲⠁⠙⠁⠑⠊

1.6.2 The Decimal Point and the Numeric Indicator: The numeric indicator is required before a decimal point that precedes a numeral when the decimal point follows a space or begins a braille line.

Example 1.6-2 .25 .5 .75

⠼⠲⠃⠑ ⠼⠲⠑ ⠼⠲⠛⠑

Reminder: When a decimal is part of a numeric label to a figure, table, section, etc., UEB is used. See 1.3.1.

1.7 General Principles: "Formatting" refers to layout on the page, such as indentations (margins), line spacing (blank lines), centering, and pagination. *The Nemeth Braille Code for Mathematics and Science Notation* specifies certain formats which are covered in these lessons and are also summarized in **Appendix C** of this course.

When an item in a UEB transcription requires the use of Nemeth Code symbols, format rules of *The Nemeth Braille Code for Mathematics and Science Notation* are to be applied to the entire transcription including those portions transcribed in UEB. When a format is not specifically addressed in the Nemeth Code, the principles provided in *Braille Formats* should be followed.

PRACTICE 1B

Instructions: Begin the list on line 1 of the braille page. Use Nemeth Code numerals for all numbers in this list. Using a 40-cell braille line, duplicate the columnar format shown. Following *Braille Formats* guidelines for the layout, you will leave a column of two blank cells between the end of the longest item in each column and the left-hand margin of the next column. These columns are unrelated therefore guide dots are not used.

592	.75	345	4.6692
206	6.4	29,254	98.6
46	59.1	1.234	3.14159
.240	0.37	1791	31,536,000
3,250	0	70.2	365.2422
8,086	987,654	.008382	273.15

INTRODUCTION TO SIGNS OF OPERATION

1.8 Signs of Operation: The most common signs of operation are listed below.

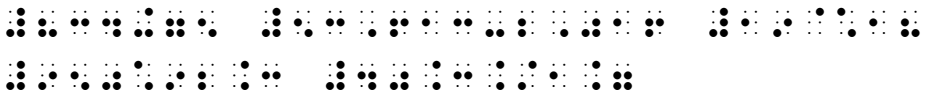
Plus	+	
Minus	−	
Multiplication (times)		
Cross (Cartesian product)	×	
Dot	·	
Division (divided by)	÷	

Since the minus sign and the hyphen are represented by the same symbol in both print and braille, the transcriber must determine the meaning of the symbols from their context.

*SYMBOL RECOGNITION See **1.13** for a discussion of the Continental comma, which is depicted by the same print symbol as the multiplication dot.*

1.8.1 Spacing with Signs of Operation: Unless otherwise stated, a sign of operation is unspaced from its related mathematical terms regardless of the print spacing. A numeric indicator is generally not needed within an unspaced expression.

Example 1.8-1 $834 + 75$ $53{,}613 - 2{,}016$ 19×18 $950 \cdot 92.3$ $40.3 \div 1.7$



1.8.2 Positive and Negative Numbers: A numeral preceded by a minus sign requires a numeric indicator when the minus sign follows a space or begins a braille line. A numeral preceded by a plus sign does not require a numeric indicator even when the plus sign follows a space or begins a braille line. *Numerals preceded by a + sign or a − sign must be brailled in Nemeth Code.*

Example 1.8-2 -3 -2 -1 0 $+1$ $+2$ $+3$



Note: These are seven separate numerals.

1.8.2.a Decimals: The numeric indicator is required between a minus sign and a decimal point that precedes a numeral when the minus sign follows a space or begins a braille line. A numeral preceded by a plus sign and a decimal point does not require a numeric indicator even when the plus sign follows a space or begins a braille line.

Example 1.8-3 −.75, −.5, −.25, 0.0, +.25, +.5



PRACTICE 1C

Instructions: Transcribe these unrelated columns using a two-column format.

+592	$9.75 + 16.22$
−7.5	$10{,}000 - 3{,}560$
404.8	19×18
−.9	$512 \cdot 63$
.708	$3{,}951 \div 7$

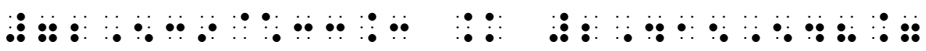
INTRODUCTION TO SIGNS OF COMPARISON

1.9 Signs of Comparison: A few signs of comparison and their braille equivalents are listed below.

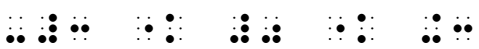
Equals	=	
Greater Than (is greater than)	>	
Less Than (is less than)	<	
Proportion (as)	::	
Ratio (is to)	:	

1.9.1 Spacing with Signs of Comparison: A space is required between a sign of comparison and a sign of operation or any other expression which precedes or follows it. *Reminder: A numeric indicator is usually required when a numeral is preceded by a space.*

Example 1.9-1 $72{,}539 \times 33.3 = 2{,}415{,}548.7$



Example 1.9-2 $-3 < 0 < +3$



Example 1.9-3 $6 : 9 :: 2 : 3$



PRACTICE 1D

Instructions: Begin each mathematical expression on a new line in cell 1.

$8.5 < 74$

$85 > 9.6$

$29 \cdot 3 = 3 \cdot 29$

$14 : 2 :: 7 : 1$

$19{,}530 - 2{,}016 \times 8.25 + 6.75 = 262{,}710.00$

MONETARY, PERCENT, AND PRIME SIGNS

1.10 Monetary Signs

The BANA decision regarding construction of monetary signs has not yet been posted but the symbols used are given in the text below.

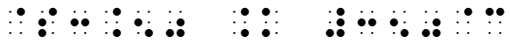
The monetary signs of UEB are used in math as well.

Cent	¢	
Dollar	$	
Euro	€	
Franc	₣	
Naira	₦	
Pound Sterling	£	
Yen	¥	

If a monetary sign is printed for which there is no established symbol, the transcriber should create one following the same "dot 4" pattern shown above.

1.10.1 Spacing with Monetary Signs: No space is left between a monetary sign and its related quantity or symbol. A number which immediately follows a monetary sign does not need a numeric indicator.

Example 1.10-1 $3.50 = 350¢



1.11 Percent and Per Mille Signs

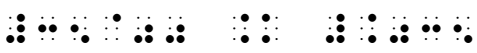
Percent sign	%	
Per mille sign	‰	

1.11.1 Spacing with Percent and Per Mille Signs: No space is left between these signs and their related quantities or symbols.

Example 1.11-1 $45\% = 0.45$



Example 1.11-2 $35‰ = .035$

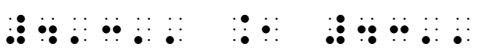


1.12 Prime Sign

Prime Sign	'	
(two prime signs)	"	

The braille symbol for the prime sign is used wherever the print symbol appears in mathematical context regardless of its meaning. When more than one prime sign is used in print, the equivalent number of signs are used in braille. Prime signs must be unspaced from each other and from the quantity to which they apply. In the following example the prime sign is used to denote feet and inches.

Example 1.12-1 $4'3" > 43"$



PRACTICE 1E

Instructions: Retain the simple vertical listing, beginning each line in cell 1.

$25¢ - 5¢ = 20¢$

$\$4.89 + 5.5\% = \$5.16$

$36‰ \times 100 = 3.6$

$5'8" = 68"$

$\$1 = £0.633456$

CONTINENTAL SYMBOLS

1.13 The Continental Comma

American Mathematical Comma	,	⠂
Continental Mathematical Comma	·	⠄

The print symbol for the continental comma is different from the comma used in the United States. The difference is <u>not</u> shown in braille—dot 6 is used to represent either comma. A transcriber's note is required to inform the reader of the Continental usage in the print edition. In this example the TN could say "The print copy uses a continental mathematical comma in the second numeral."

Example 1.13-1 27,000 = 27·000 = 27 000



1.14 The Continental Decimal Point

American Decimal Point	.	⠲
Continental Decimal Point	,	⠂

The print symbol for the continental decimal point is different from the decimal used in the United States. The difference is <u>not</u> shown in braille—the braille symbol (46) is used to represent either decimal point. A transcriber's note is required to inform the reader of the Continental usage in the print edition. In this example the TN could say "The print copy uses a continental decimal point in the second numeral."

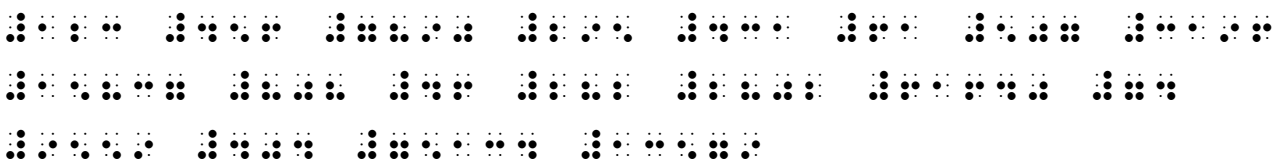
Example 1.14-1 $19.99 < £19,99



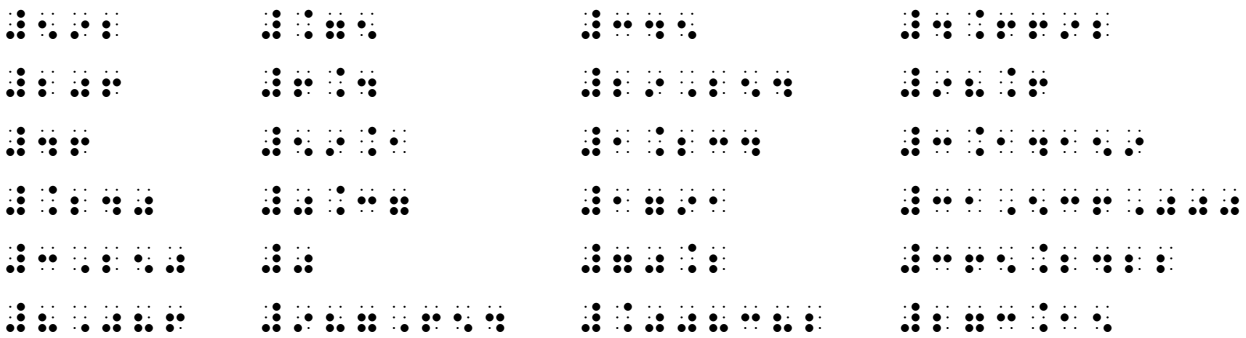
For further practice, see Appendix A—Reading Practice.

ANSWERS TO PRACTICE MATERIAL

PRACTICE 1A



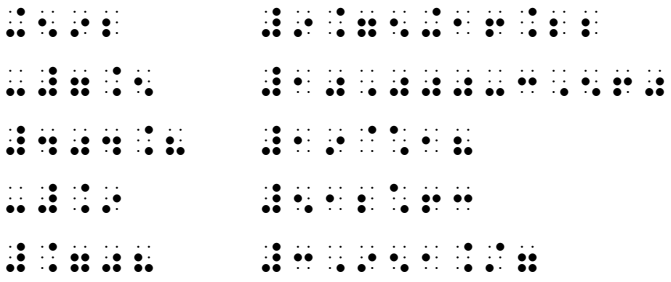
PRACTICE 1B



Fun Fact: The numbers in the rightmost column are significant scientific or mathematical numbers.

4.6692	the first six digits of one of Feigenbaum's constants from chaos theory
98.6	average healthy human body temperature in degrees Fahrenheit
3.14159	the first six digits of pi
31,536,000	the number of seconds in a year
365.2422	the number of days in a solar year
273.15	degrees Kelvin equivalent to zero degrees Celsius

PRACTICE 1C



PRACTICE 1D

⠠⠠⠠

LESSON 2

Read about this PROVISIONAL EDITION in the front matter to this book.
Check the NFB website periodically for updates to this lesson.

- *INTRODUCTION TO CODE SWITCHING*
  - *Placement of the Code Switch Indicators*
- *THE HYPHEN AND THE DASH*
- *SIGNS OF OMISSION*
- *INTRODUCTION TO IDENTIFIERS*

FORMAT
Keep Together—Mathematical Expression
Margins for Narrative (3-1)
Margins for Itemized Material with No Subdivisions (1-3)
FORMAT SUMMARY #1

INTRODUCTION TO CODE SWITCHING

2.1 A Complete Transcription: The base code used in a complete transcription is Unified English Braille ("UEB"). When mathematical content occurs anywhere in the transcription, the non-technical notation follows UEB rules while the technical notation follows the rules of the Nemeth Code. The reader will assume he is reading Unified English Braille unless signaled otherwise by the use of a code switch indicator.

Opening Nemeth Code Indicator	⠸⠩	*(followed by a space)*
Nemeth Code Terminator	⠸⠱	*(preceded by a space)*

These symbols must be listed on the Special Symbols page in the transcriber-generated portion of each volume. See ***Lesson 18*** *for further details.*

The material between the opening Nemeth Code indicator and the Nemeth Code terminator follows Nemeth Code rules. UEB symbols and indicators are not used inside of the Nemeth Code switches. The space required after the opening Nemeth Code indicator and before the Nemeth Code terminator does not represent a space in print.

The opening Nemeth Code indicator and the Nemeth Code terminator may also be referred to as "code switch indicators" or "switches".

2.2 Use of the Code Switch Indicators: Switch to Nemeth Code when mathematical material or technical notation is encountered.

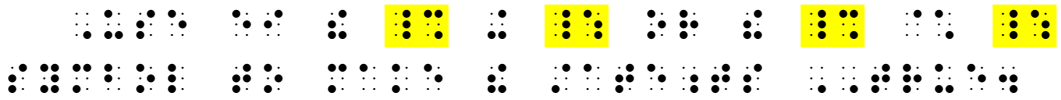
Example 2.2-1 $2 + 2 = 22$ is Grampa's favorite joke.

⠸⠩ ⠼⠃ ⠐⠖ ⠼⠃ ⠐⠶ ⠼⠃⠃ ⠸⠱ ⠠⠛⠗⠁⠍⠏⠁⠄⠎ ⠋⠁⠧⠕⠗⠊⠞⠑ ⠚⠕⠅⠑⠲

Throughout this lesson the code switch indicators are highlighted.

Switch to Nemeth Code for fragmentary expressions. The isolated mathematical symbols in the next example must use Nemeth Code symbols even though UEB symbols exist for these mathematical signs.

Example 2.2-2 Use either the + or the × symbol to make the statements TRUE.



Instructions: Treat PRACTICE 2A as a centered heading. Begin each sentence in cell 3 with any runovers beginning in cell 1.

PRACTICE 2A

What does 16 + 4 + 100 equal?

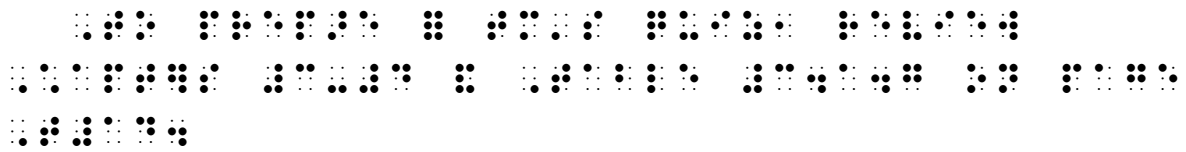
If 10 × 10 × 10 = 1,000 what does 10 × 10 × 10 × 10 equal?

Does 5 ÷ 2 name a whole number?

% ("percent") means parts per hundred; ‰ ("per mille") means parts per thousand.

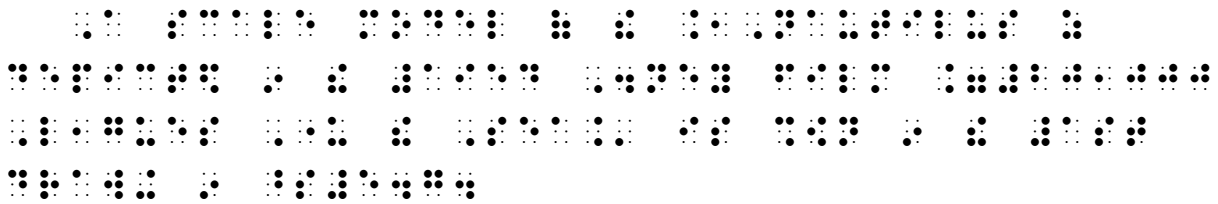
2.3 Which Code? Recall from **Lesson 1** that page numbers and labels to figures, tables, sections, etc. are brailled in UEB. Examples are shown in **1.3.1**. Here is another example that does not require code switching.

Example 2.3-1 To prepare for tomorrow's quiz, review Chapters 3-4 and Table 3.1.7 on page T14.



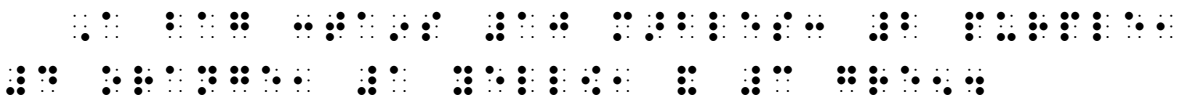
To avoid excessive code switching, freestanding numbers including numbers with internal commas or with ordinal endings may be brailled in UEB. This applies to incidental numbers encountered in narrative ...

Example 2.3-2 A scale model of the *Nautilus* as depicted in the 1954 Disney film *20,000 Leagues Under the Sea* is shown in the 1st drawing in §5.7.

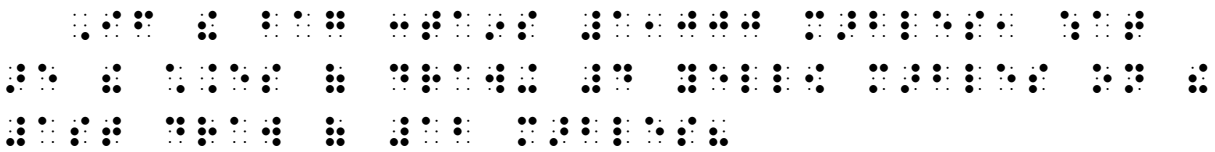


... as well as to freestanding numbers in a math problem. When the surrounding text is UEB, there is no need to switch to Nemeth Code for such numbers.

Example 2.3-3 A bag contains 10 marbles: 2 purple, 4 orange, 1 yellow, and 3 green.

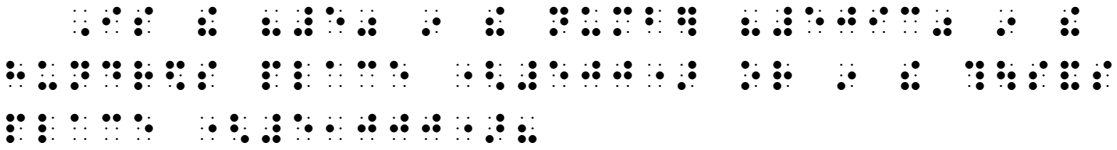


Example 2.3-4 If the bag contains 1,000 marbles, what are the chances of drawing 4 yellow marbles on the 1st draw of 12 marbles?



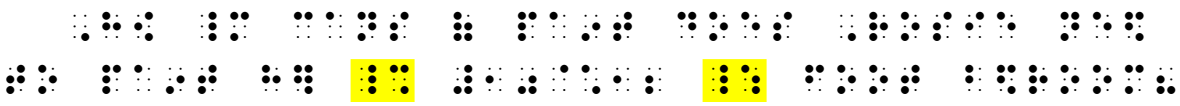
When such numbers are touching literary punctuation such as quotes or parentheses, they are still considered to be "freestanding" and may be brailled in UEB.

Example 2.3-5 Is the "5" in the number "5093" in the hundreds place (500) or in the thousands place (5,000)?



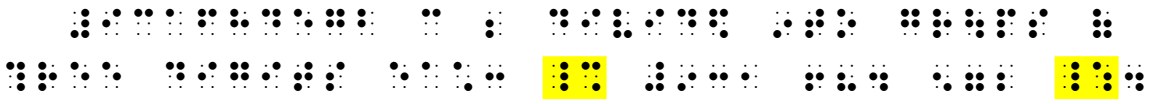
In the next example, a switch to Nemeth Code is required for the multiplication symbol (see section **2.2**, above). The numbers associated with the symbol are also brailled in Nemeth Code.

Example 2.3-6 How many cans of paint does Rosie need to paint her 10×12 foot bedroom?



2.4 Placement of Literary Punctuation: When mathematical notation ends a sentence or a phrase and Nemeth Code is terminating, first determine the function of the punctuation. If the punctuation applies to the structure of the sentence and is not actually part of the math expression, braille the Nemeth Code terminator first. The punctuation mark then will follow the termination indicator. There is no space between the terminator and the punctuation mark.

Example 2.4-1 931684572 can be divided into groups of three digits each: 931 684 572.



*The first numeral does not require a switch to Nemeth Code because it is a freestanding, unmodified numeral. The second numeral, however, requires a switch because the UEB numeric space indicator is not used in a Nemeth Code transcription. Review section **1.4.1.***

Instructions: Continue to center the heading and begin each problem in cell 3 with runovers in cell 1.

PRACTICE 2B

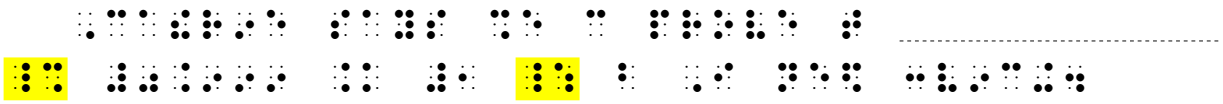
If $47 - 41 > 1$, do you suppose that $41 - 47 < 1$? Why or why not?

If $5 + 10$ equals 15, what does $10 + 5$ equal? Explain.

FORMAT

2.5 ***Keep Together*—General Principle Regarding Mathematical Expressions:** A mathematical expression that will fit on one braille line within the current margins must not be divided between lines. The entire expression is brought down to the next line.

Example 2.5-1 Catherine says she can prove that $0.999 = 1$ but I need convincing.

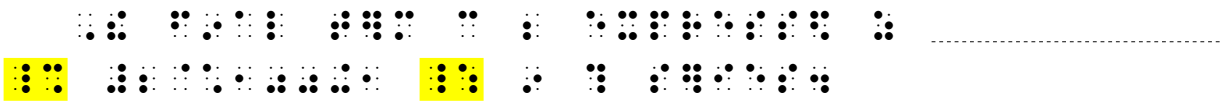


Items to the left and right of a comparison symbol, along with the symbol, constitute one "mathematical expression."

Placement of the Switch Indicators

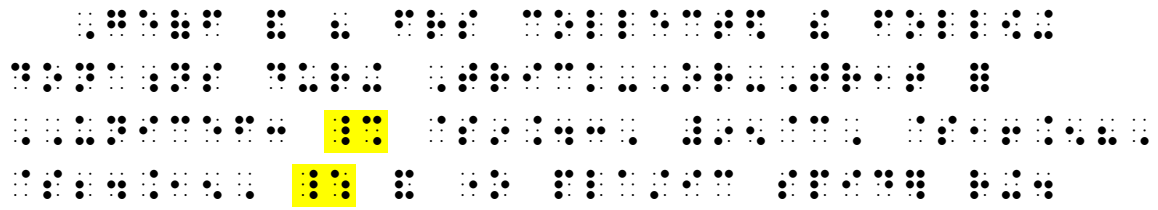
2.5.1 Keep Together If Possible: Within a paragraph, the switch indicators should appear on the same line as the expression <u>if the mathematical expression and the two indicators will fit on one braille line</u> within the current margins. In the previous example as well as the next one a new line is forced to begin with the opening Nemeth Code indicator even though the indicator could fit on the previous line.

Example 2.5-2 The final term can be expressed as $2 \times 100 + 1$ in this series.



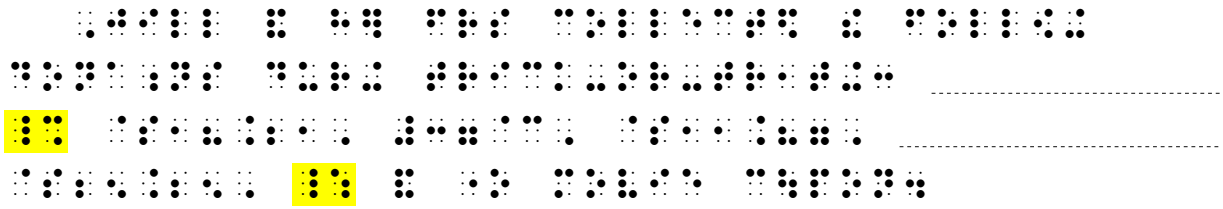
2.5.1.a If two or more math expressions are brailled between the same code switch indicators, the line may wrap at the space between the expressions even if the entire Nemeth Code portion could fit on one line.

Example 2.5-3 Geoff and his friends collected the following donations during Trick-Or-Treat for UNICEF: $9.43, 95¢, $16.58, $24.15, and one plastic spider ring.



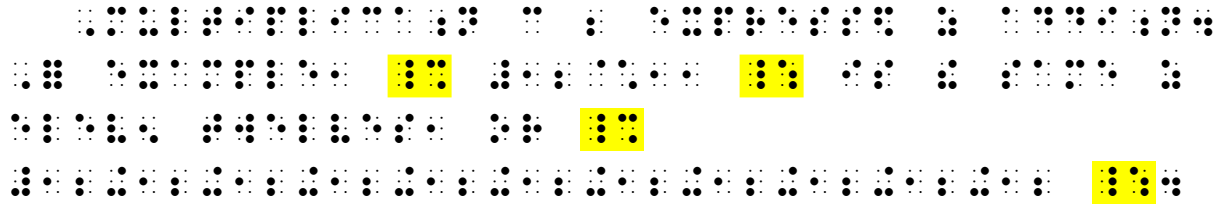
Even so, a switch indicator should not stand alone on a line if there is room for it to fall on the line with the math expression to which it applies.

Example 2.5-4 Jill and her friends collected the following donations during trick-or-treating: $18.21, 37¢, $11.87, $25.25, and one movie coupon.



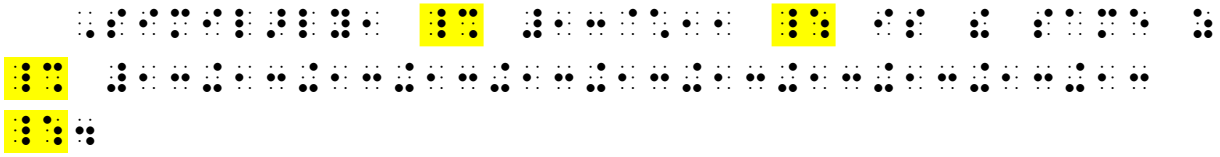
2.5.2 A Switch Indicator May Stand Alone on a Line if Unavoidable: If a math expression will fit on one line but there is not room for one or both of the switch indicators, one or both switch indicators may stand alone on a line. Keeping the mathematical expression intact on one line is the priority. Several layouts are illustrated below.

Example 2.5-5 Multiplication can be expressed as addition. For example, 12×11 is the same as eleven twelves, or 12 + 12 + 12 + 12 + 12 + 12 + 12 + 12 + 12 + 12 + 12.



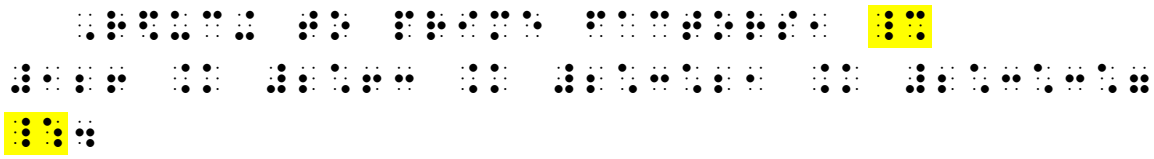
This opening Nemeth Code indicator falls on the line before the long expression.

Example 2.5-6 Similarly, 13×11 is the same as $13 + 13 + 13 + 13 + 13 + 13 + 13 + 13 + 13 + 13 + 13$.



This Nemeth Code terminator (along with the related punctuation) falls on the line after the long expression.

Example 2.5-7 Reducing to prime factors, $126 = 2 \cdot 63 = 2 \cdot 3 \cdot 21 = 2 \cdot 3 \cdot 3 \cdot 7$.



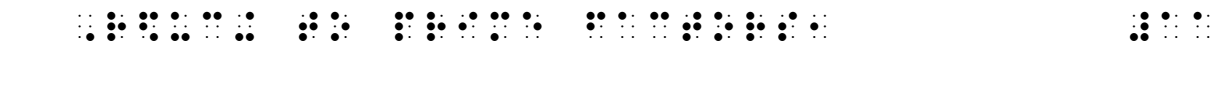
Because the expression takes up the full available line width, both the opening Nemeth Code indicator and the Nemeth Code terminator fall on separate lines.

Note: A sign of comparison links the symbols on the left side of the comparison sign with the symbols on the right side of the comparison sign and so is considered to be one mathematical expression. The math portion in the example above is one expression, not four, and should not be divided between lines if it will fit on one line within the current margins.

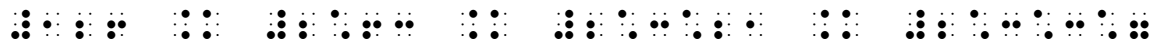
2.5.3 Switch Indicators at Page Turns: If the switch to Nemeth Code occurs at a braille page turn, keep the opening switch indicator on the same braille page as the mathematical material to which it applies. For instance, using the same example:

Example 2.5-7, cont.

Reducing to prime factors, $126 = 2 \cdot 63 = 2 \cdot 3 \cdot 21 = 2 \cdot 3 \cdot 3 \cdot 7$.

25 

1 

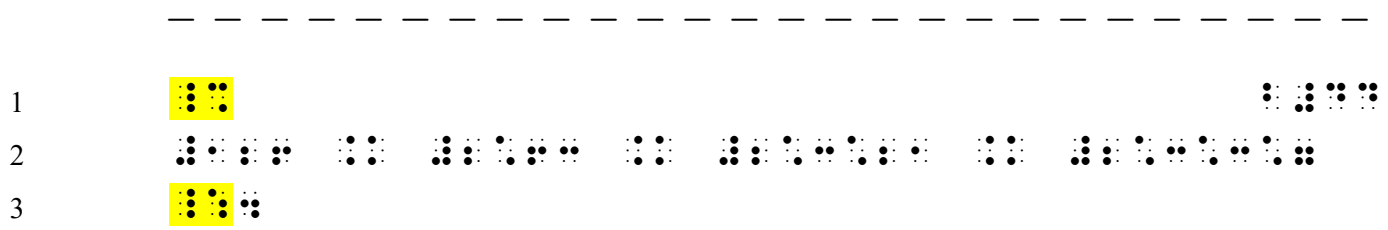
2 

3 

Unless it is impossible to arrange, both the opening Nemeth Code indicator and the Nemeth Code terminator must appear on the same page as the expression to which they apply. In the example above, the following layout would be applied if the braille page ended before the terminator could be brailled.

24 

25 



Instructions: Begin each paragraph in cell 3 with runovers in cell 1.

PRACTICE 2C

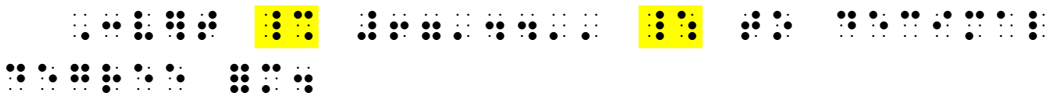
How many squares can you find on an 8 × 8 checkerboard? (*Hint:* There are more than 65 squares.) First determine how many 1 × 1 squares, then how many 2 × 2 squares, and so on, you can find.

12×11 is also the same as twelve elevens, or 11 + 11 + 11 + 11 + 11 + 11 + 11 + 11 + 11 + 11 + 11 + 11.

2.6 Consistency with Mathematical Symbols: Although UEB provides symbols for math notation and measurement, the transcriber's objective is to *maintain consistency in the appearance of the mathematical symbols throughout the transcription*. A symbol that is used in mathematical context is brailled in Nemeth Code, even in narrative context. This includes the symbols learned in **Lesson 1**—signs of operation, signs of comparison, monetary, percent, and prime signs.

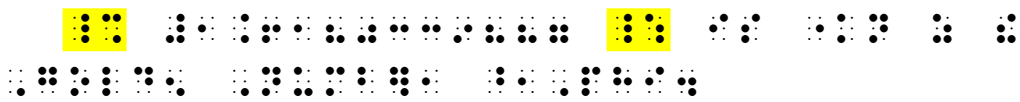
This example uses Nemeth Code symbols for prime signs representing minutes and seconds of arc. A switch to Nemeth Code is required.

Example 2.6-1 Convert 67'44'' to decimal degree form.



The next example uses the Nemeth Code decimal point in a mathematical numeral. A switch to Nemeth Code is required.

Example 2.6-2 1.6180339887 is known as the Golden Number, **Phi**.



PRACTICE 2D

With a 7% sales tax added, how much will a $6 bus ticket actually cost?

Percent to Decimal: To convert, divide the percent by 100: $78.6\% = 78.6 \div 100 = 0.786$.

Julio's lot measures 56' wide and 100' deep. The garden shed is 20'8'' back from the street.

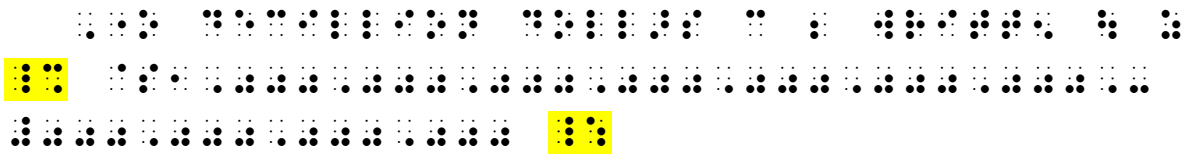
THE HYPHEN AND THE DASH

2.7 The Hyphen and the Dash As Punctuation: As punctuation marks, the hyphen and the dash will most often be found in literary context, in which case the UEB symbols and rules are applied. In mathematical context, Nemeth Code symbols are used. The hyphen is the same in either code. If a short dash occurs in mathematical context, it is brailled as shown below.

Hyphen	-	
Dash (short)	–	

2.7.1 A Hyphen May Divide a Long Numeral: If a long numeral cannot be contained on one braille line it is divided after a comma (if a comma is present) and a hyphen is inserted. *The line continuation indicator of UEB is not used in Nemeth Code.* When a numeral is divided between braille lines the numeric indicator must be used before the first digit of the numeral on the next line.

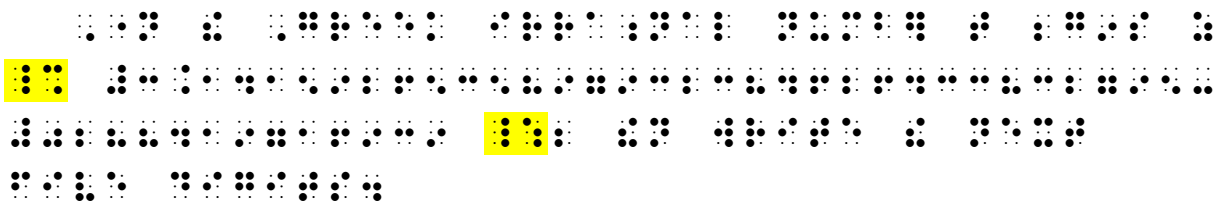
Example 2.7-1 One decillion dollars can be written out as
$1,000,000,000,000,000,000,000,000,000,000,000



Reminder: Because of the dollar sign, the number must be brailled in Nemeth Code. The opening switch indicator is placed on the same line as the expression since the whole number won't fit on one line.

2.7.1.a: If the long numeral does not contain a comma, the hyphen may be inserted after any digit.

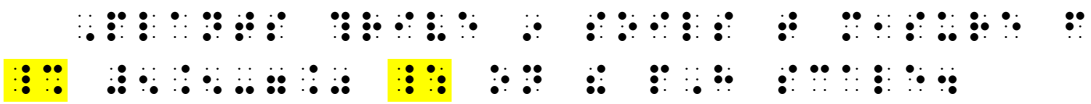
Example 2.7-2 Name the Greek irrational number that begins as 3.14159265358979323846264338327950288419716939; then write the next five digits.



Because the math number contains a decimal point it must be brailled in Nemeth Code. The semicolon applies to the sentence structure and is placed after the Nemeth Code terminator.

2.7.2 A Hyphen May Connect Numerals: If connected numerals are shown in mathematical context, the numeric indicator is <u>not</u> used after a hyphen connecting them.

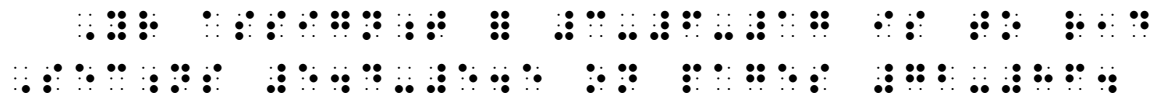
Example *2.7-3* Plants thrive in soils that measure from 5.5-7.0 on the pH scale.



Because the math numbers contain a decimal point they must be brailled in Nemeth Code.

2.7.2.a UEB Context: Numerals connected by hyphens such as dates, page number ranges, chapter or section ranges, etc. are not mathematical and therefore the rules of UEB are followed.

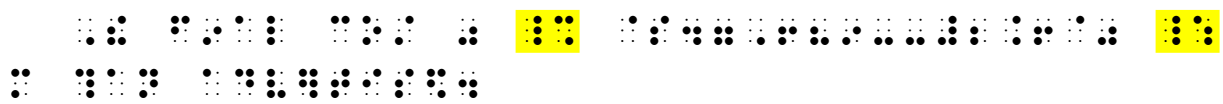
Example *2.7-4* Your assignment for 3-6-17 is to read Sections 5.4-5.5 on pages 72-86.



2.7.3 A Dash Separates: In mathematical context, the Nemeth short dash is used to represent a dash used as a mark of punctuation. A numeric indicator is required following a dash even though that number may not be preceded by a space.

The BANA Nemeth Code Technical Committee is discussing details regarding the spacing of the dash in Nemeth Code. This section will be completed after decisions are made.

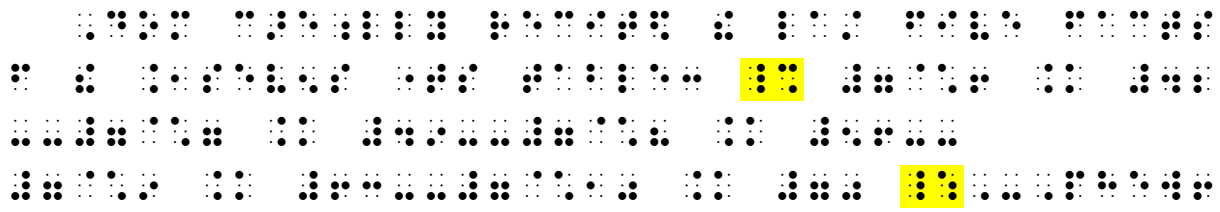
Example 2.7-4 The final cost was $47,689—2.6% more than advertised.



As punctuation, a dash implies a pause or a break in thought.

A dash may begin or end a line but the sign itself must not be divided.

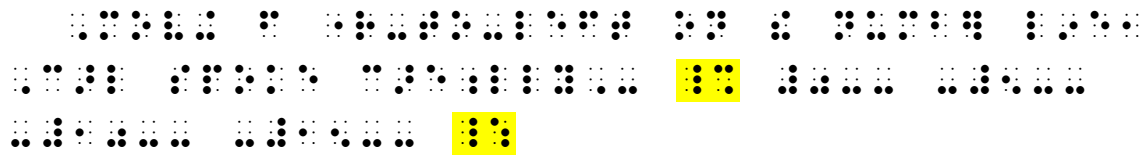
Example 2.7-5 Dom carefully recited the last five facts from the *sevens* times table: $7 \times 6 = 42$— $7 \times 7 = 49$— $7 \times 8 = 56$— $7 \times 9 = 63$— $7 \times 10 = 70$—Phew!



Notice that the UEB dash is used after the Nemeth Code terminator—unspaced, as in print, according to UEB rules. The Nemeth Code dash is used in the math.

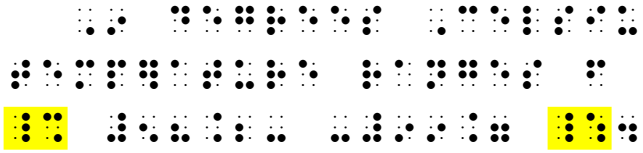
2.7.4 Hyphen, Dash, or Minus Sign? Read carefully to determine whether a symbol is a hyphen, a dash or a minus/negative sign. A space must come between a hyphen and a minus sign or between a dash and a minus sign in order to distinguish the similar constructions.

Example 2.7-6 Moving from right-to-left on the number line, Carl spoke carefully— 0— −5— −10— −15—



The UEB dash is used in the narrative; the Nemeth Code dash is used in the math. Although the math will fit all together on one line, it is four expressions, not just one, so the fullest available extent of the line is used.

Example 2.7-7 In degrees Celsius, Earth's temperature ranges from 58.2- –99.7.



Because a space must be inserted between the hyphen and the minus sign, a numeric indicator is required for –99.

PRACTICE 2E

Represent cents as decimals—$0.89—not like this—89¢.

Marcus' answer was 37¢—38¢ is the correct change.

In degrees Fahrenheit, Earth's temperature ranges from –126-136.

Expressed in seconds, the shortest meaningful interval of time (Planck Time) contains 46 decimal places: 0.00000000000000000000000000000000000000000000054 !

The BANA Nemeth Code Technical Committee is discussing details regarding the ellipsis and the long dash. The next section may change in the future.

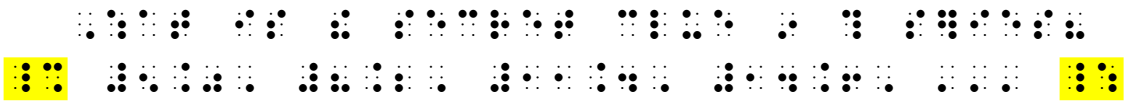
SIGNS OF OMISSION

2.8 General Use of Signs of Omission: In print, omission of mathematical or literary material may be shown by dots, a dash, a blank space, a question mark, or a combination of these or other signs devised by the author. Unless otherwise stated, the omission symbol used in braille should correspond to the print sign.

2.9 Ellipsis: A dot or series of dots in print representing an omission is an ellipsis. In literary context, the UEB ellipsis is used. In mathematical context, the ellipsis of the Nemeth Code is brailled.

Ellipsis	…	

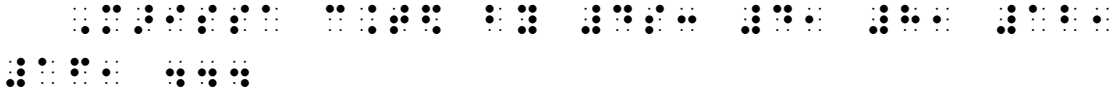
Example 2.9-1 What is the secret clue in this series? 5.0, 8.2, 11.4, 14.6, ...



The ellipsis represents further numbers in the series. Nemeth Code is used because the series requires the use of Nemeth Code for the decimals.

In literary context the UEB ellipsis is used.

Example 2.9-2 Marissa counted by 4s: 4, 8, 12, 16, ...



Even though this ellipsis represents missing numbers, because the freestanding numerals do not require a code switch, a UEB ellipsis is brailled.

2.10 Long Dash: When a dash is used to denote an omission in print, within mathematical context the Nemeth braille long dash is used.

Long Dash	_____	

Example 2.10-1 $79 \times 542 = _____$

In literary context the UEB underscore is used.

Example 2.10-2 Two and ___ are ten.

2.11 General Omission Symbol: When an omission in print is represented within switches by a question mark or by a blank space, the general omission symbol is brailled. The printed question mark can be standing alone or be shown in combination with hyphens or dashes.

General Omission Symbol	

Example 2.11-1 $92 \div 2 = ?$ $92 \div 2 = -?-$ $92 \div 2 =$ $92 \div 2 = __?__$

All four examples above are brailled as follows:

The general omission symbol follows the spacing rules of the material which it represents.

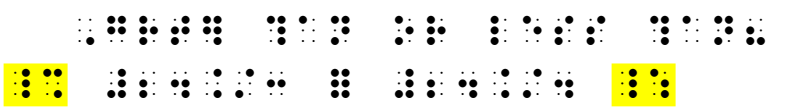
Example 2.11-2 $6 + ? = 15$

The omitted digit is unspaced from the plus sign and spaced from the equals sign.

Example 2.11-3 $1{,}5?4 + 200 = 1{,}734$

The omitted digit in the tens place is unspaced from the digits in the hundreds place and the ones place.

Example 2.11-4 Greater than or less than? $24 \div 3 \ ? \ 24 \div 4$



In the example above, the omitted comparison sign is spaced according to the rules. This is one mathematical expression and so it is forced to stay together on one line.

The number of general omission symbols used in braille must correspond to the number of omission signs used in print.

Example 2.11-5 $678 \times 27 = \underline{?}\ \underline{?}\ \underline{?}\ \underline{?}\ \underline{?}$

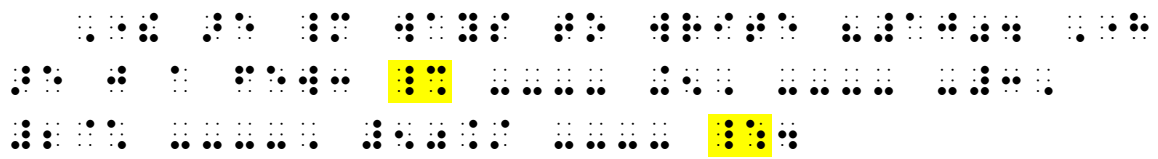


A 5-digit answer is implied by showing five omission signs.

2.12 Other Omission Symbols: Omissions are frequently shown in other ways besides a blank space, a question mark, a dash, or an ellipsis. Shape symbols used to show omissions (such as a square or a circle) are discussed in a later lesson. If the omission sign used in print has no braille equivalent in the code, the sign may be represented by a devised braille symbol or by a drawing. Devised symbols are discussed in a later lesson.

2.13 Spacing of the Ellipsis and Long Dash: The ellipsis and the long dash are preceded and followed by a space in most circumstances, even when next to an operation sign. The print copy may or may not show a space but to avoid misreading the ellipsis or the long dash as other math symbols Nemeth Code spacing rules must be followed.

Example 2.13-1 There are many ways to write "10". Here are just a few: $___ + 5$, $___ - 3$, $2 \times ___$, $50 \div ___$.



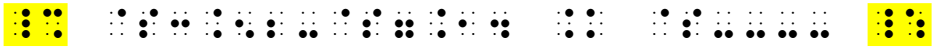
The first two expressions fit on line 2.

No space is left between an ellipsis or long dash and a related decimal point, monetary symbol, percent, or prime sign.

Example 2.13-2 $14.9 - 12.3 = ___.6$



Example 2.13-3 $\$3.52 - \$7.14 = \$\ _____$



Example 2.13-4 92% × .04 = … %



Example 2.13-5 24" = ___'



FORMAT

2.14 Paragraph Margins for Narrative Portions of Text (3-1): Narrative paragraphs in a document governed by Nemeth Code formatting begin in cell 3 with runovers in cell 1. Blocked paragraphing is not used in a technical transcription. *Reminder:* Nemeth Code formats are applied throughout a technical transcription, including portions transcribed in UEB. Each new paragraph will be indented two cells from the current margin.

PRACTICE 2F

Numerical Prefixes

Here are some examples of numeral prefixes: "Tetra-" means 4; "hexa-" means 6; "hepta-" means 7; "deca-" means 10; "dodeca-" means 12.

If a *dodecagon* is a 12-sided figure, a *dodecahedron* is a __-faced solid. A 10-faced solid is called a _____.

INTRODUCTION TO IDENTIFIERS

2.15 Terminology: Numbers or letters used to identify a problem, a mathematical expression, etc. are called *identifiers*. The material which immediately follows an identifier is called *itemized material.* Itemized subentries are referred to as *subdivisions* in the Nemeth Code.

FORMAT

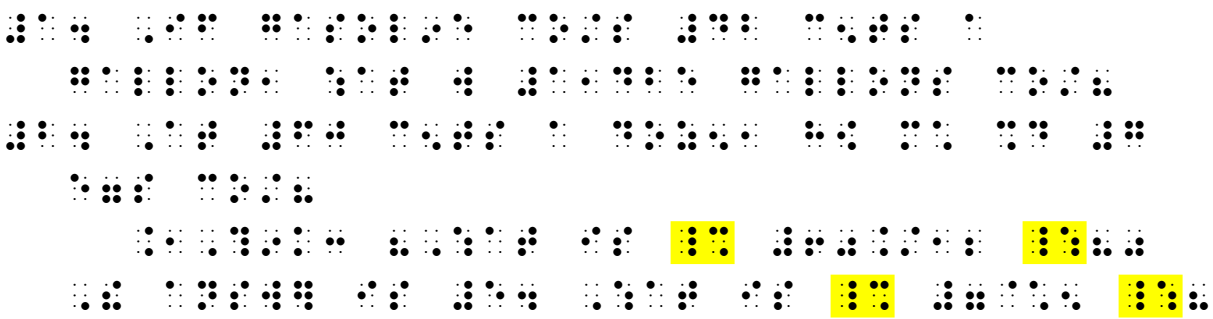
2.16 Margins for Itemized Material with No Subdivisions (1-3): The identifier begins in cell 1; runovers begin in cell 3. If the material contains more than one paragraph, each subparagraph begins in cell 5 with runovers in cell 3.

Example 2.16-1

1. If gasoline costs 42 cents a gallon, what will 1,425 gallons cost?

2. At 60 cents a dozen, how much should 7 eggs cost?

Think: "What is $60 \div 12$?" The answer is 5. What is 7×5 ?

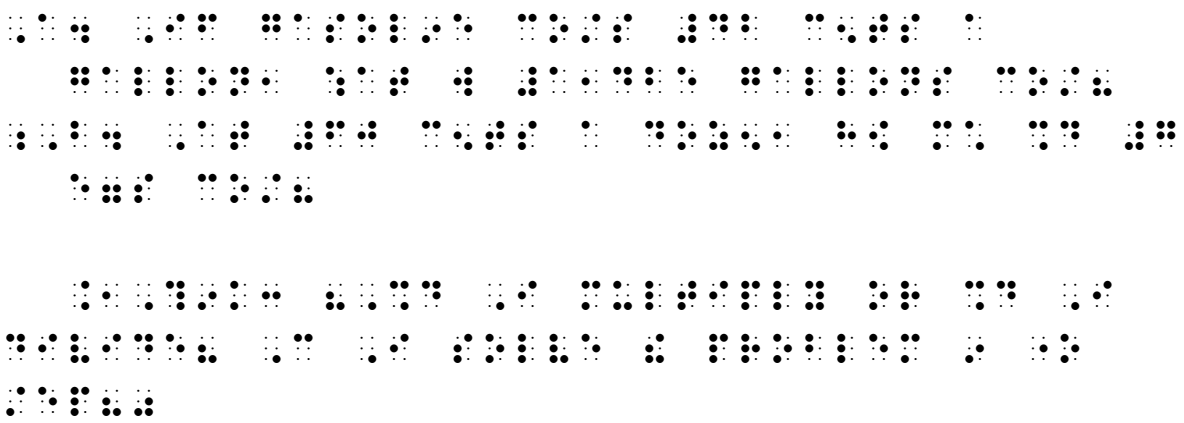


Use clues from the print layout as well as the information in the paragraph to determine if the new paragraph is indeed a continuation of the item or if it is separate material. Compare Example 2.16-1 above with the next example.

Example 2.16-2

A. If gasoline costs 42 cents a gallon, what will 1,425 gallons cost?
B. At 60 cents a dozen, how much should 7 eggs cost?

Think: "Should I multiply or should I divide? Can I solve the problem in one step?"



Notice that a blank line is left between itemized format and a 3-1 narrative paragraph.

Instructions: Format the first sentence as a narrative paragraph. Retain boldface for the paragraph heading. Insert a blank line between the narrative paragraph and the itemized material. Ignore typeface of the identifiers.

PRACTICE 2G

Refresher: These problems will test your skill with decimals.

1. First, addition: 42.6 + 37.23 + 3.215 =

2. Now subtract: 87.6 – 51.35 =

3. Try multiplication: 625.1 × 2.7 =

4. And now divide: 4.864 ÷ 3.2 =

FORMAT SUMMARY #1

Here is a summary of the Nemeth Code formats encountered so far in this course.

General Principles: When an item in a UEB transcription requires the use of Nemeth Code symbols, format rules of *The Nemeth Braille Code for Mathematics and Science Notation* are to be applied to the entire transcription including those portions transcribed in UEB. When a format is not specifically addressed in the Nemeth Code, the principles provided in *Braille Formats* should be followed.

Mathematical Expressions—Keep Together: If a mathematical expression will fit on one braille line within the current margins, it must not be divided between lines. The entire expression is brought down to the next line.

Paragraph Margins for Narrative Portions of Text (3-1): In a document governed by Nemeth Code formatting, an unitemized paragraph in explanatory portions of text begins in cell 3 and all runovers begin in cell 1. Blocked paragraphing is not used in a technical transcription.

Margins for Itemized Material with No Subdivisions (1-3): The identifier begins in cell 1; runovers begin in cell 3. If the material contains more than one paragraph, each subparagraph begins in cell 5 with runovers in cell 3.

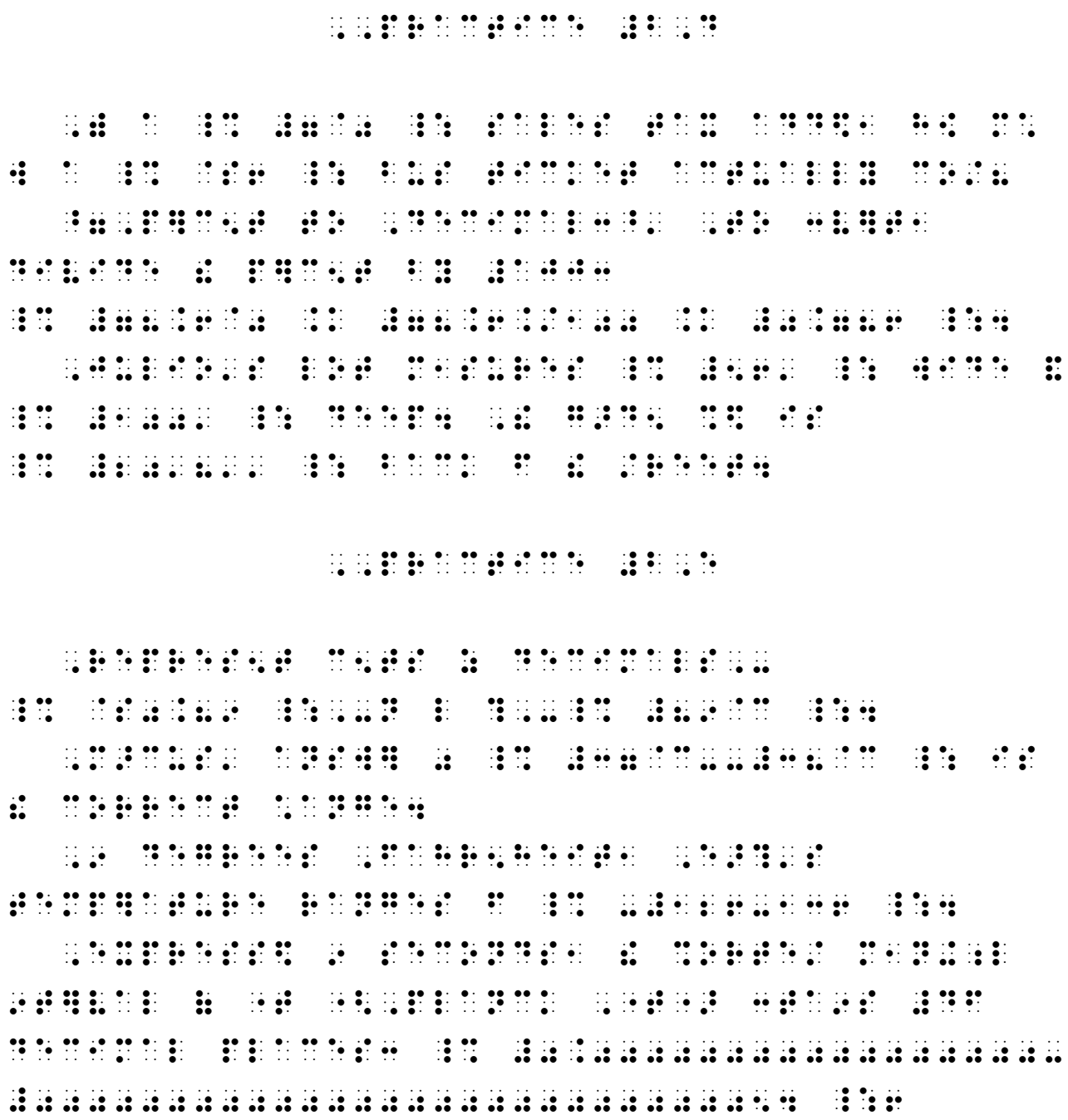
Placement of Code Switch Indicators in Narrative Context: Place the entire math expression and the two code switch indicators on the same braille line if they will fit within the current margins. If the entire string will not fit on one line, a switch indicator may stand alone on a line to allow the math expression to remain undivided. It is preferable to keep the switch indicators on the same braille page as the mathematical material to which they apply.

Blank Line: A blank line is left between itemized format and a 3-1 narrative paragraph and vice versa.

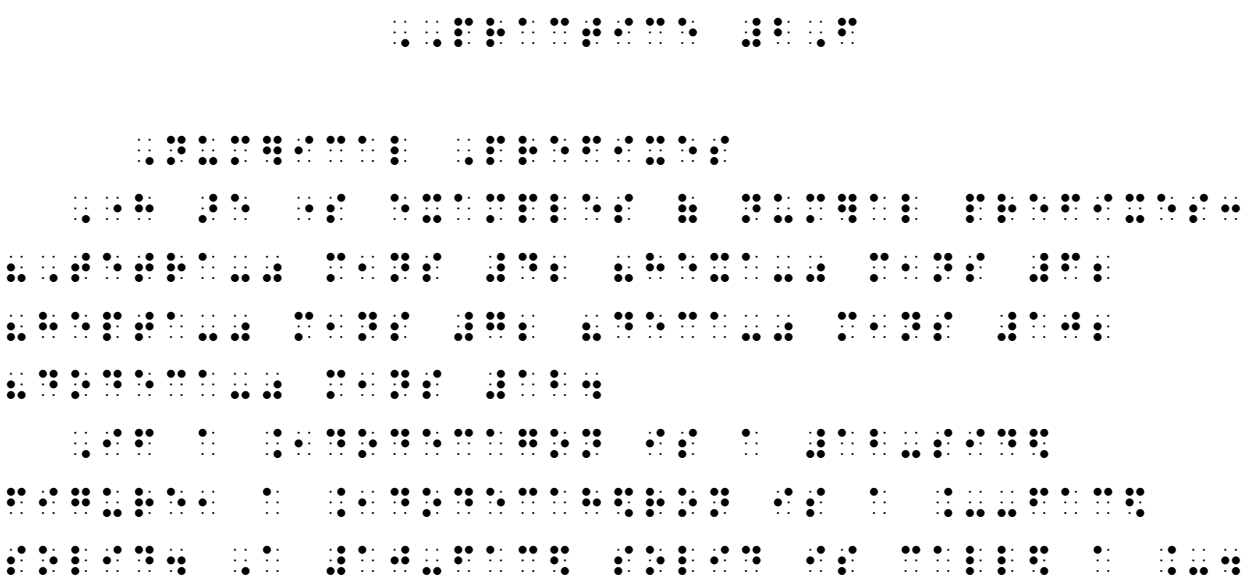
For further practice, see Appendix A—Reading Practice.

ANSWERS TO PRACTICE MATERIAL

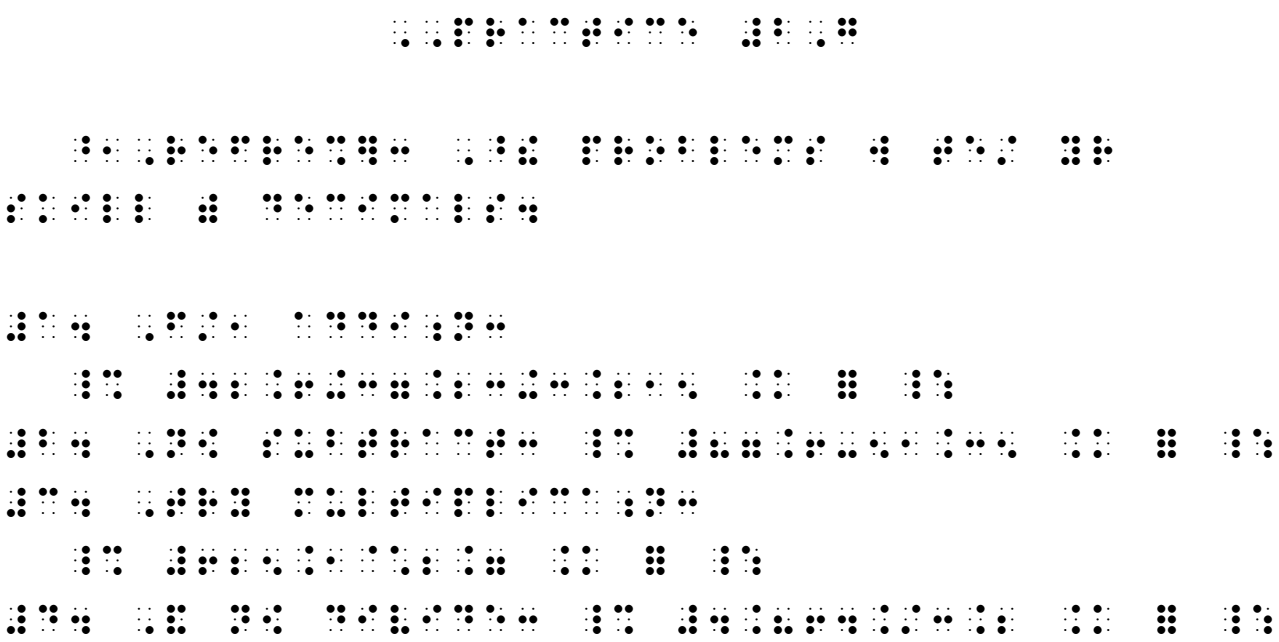
⠠⠠⠠



In the first paragraph, because the UEB dashes are unspaced (as in print), the code switch indicators may touch the dash.



PRACTICE 2F does not require any Nemeth Code switches. Rules of UEB are followed for the underscores.



EXERCISE 2

Exercise 2 will be available when this course is finished being written and is no longer "Provisional".

Proceed to Lesson 3.

LESSON 3

Read about this PROVISIONAL EDITION in the front matter to this book.
Check the NFB website periodically for updates to this lesson.

- *MORE ABOUT PUNCTUATION*
- *PUNCTUATION IN NEMETH CODE*
  - *The Punctuation Indicator*
- *INTRODUCTION TO SIGNS OF GROUPING*
  - *Code-Switching Considerations*
  - *Spacing with Signs of Grouping*
- *IDENTIFIERS, cont.*

FORMAT

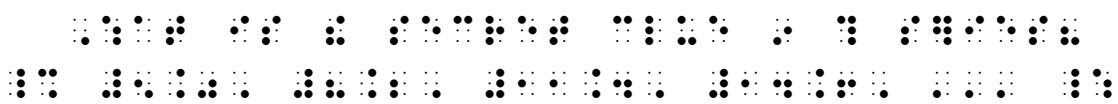
Keep Together—Hyphenated Expressions

Side-by-Side Layout

MORE ABOUT PUNCTUATION

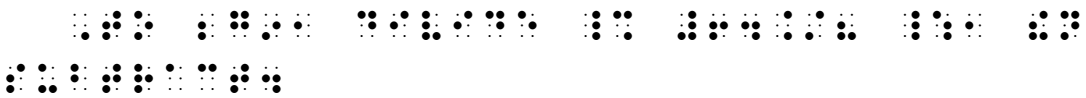
3.1 Punctuation Mode: Punctuation mode is determined by whether the punctuation occurs inside or outside of the Nemeth Code switches. The concept is simple – punctuation that occurs outside of the Nemeth Code switch indicators is brailled according to the rules of Unified English Braille; punctuation occurring within the Nemeth Code switch indicators is brailled according to the rules of the Nemeth Code. Take another look at this example from **Lesson 2**, noting that UEB punctuation is used for the question mark and the Nemeth Code comma and ellipsis are used in the mathematical portion.

Example 3.1-1 What is the secret clue in this series? 5.0, 8.2, 11.4, 14.6, ...



3.2 Spacing of UEB Punctuation and Code Switch Indicators: As shown in **Lesson 2**, punctuation that relates to the main text is placed outside of the switch indicators when the surrounding text is in UEB. There is no space between the terminator and the following punctuation.

Example 3.2-1 To begin, divide 64 ÷ 8, then subtract.



Note the use of the literary comma (dot 2) outside of the Nemeth terminator.

3.2.1 Spacing of the UEB Dash: Follow UEB spacing rules for the dash when the dash falls outside of the code switches. In the first example below, the dash is unspaced in print. In the second example, the dash is spaced in print. *Reminder:* The space before the Nemeth Code Terminator does not represent a space in print.

Example 3.2-2 Divide 64 ÷ 8—then subtract.

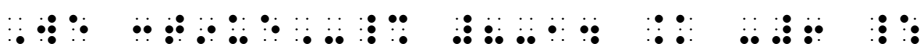


Example 3.2-3 Divide 64 ÷ 8 — then subtract.

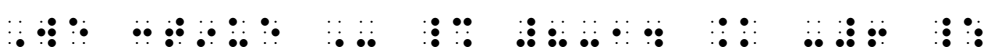


In the next example, the dash is unspaced in print. In the final example, the dash is spaced in print. *Reminder:* The space after the Opening Nemeth Code Indicator does not represent a space in print.

Example 3.2-4 We continue—8–14 = –6

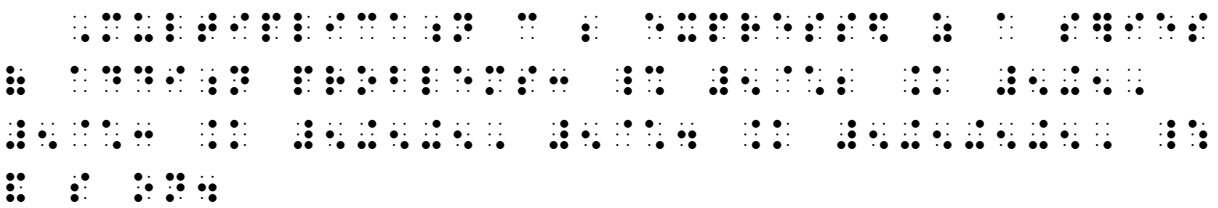


Example 3.2-5 We continue — 8–14 = –6



3.3 Nemeth Braille Punctuation: When punctuation occurs within mathematical material, Nemeth Code punctuation is used. Even when the punctuation belongs to the sentence structure, excessive code switching is avoided by using Nemeth Code punctuation. In the example below, the Nemeth Code comma (dot 6) is used within the series even though the comma itself is not "mathematical."

Example 3.3-1 Multiplication can be expressed as a series of addition problems:
$5 \times 2 = 5 + 5,\ 5 \times 3 = 5 + 5 + 5,\ 5 \times 4 = 5 + 5 + 5 + 5,$ and so on.



The last comma is placed inside the switch in order to continue the established pattern. It would not be wrong to place that comma after the Nemeth Code terminator as a dot 2 literary comma.

Instructions: Consider carefully where to place the code switch indicators and what kind of punctuation to use in these three sentences. Apply 3-1 Nemeth Code paragraphing.

PRACTICE 3A

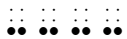
72813654, when written as 72 81 36 54, is obviously divisible by 9.

Write these numbers: 3.79, 500, –123, 2,000.88, –250,794. Now add them together.

Is the answer 4.0‰, or is it 4.0%?

PUNCTUATION IN NEMETH CODE

3.4 Background: So far we have looked at mathematical punctuation that is essentially unambiguous in mathematical context: the comma, decimal, the short and long dash, and the ellipsis. These symbols are not the same as their UEB counterparts. To review:

Mathematical Punctuation of the Nemeth Code	
Comma	
Decimal Point	
Dash (short)	
Long Dash	
Ellipsis	

When punctuation is brailled inside the Nemeth Code switches, clarification is required because many punctuation marks are represented by the same braille dots as Nemeth numerals and symbols.

Literary Punctuation Mark			**Could be misread in Nemeth Code as a ...**
Semicolon	;		*(numeral 2)*
Colon	:		*(numeral 3)*
Period	.		*(numeral 4)*
Exclamation Point	!		*(numeral 6)*
Question Mark	?		*(numeral 8)*
Quotation Marks	" "		*(numerals 8 and 0)*
Apostrophe	'		*(prime sign)*

The BANA Nemeth Code Technical Committee is discussing details regarding the treatment of "single" quotation marks in Nemeth context.

Throughout this course, isolated examples which illustrate Nemeth Code rules are marked with this icon: ⋙
Nemeth Code switch indicators are omitted from these isolated examples.

The Punctuation Indicator

3.5 Role of the Punctuation Indicator: A punctuation indicator is placed before one or more of the punctuation marks listed in the box above when such punctuation occurs in mathematical context and the punctuation is not preceded by a space. Use of the punctuation indicator assures that the braille is read as punctuation and is not misread as a mathematical symbol.

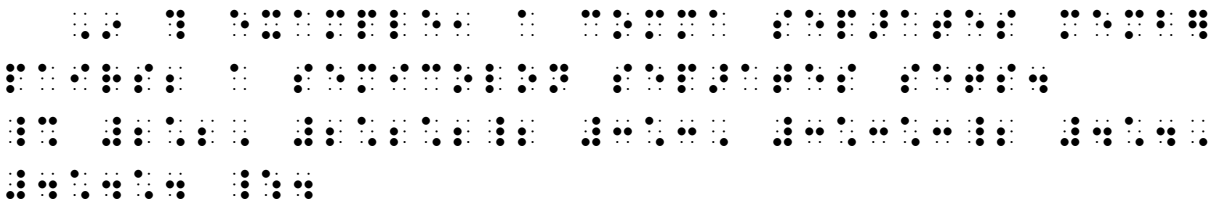
Punctuation Indicator	⠸

⋙ 2 · 2 · 2;



The punctuation indicator prevents the semicolon from being misread as the numeral 2.

Example 3.5-1 In this example, a comma separates member pairs; a semicolon separates sets. 2 · 2, 2 · 2 · 2; 3 · 3, 3 · 3 · 3; 4 · 4, 4 · 4 · 4.



Although the opening Nemeth Code indicator will fit on line 2, the first math expression will not. In general, keep the switch indicators on the same line as the mathematics to which they apply. The final period applies to the entire sentence. It is placed <u>after</u> the Nemeth Code terminator.

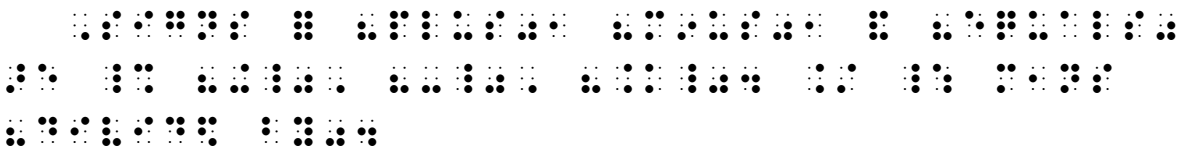
When there are two or more punctuation marks in a row following a mathematical item, only one punctuation indicator is used. If a comma is one of the punctuation marks the mathematical comma is brailled.

⋙ "−", "=".



The punctuation indicator prevents the closing quotation mark from being misread as the numeral 0 and the closing quotation mark and period from being misread as 04.

Example 3.5-2 Signs for "plus", "minus", and "equals" are "+", "−", "=". ÷ means "divided by".



A punctuation indicator is not needed for the opening quotation marks because they are preceded by a space and so will not be misread as numerals.

3.5.1 Comparison Sign in Quotes: Note that, although a space is generally left between a comparison sign and an expression which precedes or follows it, a space is not left between a comparison sign and a punctuation mark which applies to it. Observe how this applies to the equals sign in the example above.

3.5.2 Number in Quotes: A numeric indicator is required when a numeral immediately follows an opening quotation mark.

⋙ "40+10=50"

Example 3.5-3 Add 48+13 in your head. Think: "40+10=50" ... "8+3=11" ... "50 + 11 = 61".

Even though this ellipsis is not mathematical (it indicates pausing while thinking), the ellipsis of the Nemeth Code is used because it is inside the Nemeth Code switches. The final period applies to the entire sentence. It is placed after the Nemeth Code terminator.

The BANA Nemeth Code Technical Committee is discussing details regarding the treatment of digital clock time. Correct transcriptions of the examples in this section will be confirmed after approval is obtained. For now, provisional solutions are provided.

3.5.3 Clock Time: Digital clock time expressed with the use of a colon may be brailled in UEB or in Nemeth Code, depending on the context. In mathematical context, a punctuation indicator precedes the colon to prevent misreading the colon as the number 3. A numeric indicator is then required to set the reading mode back to "numbers."

3:30

UEB:

Nemeth:

Recall from **Lesson 2** that when connected numerals are shown in mathematical context, the numeric indicator is not used after a hyphen connecting them.

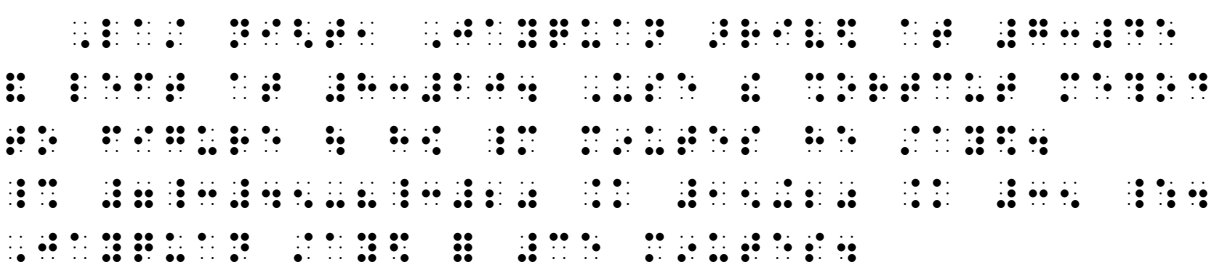
7:45-8:20

UEB:

Nemeth:

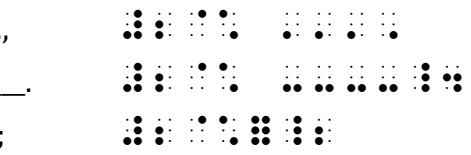
Example 3.5-2 Last night, Jayquan arrived at 7:45 and left at 8:20. Use the shortcut method to figure out how many minutes he stayed. 7:45-8:20 = 15+20 = 35. Jayquan stayed for 35 minutes.

PROVISIONAL SOLUTION



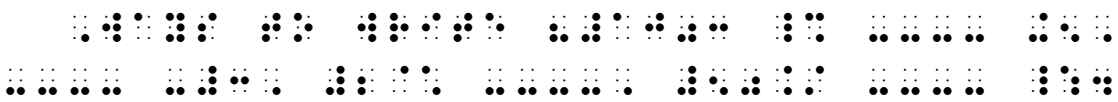
3.6 Punctuation with Omission Signs: When a sign of omission represents omitted mathematical content the appropriate omission symbol of the Nemeth Code is brailled and it is punctuated mathematically. The punctuation is unspaced from the omission symbol. (There is one exception noted after the examples.)

- 2 × ...,
- 2 × ___.
- 2 × ?;



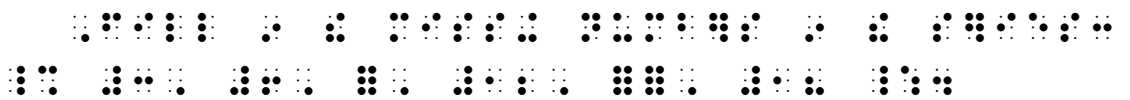
Take another look at this example from **Lesson 2**. Notice the use of the mathematical comma with the long dash.

Example 3.6-1 Ways to write "10": ___ + 5, ___ − 3, 2 × ___, 50 ÷ ___.



*Reminders from **Lesson 2**: The long dash and the ellipsis are spaced away from the multiplication symbol even though operation signs are usually unspaced.*

Example 3.6-2 Fill in the missing numbers in the series: 3, 6, ?, 12, ??, 18.

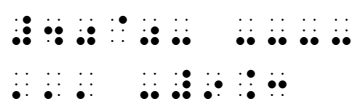


Switch Decision: The general omission symbol is a Nemeth Code symbol. The use of that symbol supports the decision to use Nemeth Code to braille the entire series even though the numerals do not necessarily require a switch.

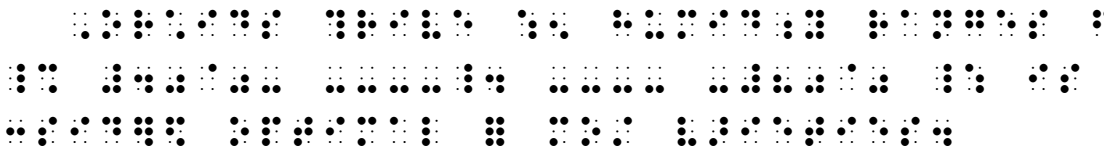
Reminder: The general omission symbol is spaced according to rules of the item it represents (in this case, a numeral). The same number of omission symbols shown in print should be used in braille.

3.6.1 SPACING EXCEPTION—The Hyphen: Although no space is left between an ellipsis and a related punctuation mark or between a long dash and a related punctuation mark, if the punctuation mark is a hyphen then a space is required.

- 40% - ___
- ... -9.3



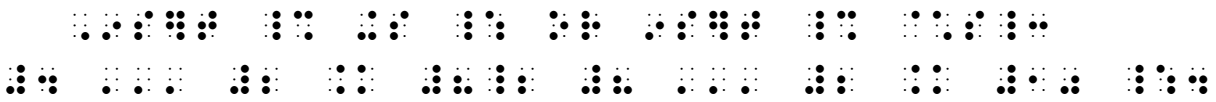
Example 3.6-3 Orchids thrive when humidity ranges from 40% - ___. ___ -80% is considered optimal for most varieties.



3.7 Punctuation and Spacing of Plural or Possessive Endings: When "s" or apostrophe-s is attached to a mathematical expression it becomes part of that expression. This means that—in mathematical context—the "s" is punctuated mathematically.

⋙ ×s: 

Example 3.7-1 Insert +s or insert ×s: $4 \ldots 2 = 8;\ 8 \ldots 2 = 10.$

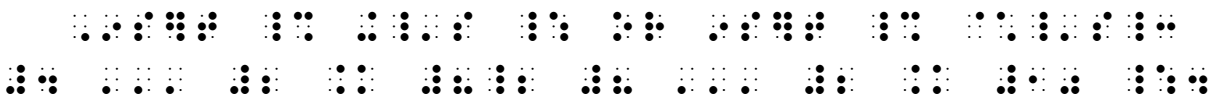


Reminder: A Nemeth Code format rule states that a mathematical expression must not be divided between braille lines if it will fit on one line within current margins. The expression "4 ... 2 = 8" must not be divided and so it begins on line 2.

A punctuation indicator is required before the apostrophe in a possessive ending "apostrophe-s". Even so, a punctuation indicator is still required before a punctuation mark that immediately follows the "s". Compare this similar example to the previous one.

⋙ ×'s: 

Example 3.7-2 Insert +'s or insert ×'s: $4 \ldots 2 = 8;\ 8 \ldots 2 = 10.$



A punctuation indicator is needed both before the apostrophe <u>and</u> before the colon in ×'s:.

The ending is unspaced from the symbol even if the symbol normally requires a space.

⋙ =s 

⋙ ='s 

Instructions: Here is a list of isolated mathematical items and punctuation marks. Following the centered heading and the blank line, braille an opening Nemeth Code indicator in cell 1 and continue with the first item in the list on the same line. Stay in Nemeth Code throughout the practice (braille the clock time in Nemeth Code). Begin each line in cell 1; begin any runovers in cell 3. Braille a Nemeth Code terminator after the completion of the last item in the list.

PRACTICE 3B

+, − ; ×, ÷ .

+'s, −'s, ×'s, ÷'s; =s, >s, <s.

" :: "

5.1, 6.22, 7.333; $8.44, $9.55; $10.66.

10:45-11:25

−16 > −___ ; 16 < ___ .

$1,400 < £ _?_

5'3" ..., 6'1"—6'2" ..., 7'0".

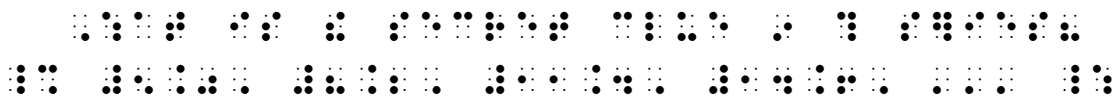
"8 · 3 = 3 · 8"

3.8 SUMMARY—Situations That Do Not Require a Punctuation Indicator: Previous examples are revisited to illustrate situations in which the punctuation indicator is not needed.

— The mathematical comma never requires a punctuation indicator.

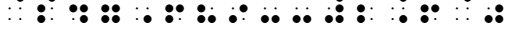
⋙ 5.0, 

Example 3.8-1 What is the secret clue in this series? 5.0, 8.2, 11.4, 14.6, ...

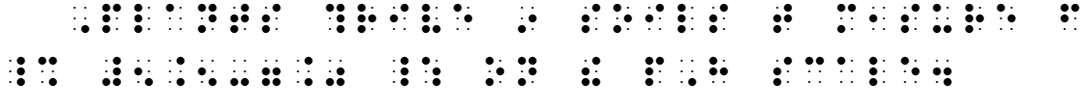


— The punctuation indicator is not used before a hyphen or a dash.

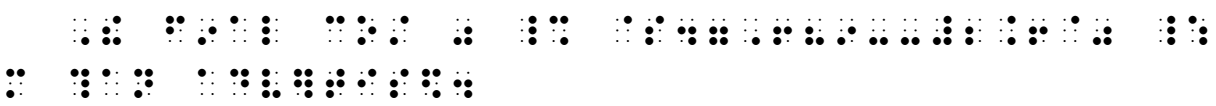
⋙ 5.5-7.0 

⋙ $47,689—2.6% 

Example 3.8-2 Plants thrive in soils that measure from 5.5-7.0 on the pH scale.



Example 3.8-3 The final cost was $47,689—2.6% more than advertised.

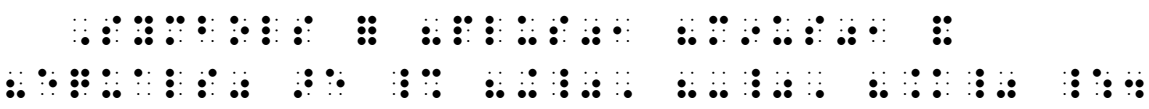


— A punctuation indicator is not needed if the first character following a space is a punctuation mark or if the punctuation mark begins on a new line.

⋙ "+", "−"

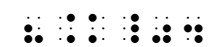


Example 3.8-4 Symbols for "plus", "minus", and "equals" are "+", "−", "=".

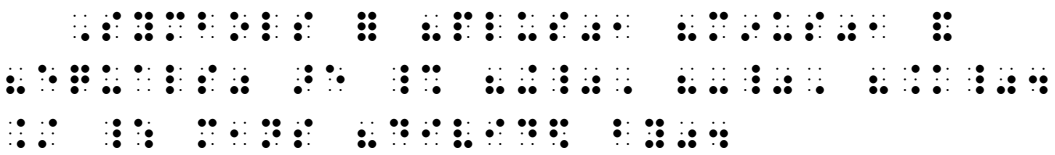


— In a sequence of punctuation marks following a mathematical expression, the punctuation indicator precedes only the first punctuation mark.

⋙ "=".

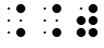


Example 3.8-5 Symbols for "plus", "minus", and "equals" are "+", "−", "=". ÷ means "divided by".



INTRODUCTION TO SIGNS OF GROUPING

3.9 Definition: In mathematical context, symbols such as parentheses, braces, and brackets are not considered to be punctuation; they are classified as *signs of grouping*. Here are some grouping signs commonly encountered in technical material.

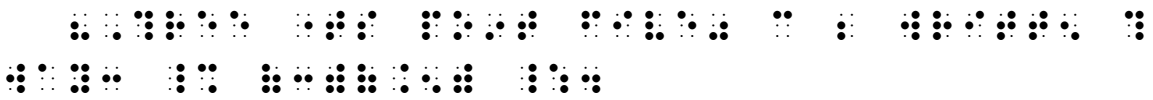
Opening Parenthesis	(	
Closing Parenthesis	)	
Opening Brace	{	
Closing Brace	}	
Opening Bracket	[	
Closing Bracket	]	
Opening Angle Bracket	⟨	
Closing Angle Bracket	⟩	
Vertical Bar	|	
Double Vertical Bar	‖	

More signs of grouping will be presented in later lessons.

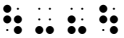
3.10 Signs of Grouping with Numerals: The numeric indicator is not used before a numeral or before a decimal point that immediately follows an opening grouping sign.

⋙ (3)(.5) 

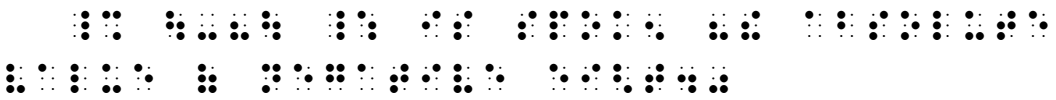
Example 3.10-1 "Three times point five" can be written this way: (3)(.5).



The numeric indicator is not used after a minus sign that immediately follows an opening grouping sign.

⋙ |−8| 

Example 3.10-2 |−8| is spoken "the absolute value of negative eight."



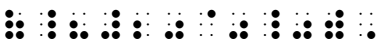
Lacking an opening grouping sign, the numeric indicator is required when the numeral is preceded by a space or begins a braille line. Compare:

⋙ 1) 

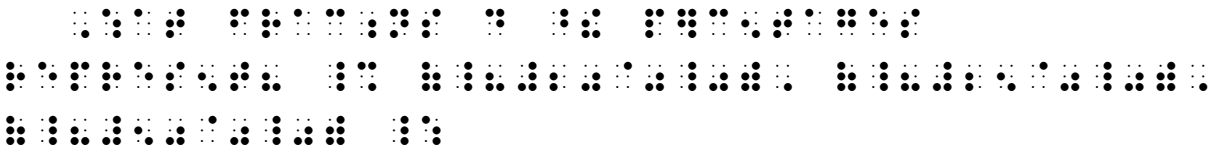
⋙ (1) 

3.11 Punctuation with Grouping Symbols: Grouping symbols of the Nemeth Code are mathematical symbols and therefore must be punctuated mathematically inside of the switch indicators. Thus, except for the mathematical comma, hyphen, and dash, a punctuation indicator must be used before a punctuation mark following a sign of grouping.

⋙ {−5}, (0), [+5]. 

⋙ ("20%"), 

Example 3.11-1 What fractions do these percentages represent? ("20%"), ("25%"), ("50%")



Reminder: A numeric indicator is required when a numeral immediately follows an opening quotation mark.

3.12 Nested Grouping Symbols: When two or more grouping signs follow one another the outer set may be printed using a taller size in order to visually distinguish the nested groupings. The braille transcription does not differentiate between the sizes—regular grouping symbols are brailled.

Example 3.12-1 $\big((4+7)-(7+4)\big)$

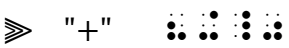


In print, the first and last parentheses are taller than the others.

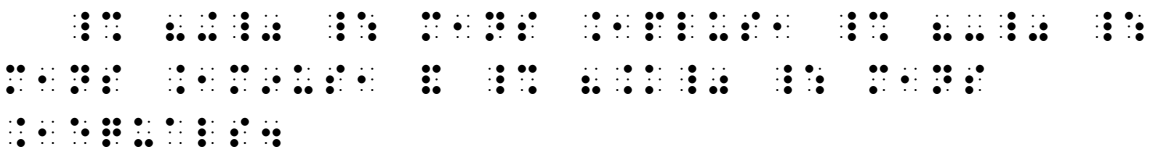
Note that this does not apply to the vertical bar since the double vertical bar is a distinct mathematical symbol. Nested vertical bars are discussed in a later lesson.

Code-Switching Considerations

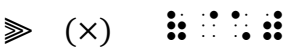
3.13 "Enclosed" Technical Material: When parentheses, brackets, braces, or quotation marks enclose isolated technical material, recommended practice is to braille the paired punctuation inside the code switches.



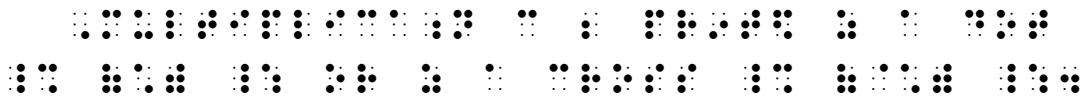
Example 3.13-1 "+" means *plus,* "–" means *minus,* and "=" means *equals.*



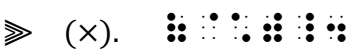
Recall that UEB symbols are not used inside of the Nemeth Code switches. Even when a sign of grouping is functioning as a punctuation mark rather than a mathematical symbol, the Nemeth Code grouping symbols are used inside the switches.



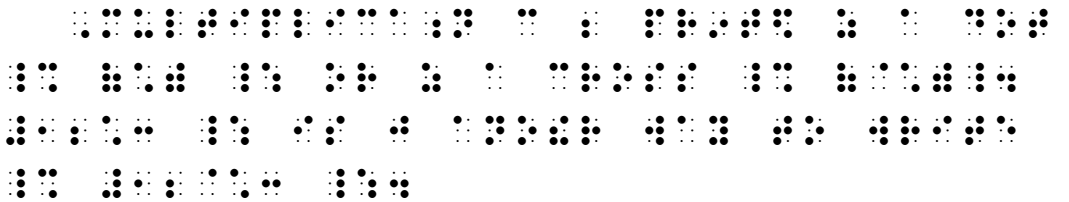
Example 3.13-2 Multiplication can be printed as a dot (·) or as a cross (×).



3.13.1 Punctuation Following a Sign of Grouping: Grouping signs of the Nemeth Code are punctuated mathematically. This rule is illustrated by expanding the previous example, continuing in Nemeth Code following the period.

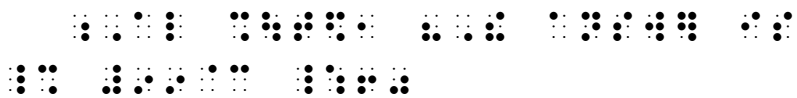


Example 3.13-3 Multiplication can be printed as a dot (·) or as a cross (×). 12 · 3 is just another way to write 12 × 3.



3.14 The Bigger Picture: On the other hand, if the punctuation applies to a larger phrase where UEB is being used, braille the paired punctuation marks in UEB.

Example 3.14-1 Al shouted, "The answer is 99¢!"

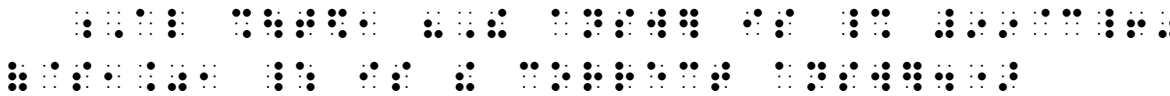


Example 3.14-2 ($1.01 is the correct answer.)



However, do not switch out of Nemeth Code merely for the sake of a punctuation mark. The practice of avoiding excessive code switching takes precedence over symmetrical pairing of punctuation marks.

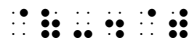
Example 3.14-3 Al shouted, "The answer is 99¢!" ($1.01 is the correct answer.)

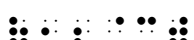


If you ask yourself, "Does the punctuation apply only to the math expression, or does it apply to the whole phrase (not just the math)?" and "Is Nemeth Code in effect both before and after the punctuation?" the best transcription interpretation will usually be clear.

Spacing with Signs of Grouping

3.15 Spacing Inside of the Grouping Signs: Unless other rules apply, no space is left between an opening or a closing sign of grouping and the material which it encloses.

⋙ $[-4]$ 

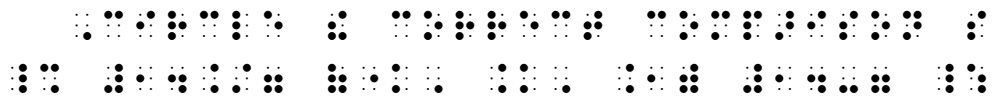
⋙ $(12¢)$ 

This includes symbols which usually require spacing—no space is left between a dash, an ellipsis, a sign of comparison, or any other symbol and its sign of grouping.

⋙ $\|\ldots\|$ 

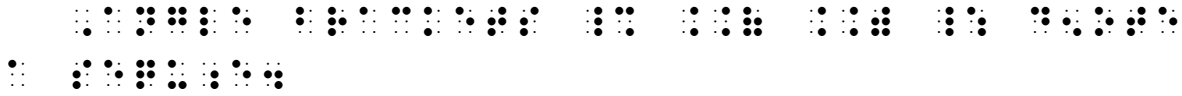
⋙ $(<, =, >)$ 

Example 3.15-1 Circle the correct comparison sign: $14 \div 7$ $(<, =, >)$ $14 - 7$



3.15.1 SPECIAL CASE—When a space is printed between an opening and a closing sign of grouping and that blank space does not represent an omission, the space between the grouping signs is included in the braille transcription.

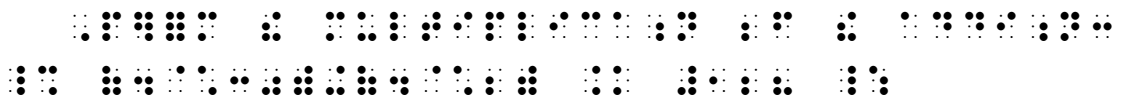
Example 3.15-2 Angle brackets ⟨ ⟩ denote a sequence.



Context will help you determine whether the print sign is an angle bracket or a "less than" or a "greater than" symbol.

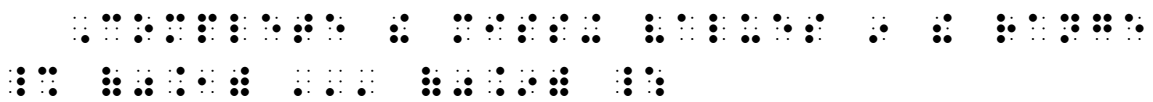
3.16 Spacing Outside of the Grouping Signs: The spacing before and after an enclosed expression is subject to the spacing rules for the signs which precede or follow the enclosure.

Example 3.16-1 Perform the multiplication before the addition: $(4 \times 30) + (4 \times 2) = 128$



Operation signs are unspaced; comparison signs are spaced.

Example 3.16-2 Complete the missing values in the range (0.1) ... (0.9)



The ellipsis is spaced.

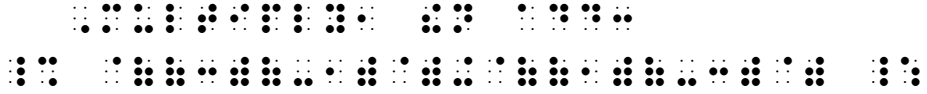
No space is left between an enclosed expression and a numeral when these items are part of the same expression unless other spacing rules apply. These items often appear to be spaced in print.

Example 3.16-3 Does $5\,(9 + 7) = (5 \cdot 9) + 7$?



No space is left between an enclosed expression and another sign of grouping when these items are part of the same expression unless other spacing rules apply. These items often appear to be spaced in print.

Example 3.16-4 Multiply, then add: $[(3)(-1)] + [(1)(-3)]$



Instructions: Format each line in print as a 3-1 paragraph in braille.

PRACTICE 3C

Is $3(-2.5) + (-4)$ the same as $3(-2.5 + (-4))$?

Use a number line to illustrate this addition problem: $[-4 - (-1)] + [-1 - (-3)]$.

$7 + (-3) + (-4) = ?$

$8 + |(-2) + (-3)| = ?$

$|2(-7.5)| + 3.2(2) = ?$

The **multiplicative identify** [sic] property is illustrated: $(83)(1) = 83$.

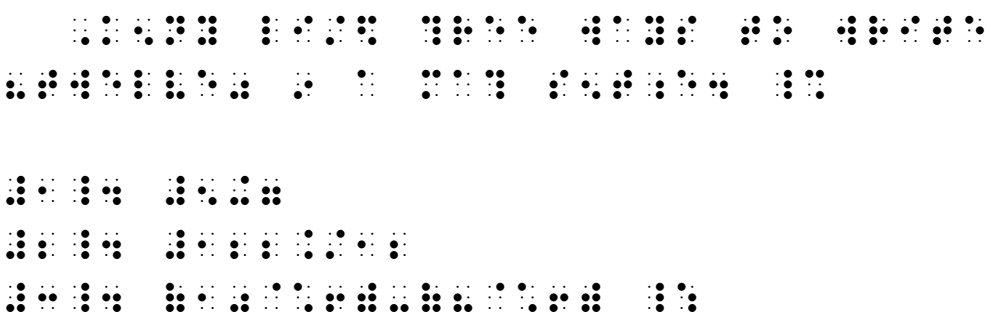
A pair of braces "{ }" indicates a set.

What is the meaning of the symbol "||" in "The answer is $\|3.1\|$"?

3.17 Code Switching and Identifiers—Maintaining Alignment: In itemized sections, the identifiers should each begin in the same cell. You have learned one format for itemized text: 1-3. In that layout, each identifier should begin in cell 1. By placing the opening Nemeth Code indicator at the end of the line of text that precedes a list of numbered items you can ensure that all identifiers begin in the same cell.

Example 3.17-1 Kenny listed three ways to write "twelve" in a math sentence.

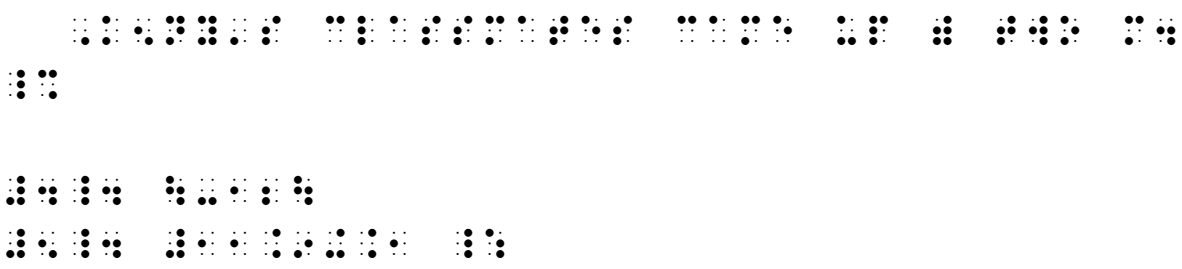
1. $5+7$
2. $144 \div 12$
3. $(10 \times 6) - (8 \times 6)$



If there is no room for the opening Nemeth Code indicator at the end of the line that precedes the identified material, place it on the next line in the appropriate runover cell. The code switch indicator does not take the place of the blank line that must precede the itemized material.

Example 3.17-2 Kenny's classmates came up with two more.

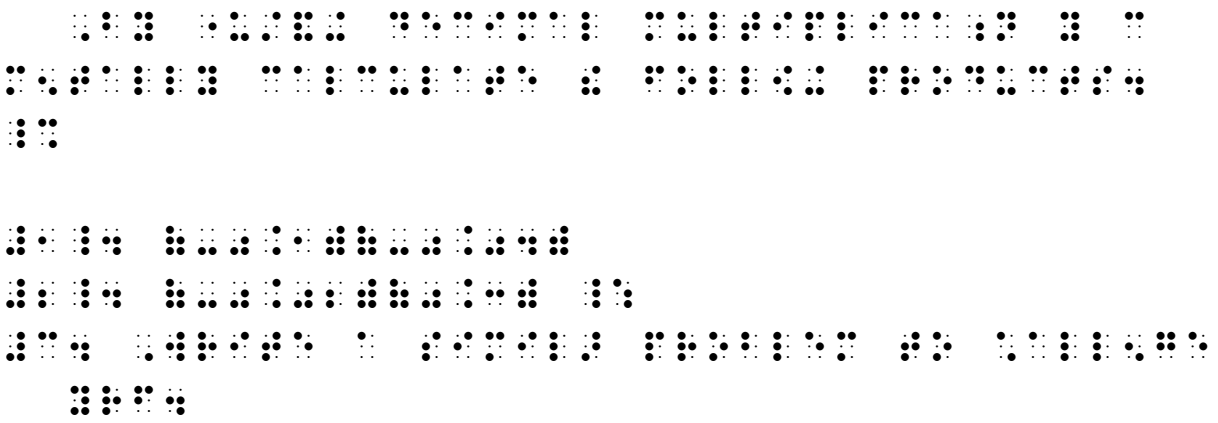
4. $|-12|$
5. $11.9 + .1$



3.18 Context Decisions and Code Switching: Even though an identifier is not part of the math, there is no need to switch out of Nemeth Code simply to braille an identifier. Notice in the two examples above that all of the itemized expressions are in Nemeth Code as well as their identifiers. If code switching is required within the itemized portion, identifiers are transcribed according to the rules for the code in use at the time regarding upper-cell or lower-cell numerals, use of indicators, and construction of related punctuation or grouping symbols. All identifiers in a section do not need to be transcribed in the same code.

Example 3.18-1 By understanding decimal multiplication you can mentally calculate the following products.

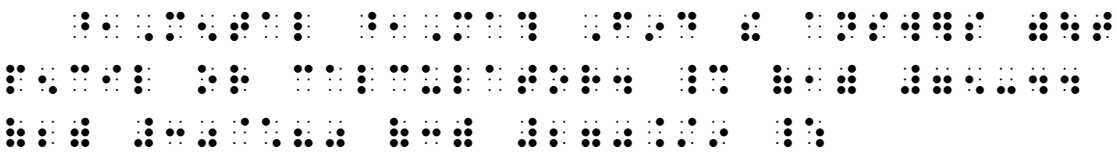
1. $(-0.1)\,(-0.04)$
2. $(-0.02)\,(0.3)$
3. Write a similar problem to challenge yourself.



The identifiers can be brailled in either code. Note how the switch indicators are placed where they are least intrusive to the flow of text and do not interfere with the alignment of the first cell of the identifiers.

Embedded identifiers may be brailled in Nemeth Code to avoid excess code switching.

Example 3.18-2 **Mental Math** Find the answers without pencil or calculator. $(1)\ 75-44\ \ (2)\ 30\times 80$ $(3)\ 270\div 9$

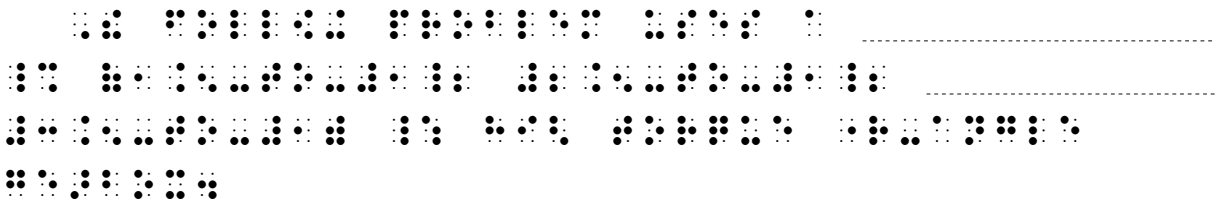


PRACTICE 3D

1) The box is 2′4″ in height.
2) $5'$ is the same as $60''$
3) $12' + 15'' = 13'3''$

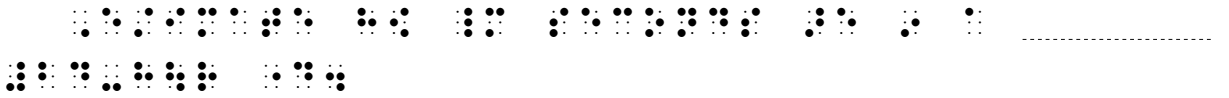
3.19 ***Keep Together*—Hyphenated Expressions:** A hyphenated expression containing one or more mathematical components must not be divided between braille lines.

Example 3.19-1 The following problem uses a (1.5-to-1; 2.5-to-1; 3.5-to-1) high torque right-angle gearbox.



Because Nemeth Code format rules are applied throughout a technical transcription, this rule also applies in UEB text when a numeral and a word are connected by a hyphen.

Example 3.19-2 Estimate how many seconds are in a 24-hour day.

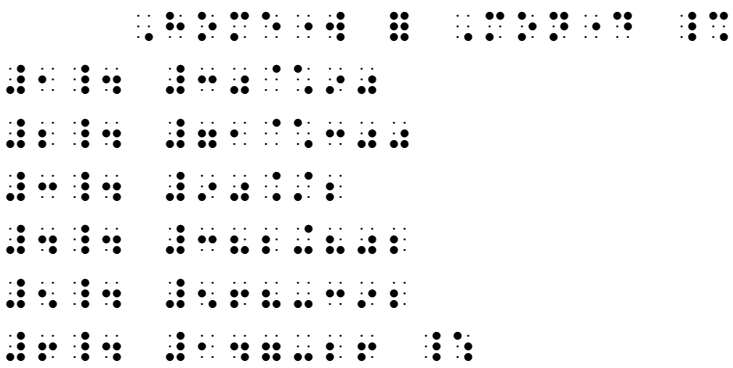


3.20 Margins for Itemized Material with No Subdivisions—Side-by-Side Layout: When the print copy arranges itemized material side-by-side across the page and there are no subdivisions, the braille format is changed so that all identifiers start in cell 1.

Example 3.20-1

Homework for Monday

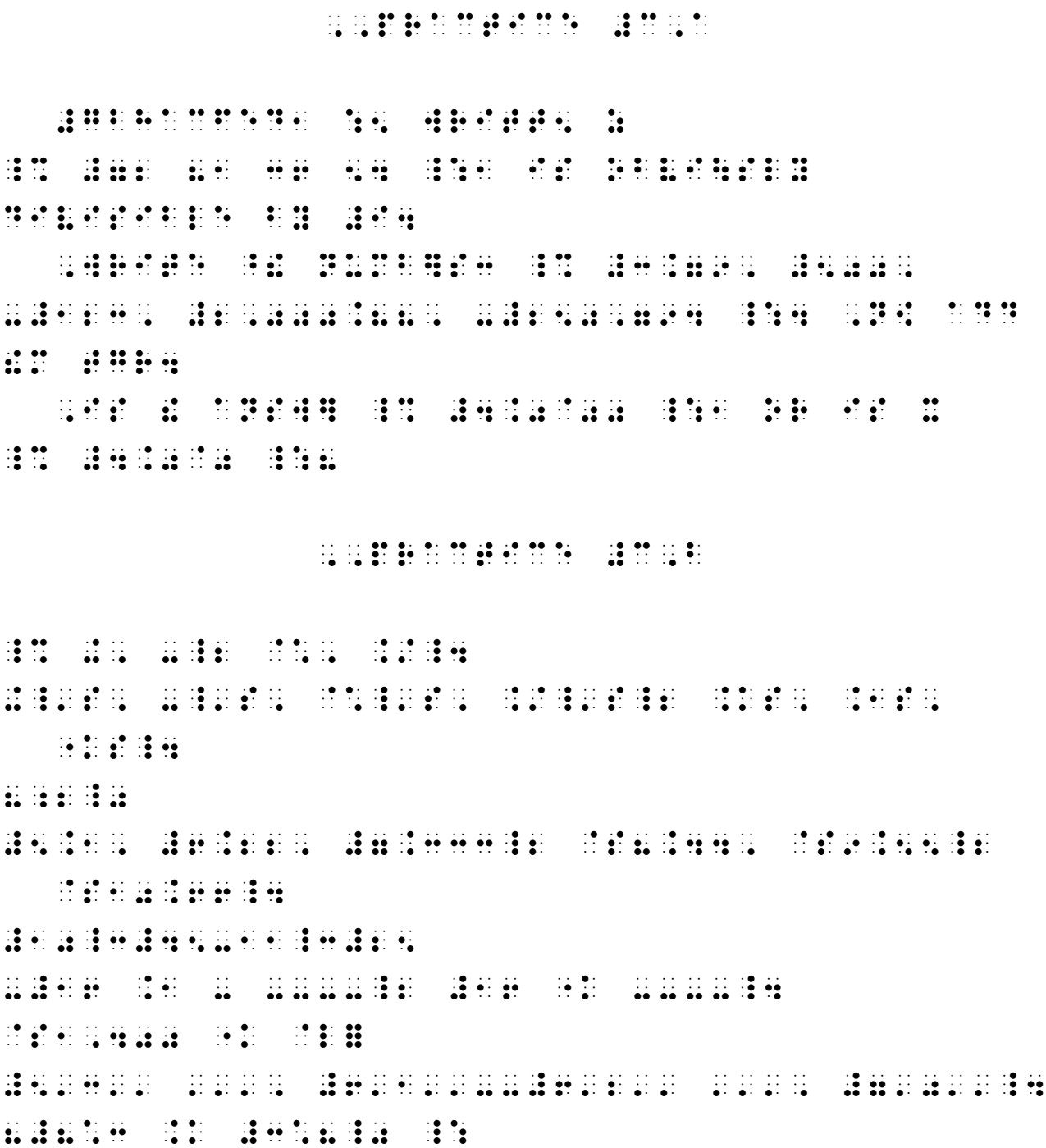
1. 30×90
2. 71×300
3. $90 \div 2$
4. $382 + 802$
5. $568 - 392$
6. $147 - 26$



Line 1: Note that the opening Nemeth Code indicator is placed at the end of the cell-5 heading. A discussion about code switches and headings occurs in a later lesson.

For further practice, see Appendix A—Reading Practice.

ANSWERS TO PRACTICE MATERIAL



LESSON 4

Read about this PROVISIONAL EDITION in the front matter to this book.
Check the NFB website periodically for updates to this lesson.

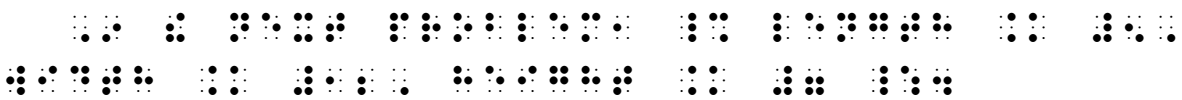
- *WORDS*
  - *Introduction to Abbreviations*
  - *Single-Word Switch Indicator*
- *Switch Indicators at Page Turns, cont.*
- *LETTERS*
  - *Introduction to the English Letter Indicator*
  - *Mathematical Letter Combinations*

FORMAT
Keep Together
FORMAT SUMMARY #2

WORDS

4.1 Words in Mathematical Context: When words are part of an equation or math expression the words are included in the technical notation—that is, the whole expression is placed inside the Nemeth switches. No contractions are used within Nemeth switches. Spacing rules of the Nemeth Code are followed.

Example 4.1-1 In the next problem, length=5, width=12, height=7.



The words are brailled without contractions. Nemeth Code requires a space before and after the comparison signs (equals signs) regardless of print spacing.

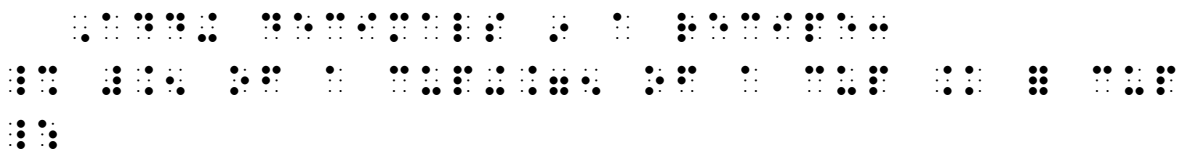
In the next example, words are substituted for values in a formula. The words are part of the math expression and are brailled without contractions. The division symbol is unspaced from the words according to Nemeth Code rules for spacing of operation signs.

Example 4.1-2 Dividend ÷ Divisor = Quotient



As part of a math problem expressed in symbols and words, the words are included in the switch.

Example 4.1-3 Adding decimals in a recipe: .5 of a cup + .75 of a cup = _?_ cup



The operation sign (+) is unspaced; the comparison sign (=) is spaced, according to the rules of the Nemeth Code.

4.1.1 Capitalization: Each fully capitalized word in technical context is preceded by the double capitalization indicator of the Nemeth Code. The UEB capitalized passage indicator is not used in Nemeth context.

Double Capitalization Indicator	⠠⠠

Example 4.1-4 TOTAL EGG COUNT = 79

⠠⠠ ⠠⠠⠉⠕⠞⠁⠇ ⠠⠠⠑⠛⠛ ⠠⠠⠉⠕⠥⠝⠞ ⠠⠠ ⠼⠛⠊

4.2 Words in Narrative: It is not necessary to include a word inside the Nemeth switches when a word is adjacent to a mathematical item. For example,

Words Labeling a Math Item

Example 4.2-1 Figure 4.7 shows Shape 4 and its reflection, Shape 4′.

⠠⠊⠛⠥⠗⠑ ⠼⠙⠲⠛ ⠩⠓⠕⠺⠎ ⠠⠎⠓⠁⠏⠑ ⠼⠙ ⠯⠊⠞⠎
⠗⠑⠋⠇⠑⠉⠞⠊⠕⠝⠂ ⠠⠎⠓⠁⠏⠑ ⠼⠙⠄ ⠲

Example 4.2-2 Chris used 25.5 cans of paint.

⠠⠉⠓⠗⠊⠎ ⠥⠎⠑⠙ ⠼⠑⠲⠢ ⠉⠁⠝⠎ ⠕⠋ ⠏⠁⠊⠝⠞⠲

Example 4.2-3 What is 5.5 percent of 72?

⠠⠺⠓⠁⠞ ⠊⠎ ⠼⠑⠲⠢ ⠏⠻⠉⠑⠝⠞ ⠕⠋ ⠼⠛⠃⠦

Units of Measure

Example 4.2-4 Logan weighed exactly 7.00 pounds at birth.

⠠⠇⠕⠛⠁⠝ ⠺⠑⠊⠛⠓⠑⠙ ⠑⠭⠁⠉⠞⠇⠽ ⠼⠛⠲⠚⠚ ⠏⠕⠥⠝⠙⠎ ⠁⠞
⠃⠊⠗⠞⠓⠲ ⠠⠊⠞ ⠺⠁⠎ ⠼⠁⠛⠲⠚⠚

Note that the number need not fall on the same line with the unit of measure when the unit of measure is a word.

Example 4.2-5 Marc's sister weighed 6 pounds 3.2 ounces.

⠠⠍⠁⠗⠉⠄⠎ ⠎⠊⠎⠞⠻ ⠺⠑⠊⠛⠓⠑⠙ ⠼⠋ ⠏⠕⠥⠝⠙⠎
⠼⠉⠲⠃ ⠕⠥⠝⠉⠑⠎⠲

4.3 Punctuation With Words: Words are not punctuated mathematically, even when they are brailled in mathematical context. For a comma, the dot 2 comma is used; for other punctuation marks, no punctuation indicator is used. The next example shows a set of class members using mathematical braces to enclose the set.

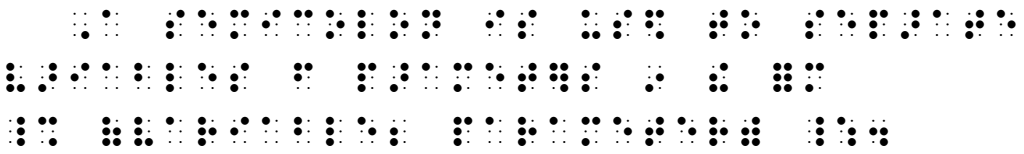
Example 4.3-1 {Richard, Daniel, Steven}



The literary comma is used when a comma follows a word, even in mathematical context.

In the next example the math form being described is illustrated with words. Because the model is a math expression, a switch to Nemeth Code is required.

Example 4.3-2 A semicolon is used to separate variables from parameters in the form (variable; parameter).



The semicolon does not require a punctuation indicator because words are punctuated in literary mode, even in mathematical context.

PRACTICE 4A

A. If 1 pound of Swiss cheese costs $2.50, how much does 4.7 pounds cost?

B. JMHS's set of high-jump champions: {Terry, Leslie, Traci}

C. The parts of a subtraction problem are named as follows: minuend – subtrahend = difference.

D. 98.6 degrees Fahrenheit is not necessarily "normal" body temperature for everyone.

E. Find the original and new prices when x=2 games and when x=7 games. Did you get the same prices as you found in problem 6?

Introduction to Abbreviations

This lesson examines abbreviations within narrative.

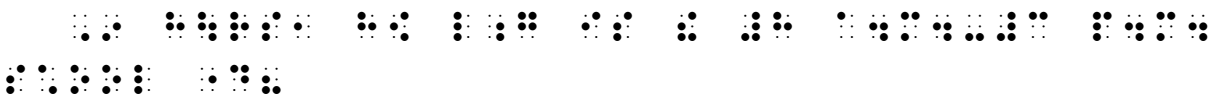
4.4 Abbreviations: An abbreviation by itself is not mathematical and does not require a switch to Nemeth Code.

Example 4.4-1 SAS means "side angle side".



Even though SAS is a special abbreviation in the field of mathematics, as used in this example it is simply an abbreviation in the narrative and is brailled in UEB.

Example 4.4-2 In hours, how long is the 8 a.m.-3 p.m. school day?



A freestanding unmodified numeral and any associated abbreviation does not require a code switch.

When an abbreviation is part of an equation or math expression, both the number and the abbreviation are placed inside the Nemeth switches.

Example 4.4-3 15 mm can be expressed as 1.5 cm.

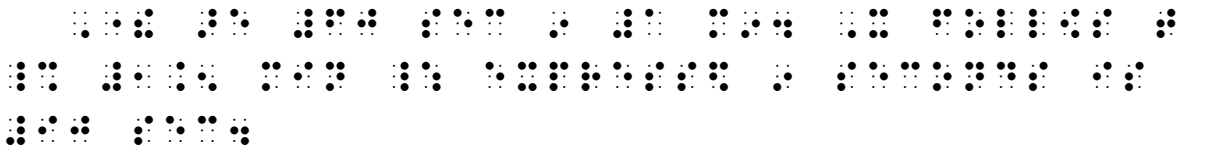


The abbreviation "cm" applies to the number "1.5" and so the abbreviation is included in the switches.

No contractions are used in an abbreviation that is brailled in Nemeth Code.

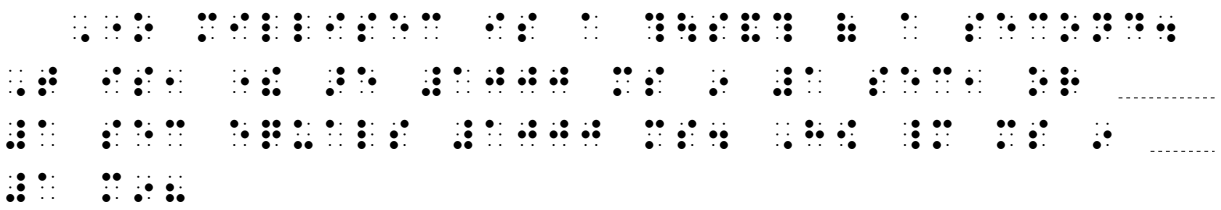
⋙ 1.5 min 

Example 4.4-4 There are 60 sec in 1 min. It follows that 1.5 min expressed in seconds is 90 sec.



4.4.1 FORMAT—Keep Together: An abbreviation and a preceding or following numeral to which it applies must not be divided between braille lines. Because Nemeth Code format rules are applied throughout a technical transcription, this rule also applies in UEB text. Note that the print copy may not follow this format, but it must be applied in the braille transcription.

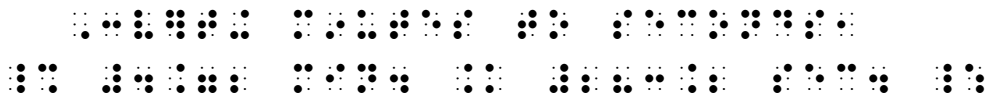
Example 4.4-5 One millisec is a thousandth of a second. That is, there are 1000 ms in 1 sec, or 1 sec equals 1000 ms. How many ms in 1 min?



4.4.2 Punctuation with Abbreviations: Abbreviations are not punctuated mathematically, even when they are brailled in mathematical context. For a comma, the dot 2 comma is used; for other punctuation marks, no punctuation indicator is used. Note in the example below that the periods following "min." and "sec." do not require a punctuation indicator.

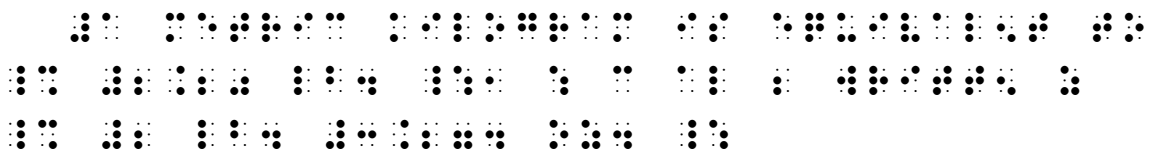
⋙ 4.72 min. = 283.2 sec.

Example 4.4-6 Converting minutes to seconds, 4.72 min. = 283.2 sec.



4.4.2.a Abbreviations with a Related Period: Examine the print copy to determine if the abbreviations include a period. If they do, the abbreviation must not be separated from its related period. If a Nemeth Code terminator follows the abbreviation, the related period is brailled before the terminator, unspaced from its abbreviation. If a period functions both with an abbreviation and as punctuation at the end of a sentence, keep the period with its abbreviation.

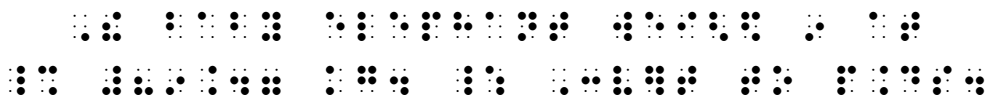
Example 4.4-7 1 metric kilogram is equivalent to 2.20 lb., which can also be written as 2 lb. 3.274 oz.



The comma is part of the sentence structure and so is placed after the Nemeth Code terminator. Although "2 lb." by itself does not require a switch, because it is part of a measurement that contains a decimal, the entire weight is brailled in Nemeth Code to maintain continuity.

In an isolated problem where there are no context clues to determine whether a period applies to the abbreviation or merely ends the sentence, assume that it relates to the abbreviation.

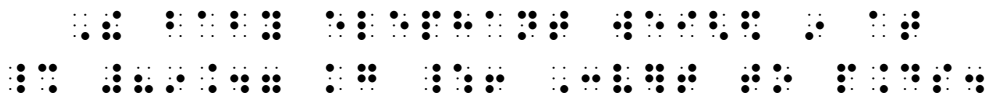
Example 4.4-8 The baby elephant weighed in at 89.47 kg. Convert to pounds.



The end-of-sentence period after "kg" is treated as if it belongs to the abbreviation because there are no context clues to the function of the period.

Compare this to the next example where it is clear that the abbreviation has no related period.

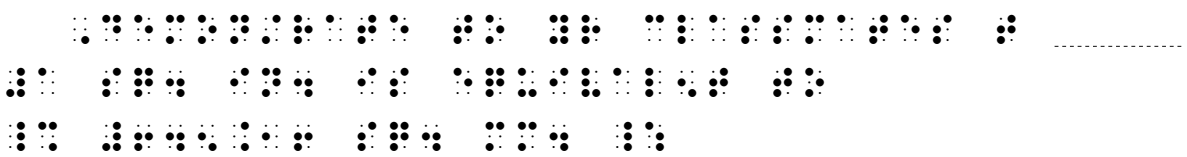
Example 4.4-9 The baby elephant weighed in at 89.47 kg! Convert to pounds.



There is no period after the abbreviation "kg". The punctuation applies to the sentence.

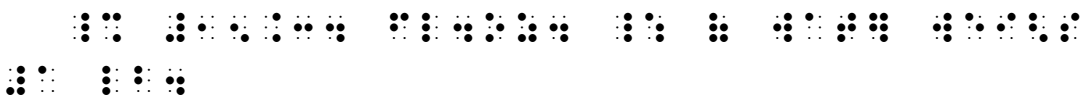
4.4.3 Spacing with Abbreviations: An abbreviation consisting of two or more components is transcribed as spaced or unspaced to conform with the print text. "Keep together" format applies to the entire abbreviation and its related numeral, even in UEB text.

Example 4.4-10 Demonstrate to your classmates that 1 sq. in. is equivalent to 645.16 sq. mm.



In print, there is a space before and after "sq."

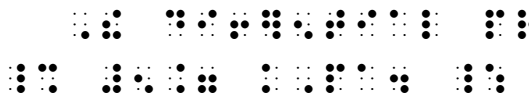
Example 4.4-11 15.34 fl.oz. of water weighs 1 lb.



In print, there is no space between "fl." and "oz."

Unless a Nemeth Code rule states otherwise, a space must be left between an abbreviation and the numeral to which it applies, even if the print copy shows no space.

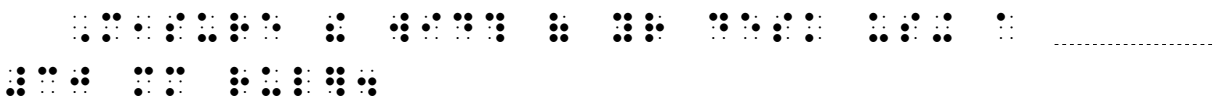
Example 4.4-12 The differential pressure is 5.7kPa.



In print, there is no space between "5.7" and the abbreviation "kPa."

This Nemeth Code spacing rule applies even in UEB text.

Example 4.4-13 Measure the width of your desk using a 30mm ruler.



In print, there is no space between "30" and "mm".

> The BANA Nemeth Code Technical Committee is discussing details regarding punctuation of the omission dash and ellipsis when these symbols indicate a missing word or abbreviation. This section will be completed after decisions are made.

4.5 Numbers with Ordinal Endings: Ordinal endings are not abbreviations. Recall from **Lesson 1** that a numeral with an ordinal ending is brailled in UEB if it occurs in literary context.

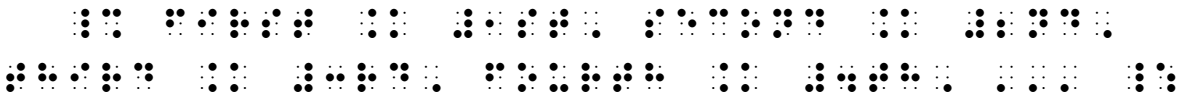
Example 4.5-1 Rearrange: 2nd, 4th, 1st, 3rd.



If the ordinal appears in mathematical context, the ordinal ending becomes part of the expression and is punctuated mathematically.

⋙ 1st, 

Example 4.5-2 first = 1st, second = 2nd, third = 3rd, fourth = 4th, ...



PRACTICE 4B

1. If 1 lb. of Gouda cheese costs $2.96, what will you pay for 2.5 lbs.?
2. If 2kg Gruyère costs £2,65, what is the cost of a wheel weighing 3kg?
3. Continuing the set of ordinals, fifth = 5th, sixth = 6th, seventh = 7th, eighth = 8th.
4. 1 kcal is equivalent to 3088.03 ft.lb.

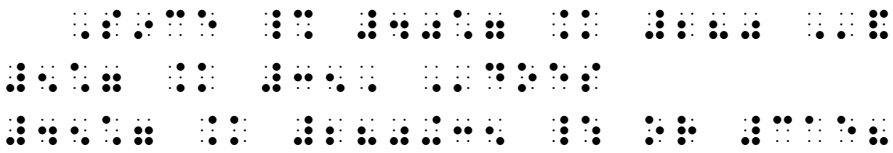
More To Come This does not complete the discussion of abbreviations in mathematical context. Single-letter abbreviations, abbreviations that use the same letters as a shortform, and further spacing rules within mathematical expressions are discussed in **Lesson 5**.

Single-Word Switch Indicator

4.6 The Single-Word Switch Indicator: Words that do not provide mathematical meaning are transcribed in UEB. To avoid use of switch indicators when a single word (or hyphenated compound word) occurs between two math expressions, the single-word switch indicator is used to indicate that the following word (or hyphenated compound word) is in UEB. The indicator is unspaced from the word. Contractions are used according to the rules of UEB. The switch is required on a single word even if the word contains no contractions. UEB typeform may be applied to the word, as appropriate. The effect of the single-word switch indicator is terminated by a space, and Nemeth Code resumes.

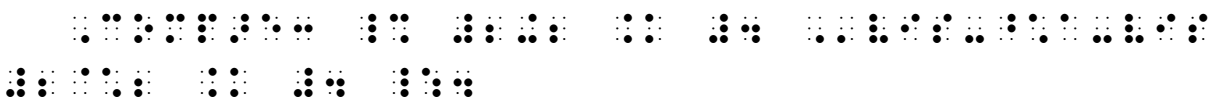
Single-Word Switch Indicator ⠸⠸

Example 4.6-1 Since $40 \cdot 7 = 280$ and $5 \cdot 7 = 35$, does $45 \cdot 7 = 280 + 35$ or 315?



The words "Since," "and," "does," and "or" are part of the sentence structure—they are not being used mathematically—and so UEB applies. The linked expression $45 \cdot 7 = 280 + 35$ *is kept together on one braille line.*

Example 4.6-2 Compare: $2 + 2 = 4$ vis-à-vis $2 \times 2 = 4$.



Until this symbol becomes widely recognized, we suggest that the single-word switch indicator be listed on the Special Symbols page in the transcriber-generated portion of each volume. See **Lesson 18** for details.

4.6.1 Consider the Context: Compare the treatment of the word "of" in these examples.

Example 4.6-3 Estimate tax: 6% of $5.25



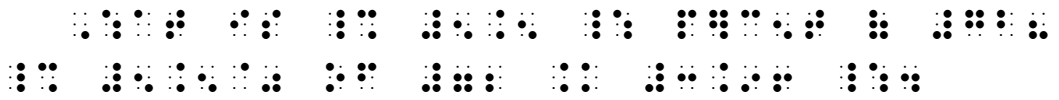
"of" is narrative and so is brailled in UEB using a single-word switch.

Example 4.6-4 6% of $5.25 = $.32



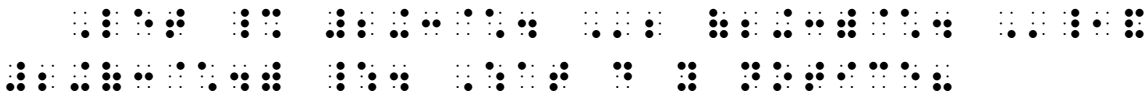
"of" is part of the math expression ($5.25 alone does not equal $.32) and so "of" is brailled as a word in Nemeth Code, without contractions.

Example 4.6-5 What is 5.5 percent of 72? 5.5% of 72 = 3.96.



4.6.2 Lower Wordsigns: A lower wordsign may be used with a single-word switch indicator without violating the lower sign rule.

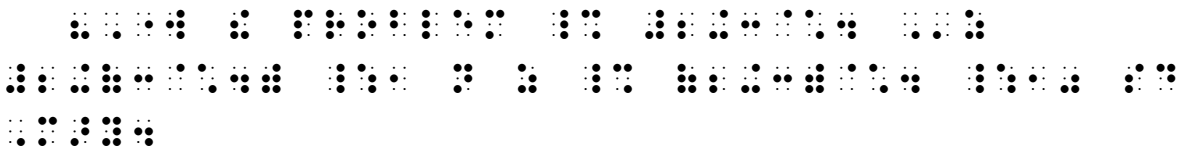
Example 4.6-6 Let 2 + 3 × 4 be (2 + 3) × 4 <u>and</u> 2 + (3 × 4). What do you notice?



The single-word switch indicator is used on the underlined word "and".

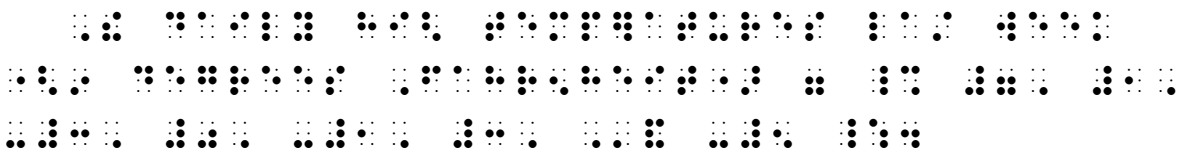
4.6.3 Two or More Words: When more than one narrative word in succession appears within mathematical context, Nemeth Code must be terminated in order to transcribe the words in UEB.

Example 4.6-7 "Work the problem 2 + 3 × 4 as 2 + (3 × 4), not as (2 + 3) × 4," said Mary.



4.6.4 Switch Considerations: Just because a numeral can be brailled in UEB does not mean it *must* be brailled in UEB. Consider how cumbersome it would be to read the next example if you used Nemeth Code only for the negative numbers.

Example 4.6-8 The daily high temperatures last week (in degrees Fahrenheit) were 7, 1, –3, 0, –1, 3, and –5.



Excessive switching is avoided by including all of the numbers (even the "unmodified" ones) inside one set of switch indicators.

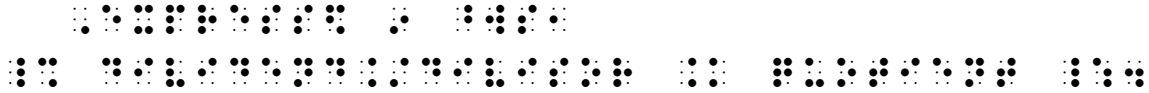
Switch Indicators at Page Turns, cont.

4.7 Review of "Keep Together" Considerations: Now that you have had more experience with switch indicators, we will consider more layout issues that occur at braille page turns. Keep the following points in mind as they relate to mathematical expressions within the narrative text.

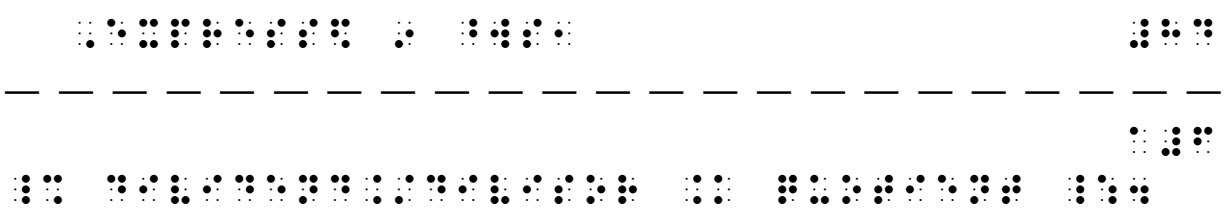
- A mathematical expression that will fit entirely on the braille line must not be divided between lines.
- If the math expression is preceded by the opening Nemeth Code indicator and followed by the Nemeth Code terminator, and if there is room on the line for both switch indicators and the expression, keep them all on the same line.
- If there is not room on the line for both switch indicators and the math expression, one of the switches will fall on a different line.
- If neither switch indicator will fit on the same line as the math expression, priority is given to keeping the math expression intact, placing each switch indicator on another line. The opening Nemeth Code indicator will be the last item on the preceding line; the Nemeth Code terminator will be the first item on the following line.
- If a page number on line 25 or line 1 does not allow the entire expression to fit on the line, the expression is brought down to the next line that has enough usable cells.
- If a math expression is preceded by the opening Nemeth Code indicator and followed by the Nemeth Code terminator and it falls at a page turn, place each switch indicator on the same page as the mathematical material to which it applies. An opening Nemeth Code indicator should not be the last item at the bottom of a braille page; a Nemeth Code Terminator should not be the first item at the top of a braille page.

The expression in the first example will fit on one braille line along with the switches and the ending punctuation ...

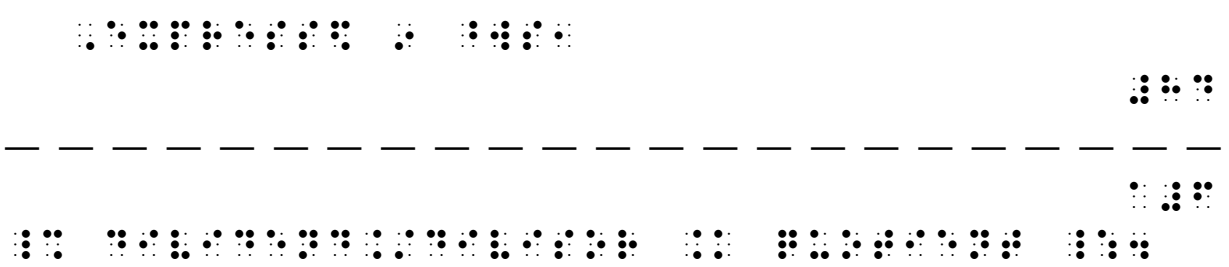
Example 4.7-1 Expressed in words, dividend ÷ divisor = quotient.



... unless a page number shares the line.

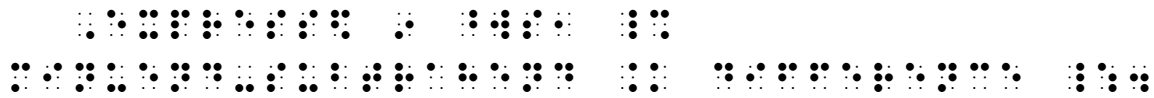


..OR..

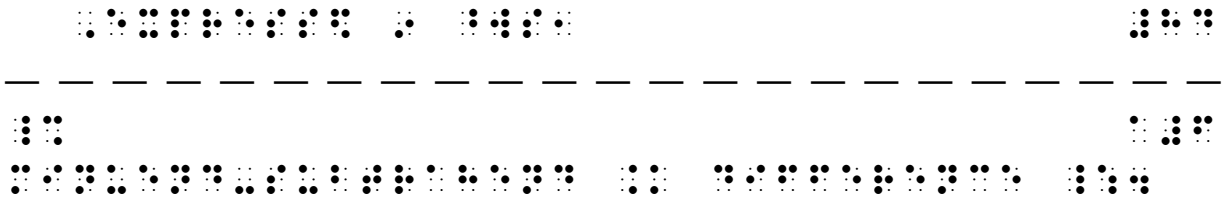


The expression in the next example will fit on one line but the code switch indicators will not. One of the indicators must be placed on a different line.

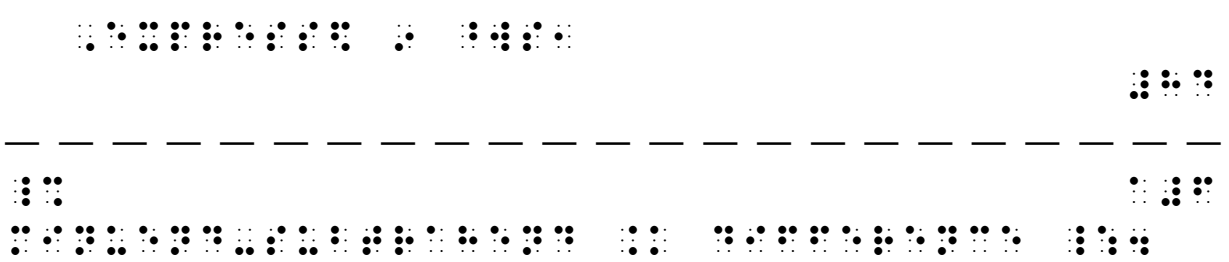
Example 4.7-2 Expressed in words, minuend – subtrahend = difference.



At a page turn, do not leave the switch indicator alone at the bottom of a braille page.

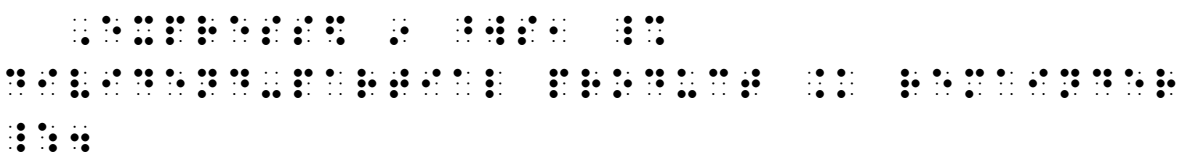


..OR..

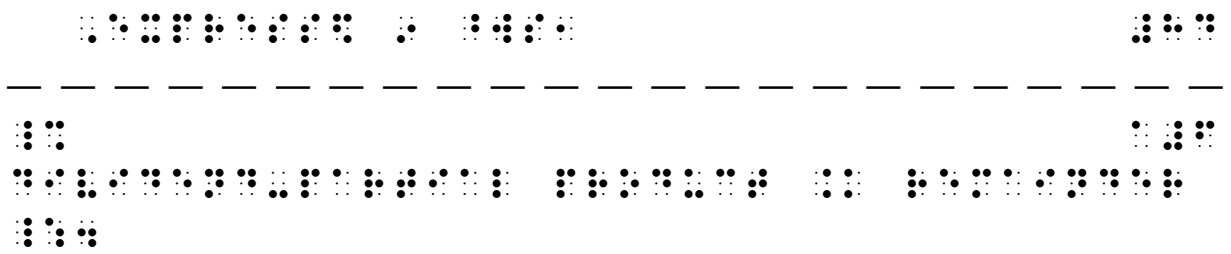


The next example is similar to the example presented in **Lesson 2**—the expression itself requires a full braille line. The switch indicators are on the preceding and following lines.

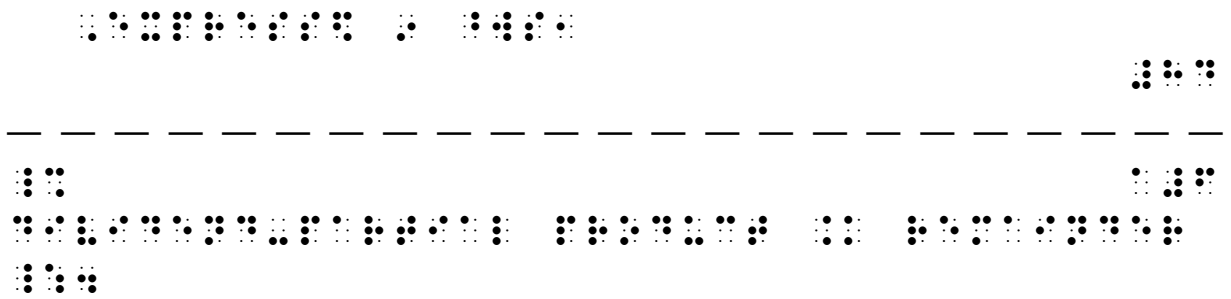
Example 4.7-3 Expressed in words, dividend – partial product = remainder.



The switch indicators must fall on the same page as the expression to which they apply.

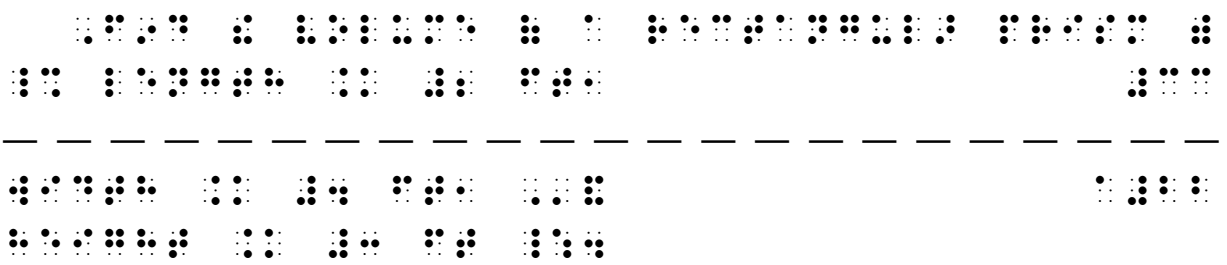


..OR..



In this final example, there is more than one expression between the code switches. Each expression is kept together on one braille line but there is no need to force the switches to be on the same page. There is no need to restate "Nemeth Code" at the page turn.

Example 4.7-4 Find the volume of a rectangular prism with length = 2 ft, width = 4 ft, and height = 3 ft.



Each of the three expressions must not be allowed to wrap before or after the equals sign or between the numeral and the abbreviation "ft". The first two occurrences of the abbreviation "ft" do not have a related period so you must assume that the period following the third occurrence does not relate to the abbreviation; it only ends the sentence and so is brailled after the Nemeth Code terminator.

Instructions: In the first paragraph, assume there is a braille page turn after the word "to". Use braille page number "55" and print page number "a44" on line 1 of the new braille page.

PRACTICE 4C

A unit of work is the foot-pound (ft-lb). One foot-pound converts to 12.000000427771 inch-pounds. How many in-lb is 4.6 ft-lb? How many ft-lb is 247.9927443 in-lb?

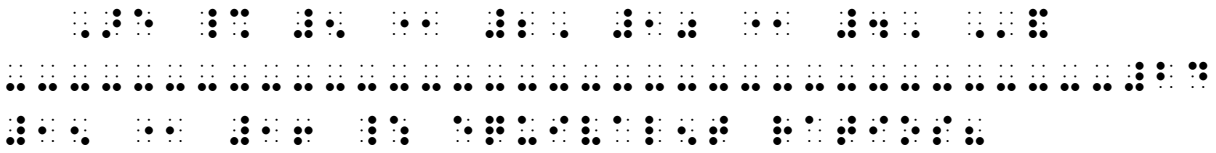
If 1 joule = 10 million ergs, and 1 megajoule = 1,000,000 joules, how many ergs is 1 megajoule? 1megajoule = ? ergs

4.8 New Print Page: The page change indicator and page number are constructed in the same way in either code. The code in place before the page change indicator remains in effect following the page change indicator.

Example 4.8-1 Are 5 : 2, 10 : 4, and

— — — — — [print page turn] — — — — — — — — —

15 : 16 equivalent ratios?



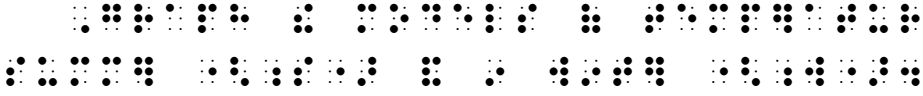
LETTERS

4.9 Single English Letters in Narrative: The language of mathematics uses single letters as mathematical characters. Special provision is made for a single English letter that has mathematical meaning when it appears within narrative. As long as the English letter is freestanding and is unmodified, it may be transcribed in UEB. The letter may touch punctuation.

Example 4.9-1 In this equation, b must be greater than a.



Example 4.9-2 Graph the models of temperatures in summer (s) and in winter (w).



A freestanding letter with an ordinal ending may also be brailled in UEB.

Example 4.9-3 Find the nth term of the arithmetic sequence.



Exception: Certain mathematical letters are characterized by a special typeface and must be brailled in Nemeth Code. Such letters will be studied in **Lesson 7**.

4.10 Single English Letters in Nemeth Code: An English letter that has mathematical meaning and which appears in technical context—that is, between Nemeth Code switches—is transcribed according to the rules of the Nemeth Code. Before illustrating the rules with examples, the definition of "single letter" as used in the Nemeth Code is presented.

4.10.1 Nemeth Code Definition of "Single Letter": To be defined as a "single letter" in the Nemeth Code several criteria must be met.

i. A "single letter" must be from the English alphabet, in regular type, and unmodified.

These <u>are</u> "single letters"	p D z R	
These <u>are not</u> "single letters"	π **D** $\bar{z}$ $\mathbb{R}$	*The first letter is not from the English alphabet, the second and fourth letters are not in regular type, the third letter is modified.*

- SPECIAL CASE: A letter representing a mathematical variable is often printed in italics but the italics are disregarded in braille. **Lesson 7** discusses typeform.

ii. Furthermore, in the print copy the letter must be both preceded by a space or by one or more punctuation marks *<u>AND</u>* followed by a space or by one or more punctuation marks.

These <u>are</u> "single letters"	"y" x, "w S"	*Each letter is preceded and followed by punctuation or by a space.*
These <u>are not</u> "single letters"	−x "wS" y+z	*The x, z, and S are not preceded by a space or by punctuation (−x is "negative x"); the y and the w are not followed by a space or by punctuation.*

- *Note:* Nemeth Code grouping symbols are not considered to be punctuation marks. Rules for letters touching grouping symbols are discussed later.

iii. Whether the leading punctuation mark is preceded by a space or not is irrelevant; whether the following punctuation mark is followed by a space or not is irrelevant.

These <u>are</u> "single letters"	"x"+"y"	*Each letter is both preceded and followed by punctuation.*

iv. If the space shown in print is not shown in braille, the letter is no longer a "single letter."

These <u>are not</u> "single letters" r + s *Although each letter is preceded and followed by a space in print, in braille the plus sign is unspaced from the letters.*

v. And finally, to be defined as a "single letter" the letter must not be an abbreviation nor can it be a word ("a", "A", "I", or "O").

These <u>are not</u> "single letters" I need 4.5 m of fabric. *"I" is a word; "m" is an abbreviation for "meters".*

Single-letter abbreviations are discussed in a later lesson.

Throughout this course, when referring to the Nemeth Code definition of a single letter, the term "single letter" is in quotation marks.

Introduction to the English Letter Indicator

When a single letter from the English alphabet is used in mathematical context it may require the use of the English letter indicator. And—even though no contractions are used in Nemeth Code—a letter combination that is the same as a shortform may require the use of a letter indicator for clarity.

English Letter Indicator ("ELI") ⠰

Several rules are in place regarding situations where the ELI is or is not used. It is important to note that the English letter indicator does <u>not</u> function in the same way as the UEB Grade 1 symbol indicator. The term "English letter indicator" clearly describes the function of the indicator.

4.11 Use of the English Letter Indicator with a "Single Letter": Except as noted in the next section, an ELI is required when a letter is a "single letter" as defined in **<u>4.10.1</u>**, above.

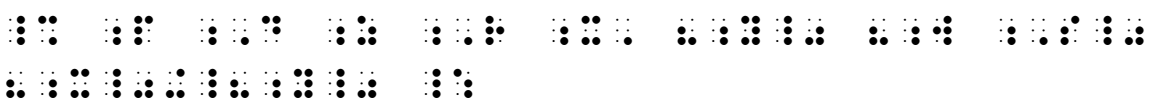
4.11.1 Capitalization of "Single Letters": To indicate a single capitalized letter, the capitalization indicator is placed between the ELI and the letter. The effect of the capitalization indicator extends only to the letter which follows it.

Capitalization Indicator ⠠

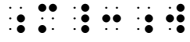
4.11.2 Punctuation of "Single Letters": A "single letter" is punctuated mathematically if the letter and the punctuation fall within the Nemeth switch indicators.

The examples from **<u>4.10.1</u>** are illustrated below, assuming mathematical context. Note the placement of the capitalization indicator as well as the use of mathematical punctuation.

Example 4.11-1 p D z R x, "y" "w S" "x"+"y"



4.11.3 Mapping Notation: In mapping notation, the symbol **:** is brailled as an unspaced colon. In the next example, because the letters n and v are each preceded AND followed by a space and/or a punctuation mark, they each require an ELI. A punctuation indicator is required before the colon.

⋙ m:W 

Example 4.11-2 m:W means "the mapping m of W."



Instructions: Demonstrate use of the English letter indicator and proper punctuation mode in the following series of single letters. Braille this practice entirely in Nemeth Code, using Example 4.11-1 as a model.

PRACTICE 4D

c, C; r, R; "l", "L"; "i, j, k"; "l"×"w"×"h"; f:R.

4.12 Nonuse of the English Letter Indicator with a "Single Letter": Even though a letter meets the criteria of "single letter" above , the ELI is <u>not used</u> when the following conditions are present.

4.12.1 Comparison Sign: If the letter is immediately preceded by a sign of comparison an ELI is not used.

⋙ 4 > x 

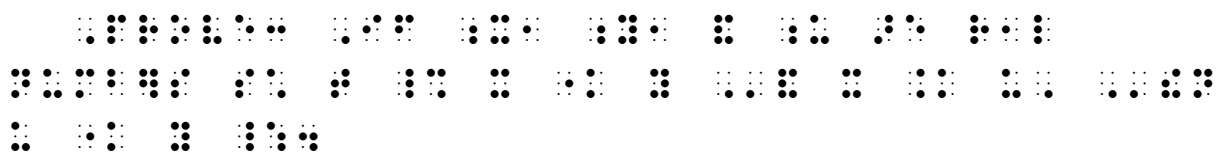
If the letter is immediately followed by a sign of comparison an ELI is not used.

⋙ x > 2 

⋙ a : b :: c : d 

⋙ "x = 3" 

Example 4.12-1 Prove: If x, y, and u are real numbers such that $x < y$ and $x = u$, then $u < y$.



Note: If punctuation comes between the letter and a sign of comparison, the letter is now a "single letter" by definition and an ELI is required.

⋙ "x" = 3



In this mapping example, the letters "p" and "s" require an ELI because they are "single letters". Letters "r" and "q" do not need an ELI because they immediately precede/follow a sign of comparison.

⋙ p:r = q:s



4.12.2 Enclosed Within Grouping Symbols: If a "single letter" is entirely enclosed within signs of grouping, the ELI is omitted.

⋙ (a)



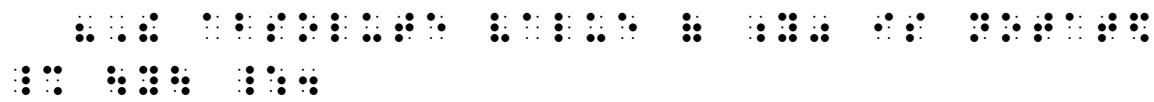
⋙ {P}



⋙ |y|



Example 4.12-2 "The absolute value of y" is notated |y|.



4.12.3 Unspaced Sequence of Terms: The ELI is not used with one or more English letters (in regular type) which occur in an unspaced sequence of mathematical symbols.

⋙ d'



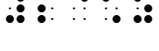
⋙ N%



⋙ 2z

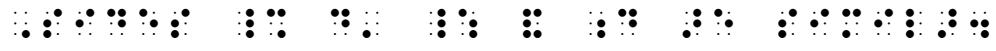


⋙ $2 \times z$

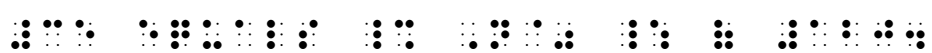


⋙ 2nth

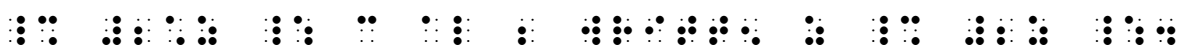
Example 4.12-3 Sides d' and d are similar.



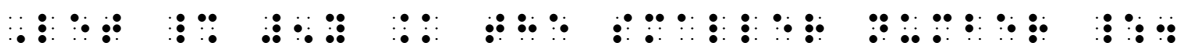
Example 4.12-4 35 equals N% of 120.



Example 4.12-5 2 · z can also be written as 2z.

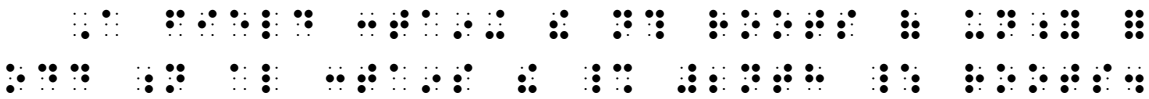


Example 4.12-6 Let 5y = the smaller number.



The words "the smaller number" are part of the math expression (they show what 5y equals) and so are included within the switches.

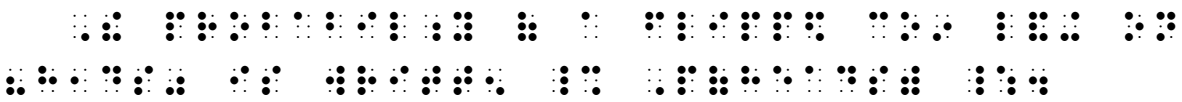
Example 4.12-7 A field containing the nth roots of unity for odd n also contains the 2nth roots.



4.12.3.a Probability Notation: In probability notation, a letter (often "P" which represents "the probability of") is followed, unspaced, by the "event" which is written between mathematical grouping symbols (often parentheses). In the next example, the event is "heads". Because the letter "P" is unspaced from the mathematical symbol "(" the letter is not a "single letter" and so an ELI is not used.

⋙ P(heads) 

Example 4.12-8 The probability of a flipped coin landing on "heads" is written P(heads).



4.13 Letters as Identifiers: Letters used as identifiers are constructed according to the rules of the code which is in effect at the time—UEB or Nemeth Code. Compare:

Print	UEB	Nemeth Code
a.		
B.		
(a)		
(B)		
c)		

Instructions: Demonstrate the use and the nonuse of the English letter indicator for "single letters" by brailling this practice entirely in Nemeth Code. To keep the identifiers beginning in the proper cell, braille an opening Nemeth Code indicator in cell 1 on the first line and begin the first item on the next line.

PRACTICE 4E

(a) $r = \text{rate}$

(b) $\text{"r"} = \text{rate}$

(c) $x,\ y,\ z < 100$

(d) $n¢ = \$4.95$

(e) $x > \text{"3"}$

(f) $a + b$

(g) $|y| = |-y|$

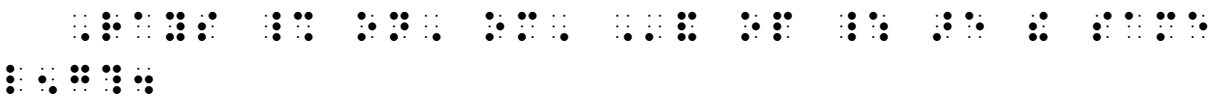
(h) $|x + y| = |x| + |y|$

(i) $P(\text{red and blue})$

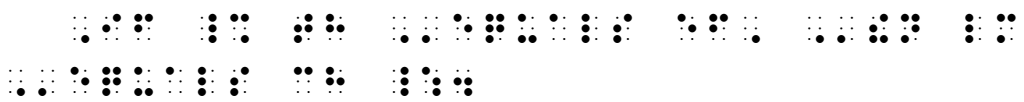
Mathematical Letter Combinations

4.14 Mathematical Letter Sequence: The option to remain in UEB to braille a mathematical letter applies only to a single freestanding English letter. A mathematical sequence of letters is a mathematical expression and must be transcribed in Nemeth Code. A mathematical letter sequence is punctuated mathematically if the punctuation falls within the Nemeth switch indicators.

Example 4.14-1 Rays on, om, and op are the same length.



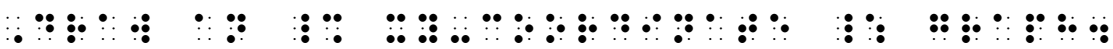
Example 4.14-2 If th equals ef, then lm equals ch.



> The BANA Nemeth Code Technical Committee is discussing details regarding the treatment of letter sequences in terms such as xy-coordinate and yz-plane. Correct transcriptions of the next two examples will be confirmed after decisions are made. For now we recommend treating the entire hyphenated expression mathematically.

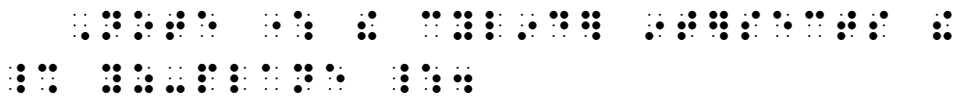
Example 4.14-3 Draw an xy-coordinate graph.

PROVISIONAL SOLUTION:



Example 4.14-4 Note where the cylinder intersects the yz-plane.

PROVISIONAL SOLUTION:



4.15 Capitalized Letter Sequence: Each capitalized letter in a mathematical sequence of letters must be capitalized individually.

⋙ PQRS 

Example 4.15-1 Prove PQRS is a rhombus.

4.16 Shortform Letter Combinations: If the mathematical letter sequence corresponds to a shortform of UEB, further rules apply. To be a "shortform letter combination" in mathematical context the letters must correspond to a shortform of UEB, must not represent a word or an abbreviation, and all of its letters must be lowercase.

These <u>are</u> "shortform letter combinations"	ab ac dcl
These <u>are not</u> "shortform letter combinations"	Ab ad DCL

Abbreviations that use the same letters as a shortform are discussed in a later lesson.

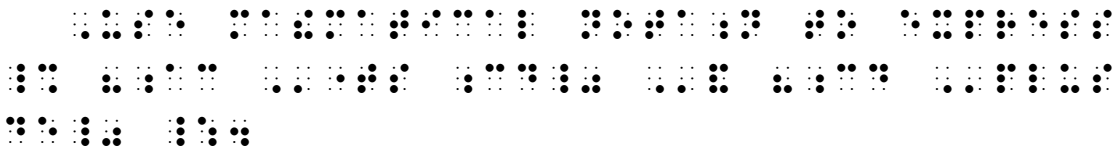
Rules regarding the use or nonuse of the ELI with a "shortform letter combination" are similar to the rules which apply to "single letters".

4.16.1 Use of the English Letter Indicator with a "Shortform Letter Combination": The ELI is used when a shortform letter combination ...

i. is in regular type;
ii. is unmodified;
iii. is preceded by a space or by one or more punctuation marks <u>AND</u> followed by a space or by one or more punctuation marks. Nemeth grouping symbols are not punctuation marks.

The effect of the indicator extends to the entire shortform following it.

Example 4.16-1 Use mathematical notation to express "ac times cd" and "cd plus de".



Two-letter combinations must be brailled in Nemeth Code, but only those letter combinations that are the same as a shortform require an ELI. Mathematical punctuation is used.

4.16.1.a Ordinals: The Nemeth Code rules regarding the use of the ELI are followed when an ordinal ending is present. If an ELI is required without the ordinal ending, it is still necessary. A switch to Nemeth Code is required because of the two-letter mathematical items.

Example 4.16-2 Mark the abth and jkth columns.

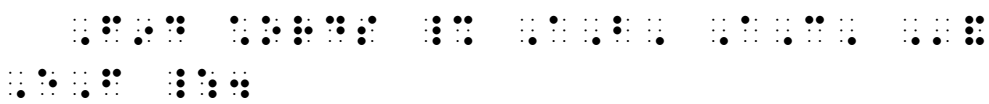


Only the letter combination that is the same as a shortform requires an ELI.

4.16.2 Nonuse of the English Letter Indicator with a "Shortform Letter Combination": When the following conditions are present, the ELI is not used even though a letter combination meets the criteria above. For English letters in regular type, an ELI is <u>not</u> used ...

... if the shortform contains any capitalized letters;

Example 4.16-3 Find chords AB, AC, and EF.

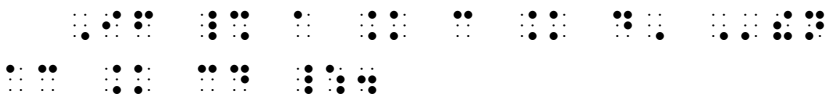


Example 4.16-4 Wd means "W times d".



... if the shortform letter combination immediately precedes or follows a sign of comparison.

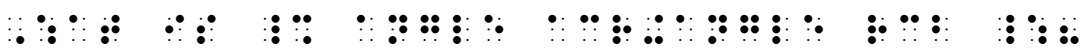
Example 4.16-5 If a = c = d, then ac = cd.



It also follows that ...

... if the shortform letter combination is not both preceded and followed by a space or by punctuation, an ELI is not needed.

Example 4.16-6 What is angle acr + angle rcb?

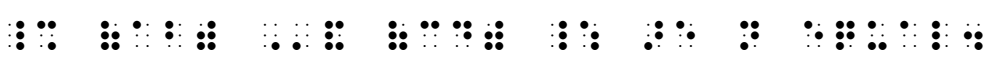


Example 4.16-7 3g × 3r × 3t = 27grt.



... if the shortform is in direct contact with both an opening and a closing grouping sign, no ELI is used.

Example 4.16-8 (ab) and (cd) are not equal.



Instructions: These examples illustrate both use and nonuse of the English letter indicator with a "single letter" or a "shortform letter combination". Explain your decisions.

PRACTICE 4F

(A) Prove: If $a < b$ and $c < 0$, then $ac > bc$. Verify your proof by determining ac and bc when $a = 5$, $b = 7$, and $c = -4$.

(B) $j = 1, 2, ..., n$

(C) 40% of $N = 120$

(D) 40% of "N" $= 120$

(E) If "rcv = rjc" does "v" = "j"?

FORMAT SUMMARY #2

Here is a summary of the Nemeth Code formats encountered in Lessons 3 and 4.

Side-by-Side Items in Itemized Material with No Subdivisions: When unsubdivided itemized material is arranged side by side across the page in print, the braille format must be changed so that all identifiers start in cell 1.

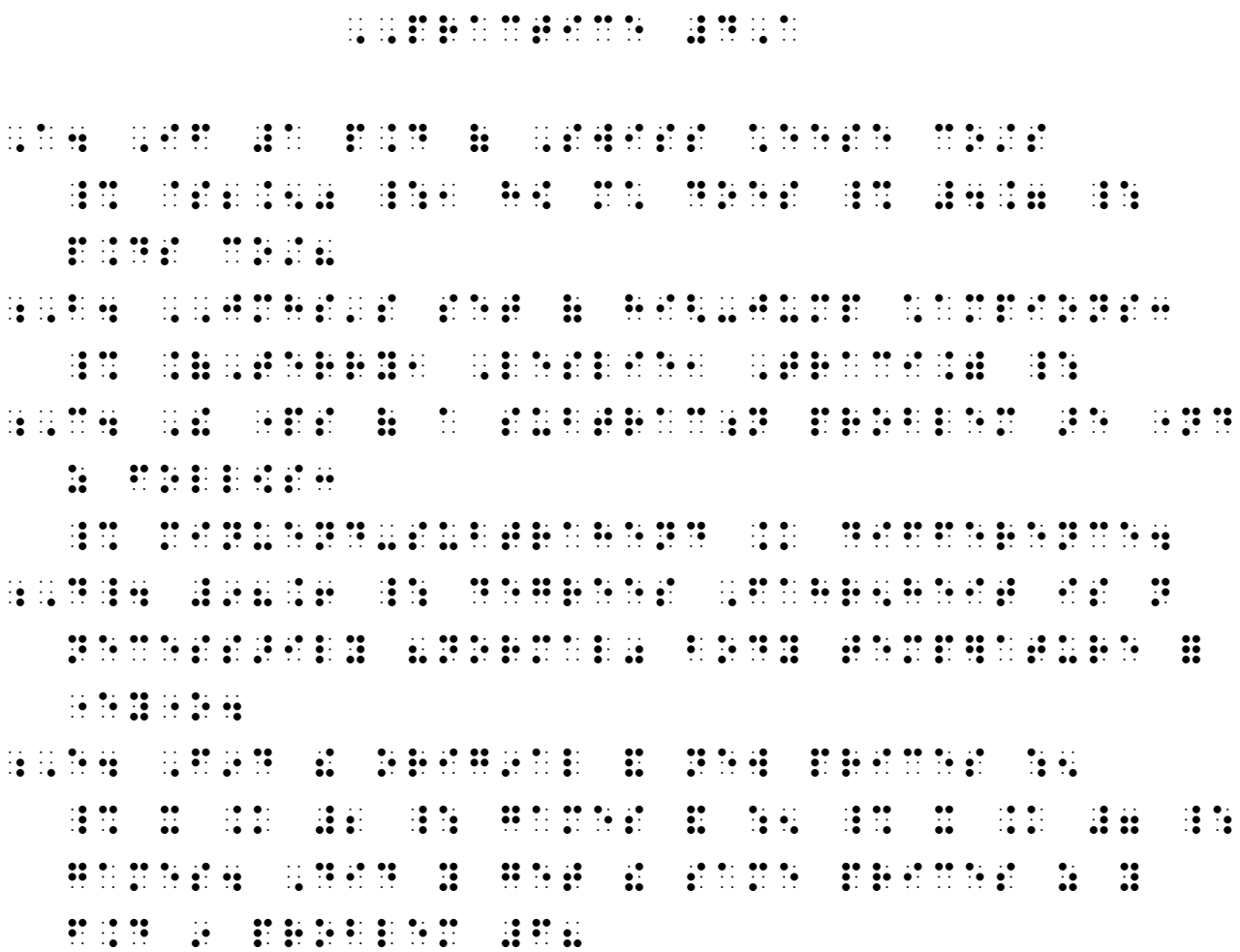
Keep Together—Hyphenated Expressions: A hyphenated expression containing one or more mathematical components must not be divided between braille lines.

Keep Together—Mathematical Expression: If a page number on line 25 or line 1 does not allow the entire mathematical expression to fit on the line, the expression must be brought down to the next line that has enough usable cells. If the expression will fit on one line but the code switch indicators will not, one or both of the indicators can be placed on a different line.

Keep Together—Abbreviation: An abbreviation and a preceding or following numeral to which it applies must not be divided between braille lines.

For further practice, see Appendix A—Reading Practice.

ANSWERS TO PRACTICE MATERIAL

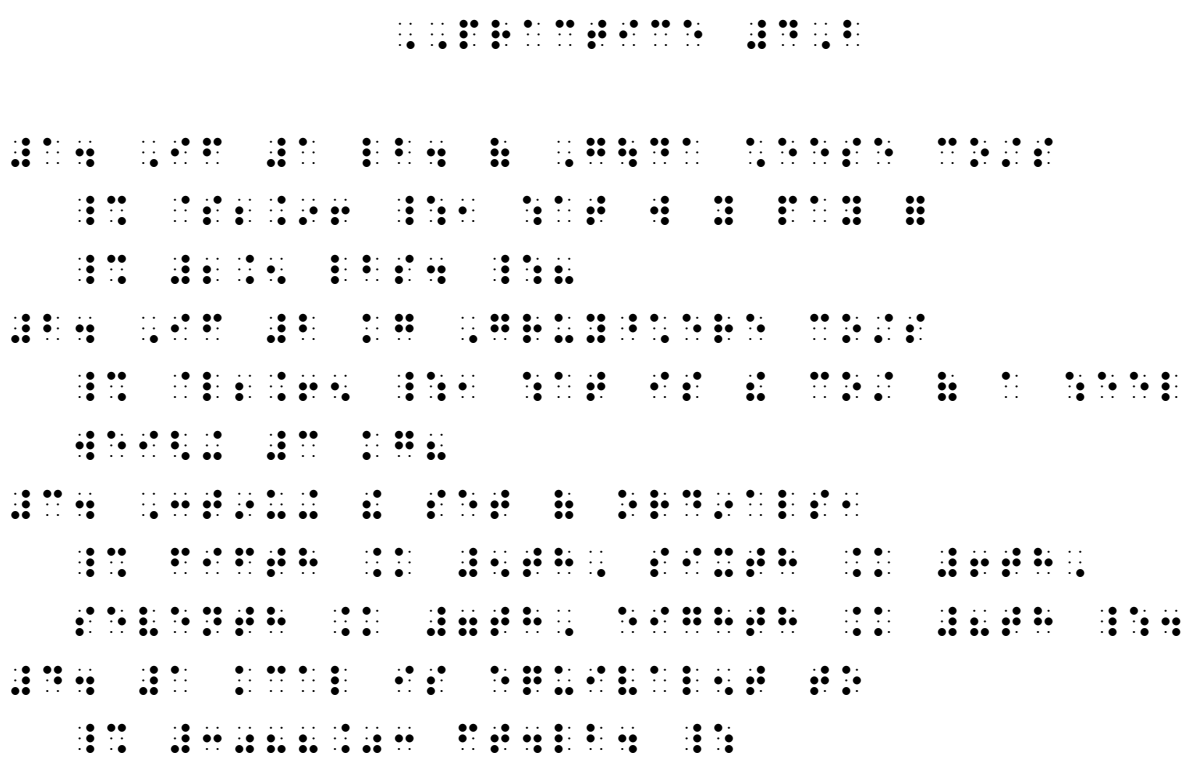


A. *A number and a related word ("4.7 pounds") do not have to fall together on the same line.*

B. *Words are punctuated with the dot 2 comma, even in mathematical context.*

C. *Following Nemeth Code spacing rules, the operation sign is unspaced from the words "minuend" and "subtrahend". Words are brailled without contractions in Nemeth Code. The period following the word "difference" does not need a punctuation indicator even though Nemeth Code has not yet terminated.*

D. *The identifier is in Nemeth Code—a punctuation indicator is needed before the period following the single letter "D". Nemeth Code terminates after "98.6".*

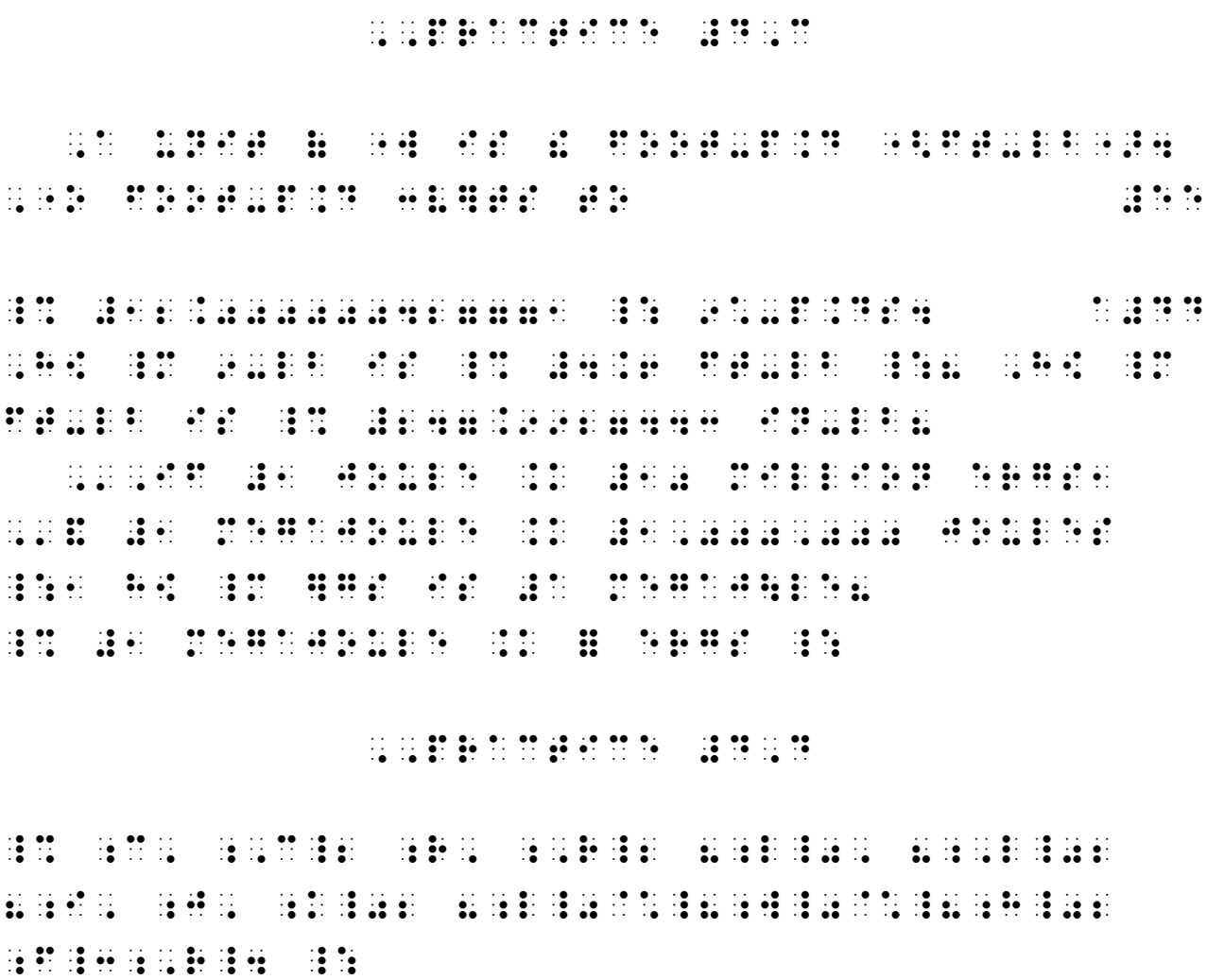


1. *A number and a related abbreviation ("2.5 lbs.") must not be separated between lines.*

2. *There must be a space between the number and the abbreviation "kg". The continental decimal point is brailled as (46).*

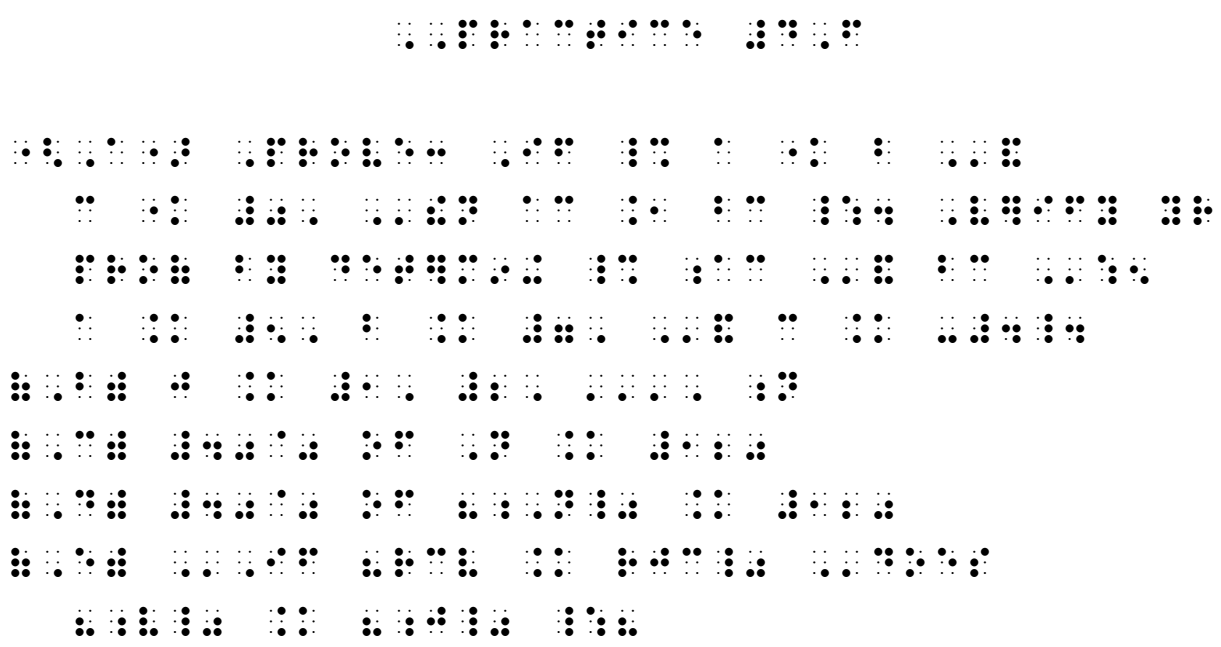
3. *Care is taken to ensure that each linked expression is not divided between braille lines. The ordinals are punctuated mathematically within the code switches.*

4. *The spacing of "ft.lb." matches print spacing (unspaced).*



Although the final period may be brailled outside of the terminator, because each of the other letters is followed by punctuation, it is logical to complete the sentence in Nemeth Code.

⠠⠠⠠



(A) Single letters and shortform letter combinations that fall before and after signs of comparison need no ELI. Two-letter mathematical expressions "ab" and "bc" must be brailled in Nemeth Code. "ac" needs an ELI when preceded and followed by a space because it is the same letter configuration as a shortform of UEB; "bc" is not a shortform letter combination and so does not need an ELI.

(B) Nemeth Code continues and so the identifier is brailled in Nemeth Code. No ELI is needed when a single letter is enclosed between grouping signs. Letter "j" is followed by a comparison sign—no ELI. Letter "n" is preceded and followed by a space—ELI required.

(C) Letter "N" is followed by a sign of comparison—no ELI.

(D) Letter "N" is preceded and followed by punctuation—ELI required even though equals sign follows.

(E) Nemeth Code continues. Identifier (E) is in Nemeth Code. The word "If" uses single-word switch indicator. Even though "rcv" and "rjc" are shortform letter combinations, an ELI is not needed because they are next to a comparison sign. On the other hand, letters "v" and "j" are each preceded and followed by punctuation—ELI is required despite the proximity of the equals sign.

EXERCISE 4

Exercise 4 will be available when this course is finished being written and is no longer "Provisional".

Proceed to Lesson 5.

LESSON 5

Read about this PROVISIONAL EDITION in the front matter to this book.
Check the NFB website periodically for updates to this lesson.

- *MORE ABOUT LETTERS*
  - *Variables*
  - *Roman Numerals*
  - *Non-Decimal Bases*
- *OTHER ALPHABETS*
- *ENCLOSED LISTS*
- *MORE ABOUT ENGLISH LETTERS*
- *MORE ABOUT ABBREVIATIONS*
- *CODE SWITCHING, cont.*

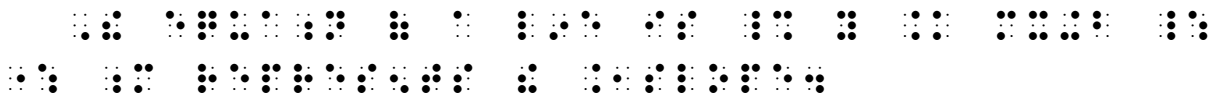
FORMAT
Keep Together—Abbreviation
Keep Together—Enclosed List

MORE ABOUT LETTERS

Variables

5.1 Mathematical Variables: An alphabetic character that represents an unspecified number is called a *variable*. Variables are usually printed in italics uniformly throughout a technical document or textbook. In both UEB and Nemeth braille, italics applied to a variable are disregarded unless other circumstances require the typeface to be retained.

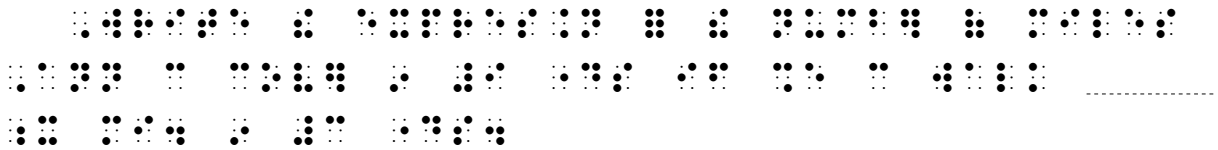
Example 5.1-1 The equation of a line is $y = mx + b$ where m represents the *slope*.



Reminder: Line 1 "y": The English letter indicator is not used when a "single letter" immediately precedes a sign of comparison. Line 2 "m": A freestanding letter may be brailled in UEB.

5.1.1 FORMAT—Keep Together: An abbreviation and a letter to which it applies must not be divided between braille lines. This format rule also applies in UEB text.

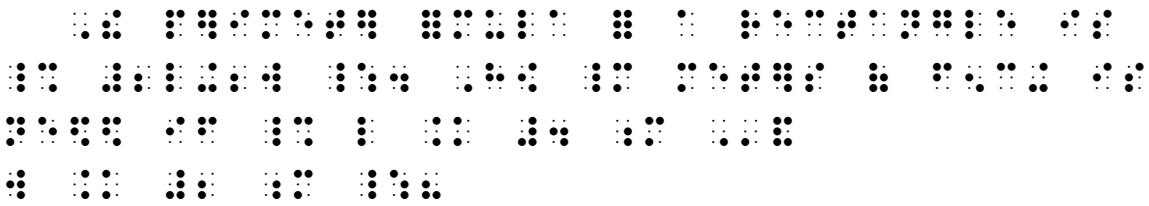
Example 5.1-2 Write the expression for the number of miles Ann can cover in 9 days if she can walk x mi. in 3 days.



"x" and "mi." must be brailled on the same line.

5.1.2 Abbreviation or Variable? The letter chosen to represent a variable is often based on the subject matter. In the next example, $2l + 2w$, the variables l and w represent unknown measurements for *length* and *width.* The letters *l* and *w* are chosen to aid in recognition of the parts of the formula, they are not abbreviations for the words length and width. Keep in mind that a variable represents a numerical value. A value will be "plugged into" the formula in place of the variable to come up with a solution. In contrast, an abbreviation represents a word—it has no numerical value. You can often answer the question "abbreviation or variable?" by noticing the typeform. In a formal publication, a variable will be printed in italics; an abbreviation will be in normal typeface.

Example 5.1-3 The perimeter formula for a rectangle is $2l + 2w$. How many meters of fencing is needed if $l = 4$ m and $w = 2$ m?



l and w are variables; m *is the abbreviation for "meters".*

PRACTICE 5A

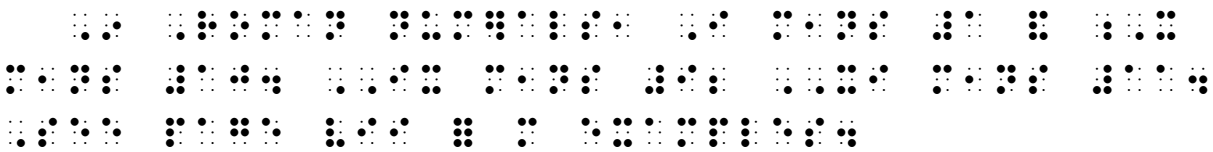
1. Express y in terms of x if $2x - 3y = 12$.
2. If $A = l \times l$, what is the length (l) of a side in inches if the area (A) of a square is 7.3 sq.ft.?
3. It is much easier to remember $A = lw$ (Area = length × width) than it is to remember $B = jt$ when trying to figure out how much carpet to buy for the living room.
4. What is the area A of trapezoid T with upper base $a = 3$ m, lower base $b = 6$ m, and height $h = 13$ m?

Roman Numerals

The BANA Nemeth Code Technical Committee is discussing details regarding the treatment of isolated Roman numerals in narrative context. Correct transcriptions of the examples in this section will be confirmed after decisions are made. For now, follow these provisional guidelines.

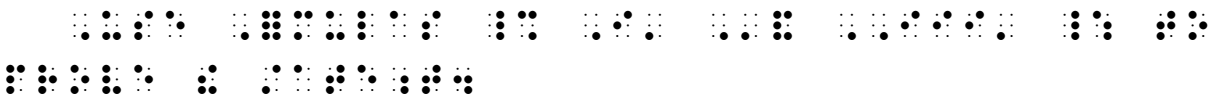
5.2 Letter or Numeral? Roman numerals are notated as certain capitalized and uncapitalized letters. Roman numerals can be brailled in UEB or in Nemeth Code, using context clues to decide whether or not to switch, just as you do with Arabic numerals. The first example below does not require code switching.

Example 5.2-1 In Roman numerals, I means 1 and X means 10. IX means 9; XI means 11. See page vii for more examples.



5.3 Transcribing Roman Numerals in Mathematical Context: When Roman numerals are part of a mathematical expression, or when a mathematical symbol is associated with a Roman numeral, a switch to Nemeth Code is required.

Example 5.3-1 Use Formulas I' and III' to prove the statement.



*The presence of a prime sign requires a switch to Nemeth Code. See **4.2** regarding words labeling a mathematical item.*

5.3.1 Roman Numerals Consisting of Lowercase Letters: In Nemeth Code, an English letter indicator (ELI) is used before *any lowercase* Roman numeral if the Roman numeral in braille is preceded by a space or by one or more punctuation marks and followed by a space or by one or more punctuation marks.

English Letter Indicator 

⋙ i ii iii iv v 

Use of the ELI with a lowercase Roman numeral follows the same rules that apply to the use of the ELI for any single English letter. In this regard, a lowercase Roman numeral is treated as "one letter" even when it consists of more than one letter.

5.3.2 Roman Numerals Consisting of One Capital Letter: In Nemeth Code, an ELI and a single capitalization indicator are used before a Roman numeral consisting of a *single capitalized* letter if the Roman numeral in braille is preceded by a space or by one or more punctuation marks and followed by a space or by one or more punctuation marks.

English Letter Indicator ("ELI") ⠰ **Single Capitalization Indicator** ⠠

⋙ I V X L C D M ⠰⠠⠊ ⠰⠠⠧ ⠰⠠⠭ ⠰⠠⠇ ⠰⠠⠉ ⠰⠠⠙ ⠰⠠⠍

The ELI is or is not used with single-letter uppercase Roman numerals according to the same rules that apply to the use/nonuse of the ELI for any single English letter.

5.3.3 Roman Numerals Consisting of Two or More Capital Letters: The double capitalization indicator of the Nemeth Code is used before a Roman numeral consisting of two or more unspaced capitalized letters.

Double Capitalization Indicator ⠠⠠

⋙ IV DC MMXVII ⠠⠠⠊⠧ ⠠⠠⠙⠉ ⠠⠠⠍⠍⠭⠧⠊⠊

In mathematical context, an ELI is not used for Roman numerals consisting of two or more capital letters, even if the letter combination is the same as a shortform.

⋙ CD DCV ⠠⠠⠉⠙ ⠠⠠⠙⠉⠧

5.3.4 Punctuation with Roman Numerals: A Roman numeral is punctuated mathematically if the punctuation falls inside the switches. The presence of punctuation does not change the rules regarding use of the ELI.

⋙ "I, IV, V, MMXVI" ⠰⠠⠶⠊⠂ ⠰⠠⠊⠧⠂ ⠰⠠⠧⠂ ⠰⠠⠍⠍⠭⠧⠊⠶

⋙ "i", "ii", "iii". ⠰⠶⠊⠂ ⠰⠶⠊⠊⠂ ⠰⠶⠊⠊⠊⠶⠲

Example 5.3-2 In Roman numerals, “ix” = 9, “L” = 50, and “C” = 100.

⠠⠊⠭ ⠠⠠⠇ ⠠⠠⠉ ⠠⠠⠊⠭ ⠨⠅ ⠠⠠⠇ ⠨⠅ ⠠⠠⠉ ⠨⠅

An ELI is required for a lowercase Roman numeral and for a single-letter Roman numeral when preceded and followed by a punctuation mark.

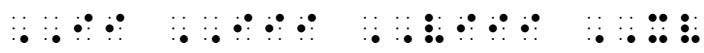
Example 5.3-3 In Roman numerals "CD" = 400 and "DCV" = 605.

⠠⠊⠭ ⠠⠠⠉⠙ ⠨⠅ ⠠⠠⠙⠉⠧ ⠨⠅ ⠠⠠⠍⠍⠭⠧⠊

Reminder: A multi-letter capitalized Roman numeral in Nemeth Code does not use an ELI even when the letter combination is the same as a shortform.

5.3.5 Nonuse of the English Letter Indicator with Roman Numerals: In Nemeth Code, the ELI is not used with a Roman numeral if the numeral ...

i. ... consists of two or more unspaced capitalized letters in regular type;

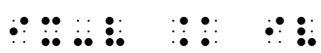
⋙ II III VIII XV 

ii. ... immediately precedes or follows a sign of comparison;

⋙ X = 10 

⋙ 1000 = M

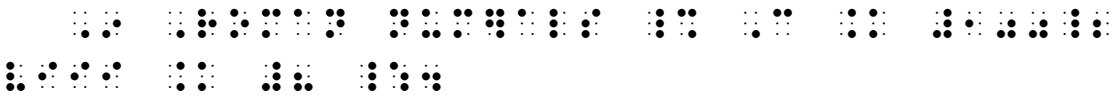
iii. ... is in an expression consisting of a sequence of unspaced mathematical symbols.

⋙ ix − v = iv 

iv. ... is entirely enclosed between grouping signs.

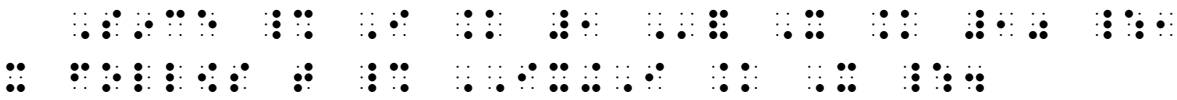
⋙ (ii) 

Example 5.3-4 In Roman numerals C = 100; viii = 8.



No ELIs: The Roman numerals immediately precede a sign of comparison (Reason ii).

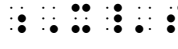
Example 5.3-5 Since I = 1 and X = 10, it follows that IX + I = X.



In IX + I, notice that "I" has its own capitalization indicator. Also note that IX and I are unspaced from the plus sign, following the same spacing rules for numerals and a sign of operation. No ELIs: Four of the Roman numerals immediately precede or immediately follow a sign of comparison (Reason ii); "IX" is in an expression consisting of a sequence of unspaced mathematical symbols (Reason iii).

5.3.6 Possessive Endings: When a Roman numeral has a possessive ending, the ELI is used or is not used as though the ending was not present.

⋙ v's 

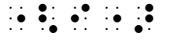
⋙ X's 

⋙ xix's and XIX's 

The BANA Nemeth Code Technical Committee is discussing details regarding plural and ordinal endings with Roman numerals. This section will be completed after decisions are made.

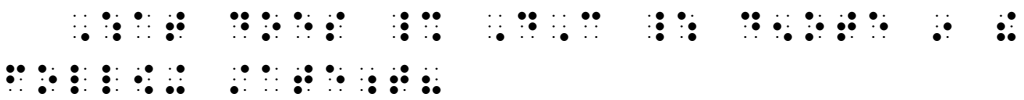
5.3.7 Large Roman Numerals: Roman numerals starting with 5,000 include a line over the numeral in print. This notation will be discussed in a later lesson.

5.4 Roman Numerals Used as Identifiers: Identifiers are transcribed according to the rules for the code in use. Compare these isolated examples of Roman numeral identifiers, noting the use of indicators and the construction of the punctuation or grouping symbols.

	UEB	Nemeth Braille
i.		
ii.		
(i)		
(ii)		

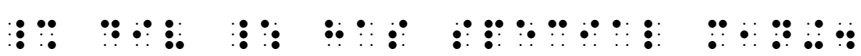
5.5 Mathematical Letter Combinations Similar to Roman Numerals: When it is unclear whether a mathematical letter combination is a Roman numeral, the combination is treated as if it were <u>not</u> a Roman numeral. In such cases, the letter combination is brailled in Nemeth Code. The letters are treated individually and the ELI is used or is not used in accordance with the rules for English letters. (Review **4.14** in Lesson 4.)

Example 5.5-1 What does DC denote in the following statement?



Out of context it is not clear whether "DC" means the Roman numeral "600" or if it is referring to a line segment. Therefore, the letters are transcribed as a mathematical letter sequence. A switch to Nemeth Code is required.

Example 5.5-2 div has special meaning.



Out of context it is not clear whether "div" means the Roman numeral "504" or if it is a special mathematical term. Therefore, the letters are transcribed as a mathematical letter sequence. A switch to Nemeth Code is required.

PRACTICE 5B

i. Triangle ABC in Quadrant IV is reflected in Quadrant III as Triangle A'B'C'.

ii. iv + vi = x

iii. X = 10, L = 50, C = 100, and D = 500.

iv. Review items v and vi.

v. Explain why MC = 1100, but CM = 900.

vi. Read items i, i′, ii, and ii′.

Non-Decimal Bases

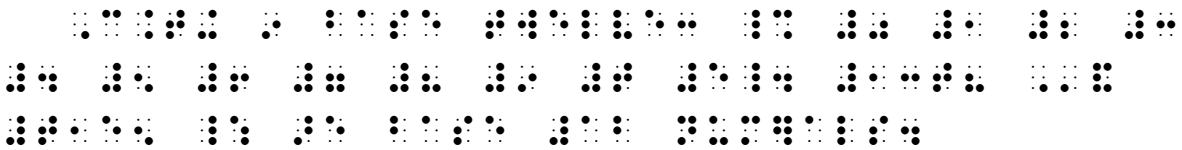
5.6 Letters Used to Represent Numerals in Non-Decimal Bases: When a system of numeration is to a base larger than 10, additional digits are devised to represent digits beyond the ten Arabic numerals. A common technique for providing additional digits is to use letters. For example, in base 12 "t" or "T" may represent ten and "e" or "E" may represent eleven. These letters do not function as letters – they are digits and are indicated as such by use of the numeric indicator. Only uncapitalized letters are used in braille, even when capital letters are used to represent non-decimal numerals in print.

⋙ t or T ⠼⠞

⋙ e or E ⠼⠑

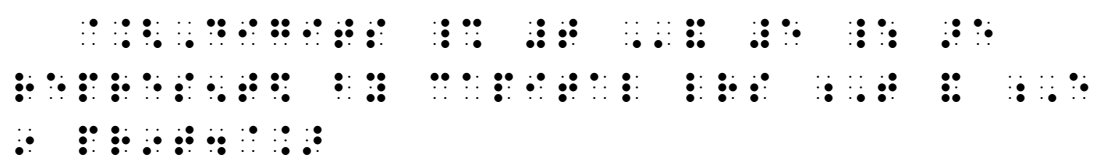
The rules regarding the use (or nonuse) of the numeric indicator for non-decimal digits are the same as the rules for the ten Arabic numerals 0 through 9. Numerals in non-decimal bases are mathematical symbols and are punctuated accordingly.

Example 5.6-1 Counting in base twelve: 0 1 2 3 4 5 6 7 8 9 T E. 13T8 and T1E5 are base 12 numerals.



5.6.1 Transcriber's Note Required: If the print copy uses capital letters, a transcriber's note is required to inform the reader of a change in capitalization in the braille transcription. Transcriber's notes are written outside of the Nemeth Code switch indicators, following UEB rules. The note itself can contain mathematical material, in which case code switching occurs within the note, but Nemeth Code must be terminated before the closing transcriber's note indicator is brailled.

Example 5.6-2 Digits t and e are represented by capital letters T and E in print.



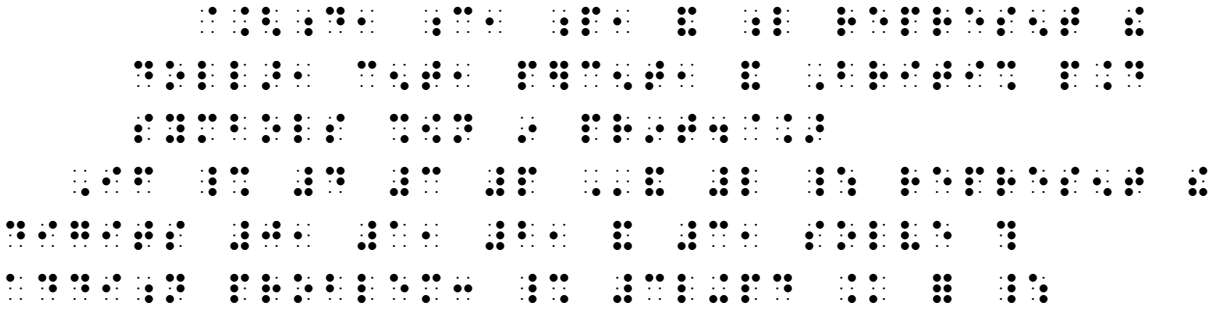
Or simply this:

Example 5.6-3 t and e are capitalized in print.



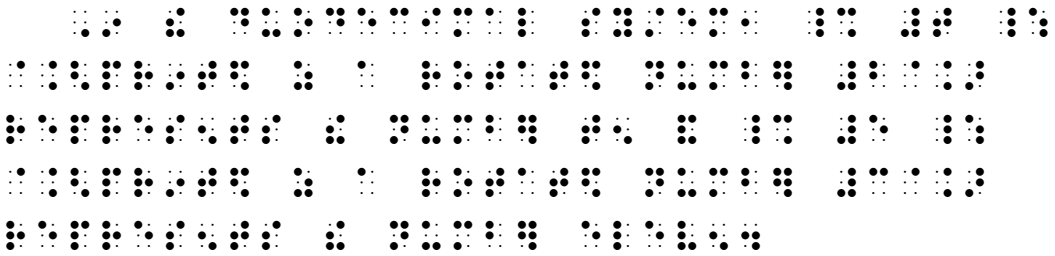
5.7 Non-Alphabetic Symbols Used to Represent Numerals: If symbols other than letters represent digits, the transcriber should choose one-cell symbols to represent the special signs. The preferred method is to select letters of the English alphabet in a similar manner as described above. A transcriber's note must specify the meanings assigned to these letters.

Example 5.7-1 If $ ¢ % and £ represent the digits 0, 1, 2, and 3, solve this addition problem: ¢£ + %$ = ?



If the print sign lacks a symbol in the Code, the transcriber's note should include a drawing or a description in order to identify it.

Example 5.7-2 In the duodecimal system, ⅀ represents the number ten and Ɛ represents the number eleven.



The two print symbols are described in embedded transcriber's notes:"printed as a rotated number 2 " and "printed as a rotated number 3 ".

PRACTICE 5C

I. In the hexadecimal system (base 16), the number "one thousand" is written as 3e8.
II. Convert hex 7A1 to decimal numeration.

OTHER ALPHABETS

5.8 Alphabetic Indicators: The language of mathematics uses letters from more than just the English (Roman) alphabet. Specific provision is made in the Nemeth Code for the transcription of the letters of the German, Greek, Hebrew, and Russian (Cyrillic) alphabets. Each alphabet has a unique alphabetic indicator.

5.8.1 Code Switching and Use of Letter Indicators: The English letter indicator was introduced in **Lesson 4**. Recall that switching to Nemeth Code to braille an English letter is not always required, and that the Nemeth Code English letter indicator may be omitted in certain circumstances. *In contrast, an alphabetic indicator is always required to identify a letter from the German, Greek, Hebrew, or Russian alphabets and a switch to Nemeth Code is always required for such letters even if UEB has a symbol for the letter.*

5.8.2 Capitalization and Punctuation: When a letter from any alphabet is capitalized in Nemeth Code, the capitalization indicator (dot 6) is placed between the alphabetic indicator and the letter. Letters are individually capitalized—the effect of the capitalization indicator extends only to the letter which follows it. In a technical transcription, letters from the German, Greek, Hebrew, and Russian alphabets are mathematical symbols and so are punctuated mathematically when the punctuation falls within the Nemeth Code switch.

Certain letters have unique mathematical applications. If you are unsure of a letter, find an expert who can identify it. Do not guess.

The Greek Alphabet

5.9 Greek Alphabet: Many letters from the Greek alphabet are used in mathematics and science. The following indicator identifies a letter as being from the Greek alphabet.

Greek Letter Indicator (standard form) 

This symbol is read as the Greek letter indicator only when immediately followed by a letter or by the capitalization indicator and a letter. The Nemeth Code table of Greek letters is reproduced on the next page.

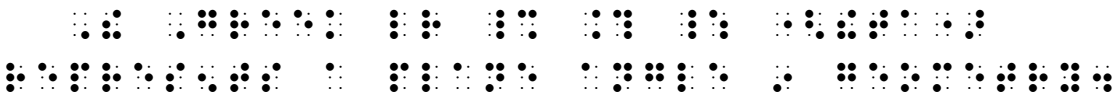
GREEK ALPHABET TABLE

Name of letter	Regular uncapitalized	Regular capitalized	Alternative form
alpha	α	A	$\propto$
beta	β	B	β
gamma	γ	Γ	
delta	δ	Δ	
epsilon	ϵ	E	
zeta	ζ	Z	
eta	η	H	
theta	θ	Θ	ϑ
iota	ι	I	
kappa	κ	K	
lambda	λ	Λ	
mu	μ	M	
nu	ν	N	
xi	ξ	Ξ	
omicron	o	O	
pi	π	Π	
rho	ρ	P	
sigma	σ	Σ	ς
tau	τ	T	
upsilon	υ	Υ	
phi	ϕ	Φ	φ
chi	χ	X	
psi	ψ	Ψ	
omega	ω	Ω	
sampi	ϡ		
stigma	ϛ		
vau	Ϝ		
koph (qoph)	Ϙ		

5.9.1 Code Switching with Greek Letters: Even though the uncapitalized form of the Greek letters in Nemeth Code is identical to the uncapitalized form in UEB, you must switch to Nemeth Code when a Greek letter appears in a technical transcription, even in narrative context. Greek letters used in the following examples are listed in the box below. Notice the placement of the capitalization indicator in the two capitalized letters. As stated in 5.8.2, the capitalization indicator is placed *between* the alphabetic indicator and the letter.

Delta	Δ	
pi	π	
Sigma	Σ	
theta	θ	

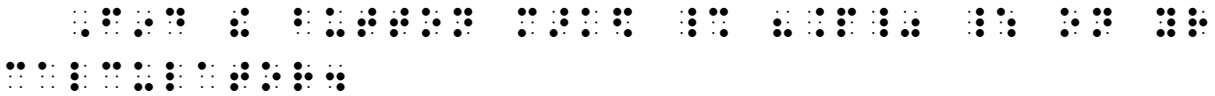
Example 5.9-1 The Greek letter θ (theta) represents a plane angle in geometry.



Example 5.9-2 $\pi < 0 < 2\pi$



Example 5.9-3 Find the button marked "π" on your calculator.



Greek letters are mathematical symbols and so are punctuated mathematically.

5.9.2 Alternate Form of Greek Letters: The Greek alphabet table shows an alternate uncapitalized print form for five of the Greek letters. The following indicator is used to identify the alternate forms.

Greek Letter Indicator (alternate form)	

The alternative form is used in braille only when both forms—standard and alternative—appear in the same print text. If a Greek letter is represented by its alternative form *instead of* its standard form throughout the print text—that is, only one form of the letter is used throughout—the symbol for the standard form is used in braille. Include a transcriber's note at the beginning of the text to inform the reader of the change in braille usage. For example,

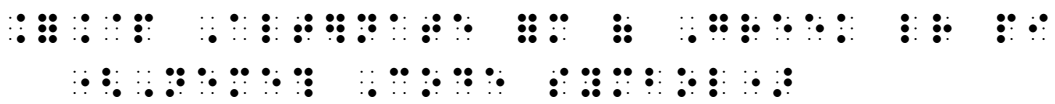
Example 5.9-4 *(Suggested wording for the Transcriber's Notes page is shown below.)*

> The alternate form of the Greek letter theta is used exclusively in print. In braille, the standard form is used.

If a text shows an alternate form of a Greek letter that does not appear in the table, follow the guidelines above to determine if you should substitute the regular form or if you should use the alternate Greek letter indicator. If the alternate form is used, list those symbols on the Special Symbols page since they do not appear in the Nemeth Code. For example, here are four alternate forms that do not appear in the braille table:

epsilon ε kappa $\varkappa$ pi ϖ rho ϱ

Example 5.9-5 *(Special Symbols page)*



If the letter's identity is not clear from context, consult an expert in the field in order to determine its designation.

PRACTICE 5D

Δx means "the change in x" and Δy means "the change in y". When x increases by Δx, y increases by Δy as expressed in the equation $y = \Delta y = f(x + \Delta x)$.

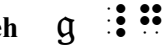
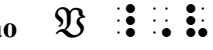
Although the handwritten form of phi (φ) may be found in source materials, only the standard form (ϕ) is used in this book.

The German Alphabet

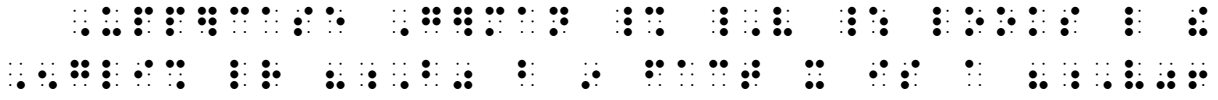
5.10 German Alphabet: German letters used as mathematical symbols must be brailled in Nemeth Code. The following indicator is used to identify a letter as being from the German alphabet.

German Letter Indicator

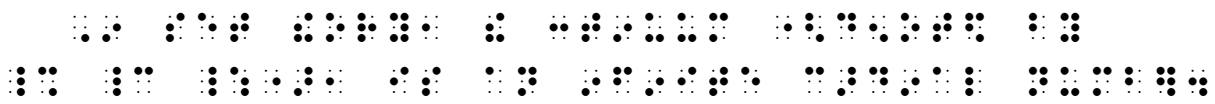
German letters are most commonly encountered in specialized fields of mathematics and science. Refer to the table of German letters in *The Nemeth Code for Mathematics and Science Notation* to determine the identity of the letter and see its braille equivalent. Three examples are shown below.

tseh $\mathfrak{c}$ 	**gheh** $\mathfrak{g}$ 	**Fao** $\mathfrak{V}$ 

Example 5.10-1 Uppercase German $\mathfrak{V}$ looks like the English letter "B" but in fact it is a "V"!



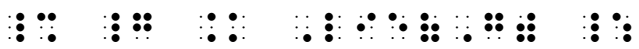
Example 5.10-2 In set theory, the continuum (denoted by $\mathfrak{c}$), is an infinite cardinal number.



German letters may be encountered in the study of set theory.

The German letter is sometimes associated with the same letter from the Roman (English) alphabet, which may help you identify it.

Example 5.10-3 $\mathfrak{g} = \mathrm{Lie}(G)$



German letters may be encountered in the study of Lie algebra.

The Hebrew Alphabet

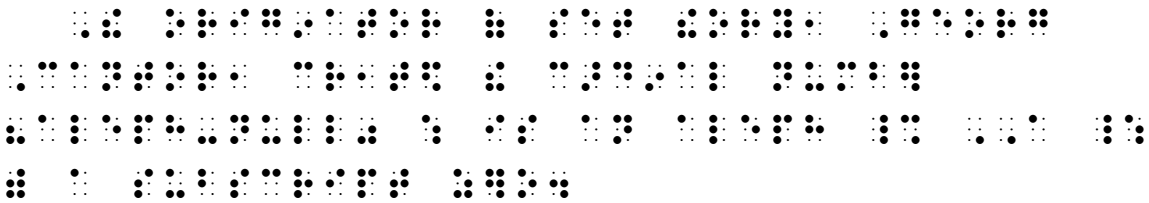
5.11 Hebrew Alphabet: Hebrew letters used as mathematical symbols must be brailled in Nemeth Code. The following indicator is used to identify a letter as being from the Hebrew alphabet.

Hebrew Letter Indicator ⠨⠄

The Hebrew alphabet has no capitalized form. The letter most commonly-encountered in technical material is the aleph: ℵ The aleph is usually written with a subscript, and so will be further discussed in the lesson on level indicators. For a complete list of the Hebrew letters and their braille equivalents, see *The Nemeth Code for Mathematics and Science Notation.*

aleph ℵ ⠨⠄⠆

Example 5.11-1 The originator of set theory, Georg Cantor, created the cardinal number "aleph-null" which is an aleph ℵ with a subscript zero.

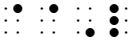
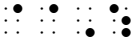


The Russian Alphabet

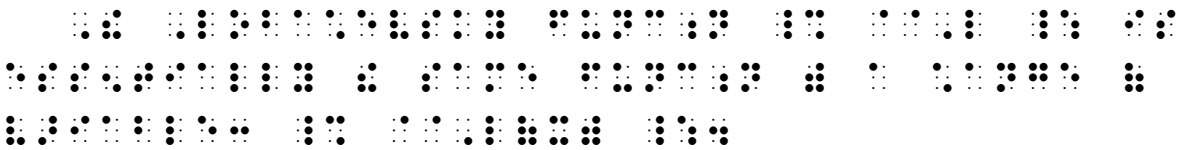
5.12 Russian Alphabet: Russian (Cyrillic) letters used as mathematical symbols must be brailled in Nemeth Code. The following indicator is used to identify a letter as being from the Russian alphabet.

Russian (Cyrillic) Letter Indicator	

Two Cyrillic letters in common usage are Sha which is used in number theory and Ell (also the lowercase ell) which is used in hyperbolic geometry. The Sha usually keeps company with bold and double-struck letters, and so will be further discussed in **Lesson 7**. For a complete list of the Cyrillic letters and their braille equivalents, see *The Nemeth Code for Mathematics and Science Notation.*

ell	л	
Ell	Л	
Shah	Ш	

Example 5.12-1 The Lobachevsky function Л is essentially the same function with a change of variable: $Л(x)$.



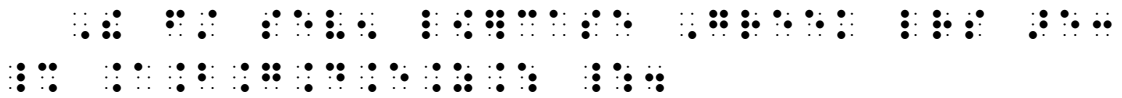
Use the Russian alphabet table in your Nemeth Braille codebook to determine how to braille the Cyrillic letters in sentence 2.

PRACTICE 5E

1) Be sure to differentiate between the Cyrillic letters "ell" Л and "Ell" Л and the Greek letters "pi" π and "Pi" П.
2) The first ten uncapitalized Cyrillic letters are: ah а, beh б, veh в, gheh г, deh д, yeh е, zheh ж, zeh з, ee и, and kah к.

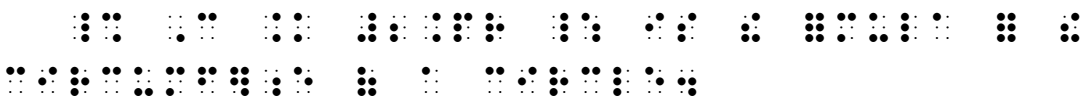
5.13 A Sequence of Unspaced Letters: The effect of an alphabetic indicator extends only to the letter which follows it. Thus, in a sequence of unspaced letters from non-Roman alphabets, the appropriate alphabetic indicator is used before each letter.

Example 5.13-1 The first seven lowercase Greek letters are: $\alpha\beta\gamma\delta\varepsilon\zeta\eta$.

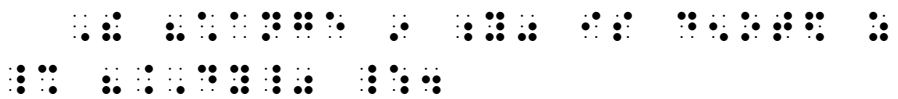


An English letter in regular type which appears in an unspaced sequence of terms does not require a letter indicator.

Example 5.13-2 $C = 2\pi r$ is the formula for the circumference of a circle.



Example 5.13-3 The "change in y" is denoted as "Δy".



5.13.1 Derivatives: The English letter combinations "dx", "dy", etc. often used in differential notation are usually spaced away from surrounding characters in print in order to enhance recognition. The space is omitted in braille unless another Nemeth Code rule requires a space. Print may show the letter d in italics or in regular type; either way, the letter is not italicized in braille.

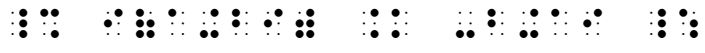
Example 5.13-4 $(x + y)\,\mathrm{d}x\,\mathrm{d}y =$ ___



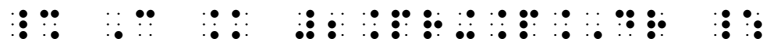
In print there is a space before each "d".

5.14 Mathematical Constant: A mathematical constant is a special number whose value is non-varying ("constant") and is represented by a certain alphabetic character. Two common examples are the Greek lowercase pi π and the English letter *i*. Constants are usually printed in italics uniformly throughout a technical document. In both UEB and Nemeth braille, constants are brailled as regular type unless other circumstances require the typeface to be maintained. (Typeforms are presented in a later lesson.)

Example 5.14-1 $i(a + bi) = -b + ai$



Example 5.14-2 $C = 2\pi r + \pi\Delta r$



Revisiting Code-Switching Considerations: Placement of code switches is the challenge in this practice. Apply normal (3-1) paragraphing margins to each sentence in the practice. Judicious placement of code switches will result in clarity for the reader. After brailling each sentence, write down your reasons for switching where you did. Then compare your decisions to the answer key and commentary at the end of the lesson.

PRACTICE 5F

Variables a and b are inversely related.

There exists a constant N such that no bit of Ω after the Nth can be proven to be 1 or 0.

Randall replied, "$12-n, 11-n, 10-n$ ... which is correct?"

"$12-n, 11-n, 10-n$, ..."

Which is correct: "$4x+3y$," "$3x+4y$," or "$4x+4y$"?

What is the remainder when 101 is divided by 3 ($101 \div 3$)?

The result is $(ax+by)(cx+dy)$, where all components are real.

($4x+3y$ is the denominator.)

ENCLOSED LISTS

5.15 SPECIAL CASE—Definition of "Enclosed List" Special provision is made for the transcription of a sequence of mathematical items enclosed within grouping signs. All of the following must be true in order to apply this rule.

i. The sequence must begin and end with a sign of grouping. The grouping signs do not have to be of the same kind.

ii. The list must have at least two items and the items must be separated by commas.

iii. An item of the list may be any sign used for omission – for example, an ellipsis or a long dash.

iv. The list cannot contain any punctuation mark other than the separating commas. (The omission ellipsis or long dash are not considered to be punctuation.)

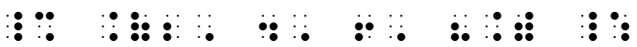
v. The list cannot contain any words, abbreviations, ordinal endings, or plural endings.

vi. The list cannot contain a sign of comparison.

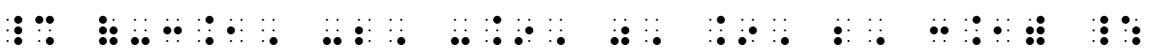
An enclosed list must be brailled in Nemeth Code, within the code switches, even if the items in the list are simple numerals or letters. The enclosure symbols are part of the mathematical notation, using the appropriate Nemeth Code grouping symbols.

5.15.1 Nonuse of the Numeric Indicator in an "Enclosed List": A numeric indicator is not used before a numeral or before a decimal point and a numeral in an "enclosed list".

Example 5.15-1 $\{2, 4, 6, 8\}$



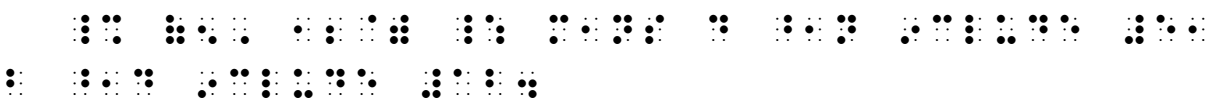
Example 5.15-2 (−3.1, −2, −.9, 0, .9, 2, 3.1)



Example 5.15-3 Consider the set $\{5 + 5, 10 + 10\}$.

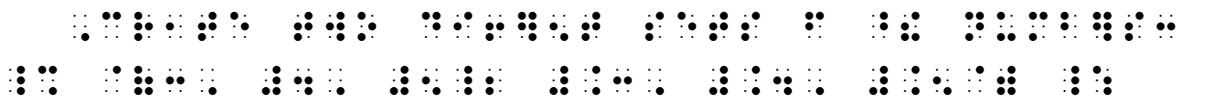


Example 5.15-4 (5, 12] means do **not** include 5, but **do** include 12.



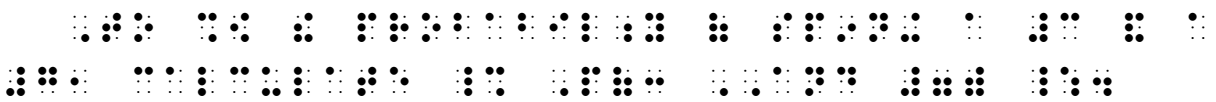
The next two examples do not satisfy the definition of an "enclosed list". A numeric indicator is brailled where required.

Example 5.15-5 Create two different sets from these numbers: [3, 4, 5; .3, .4, .5]



This is not an "enclosed list" because a semicolon is used. A numeric indicator is required before all but the first numeral.

Example 5.15-6 To show the probability of spinning a 3 and a 7, calculate P(3 AND 7).



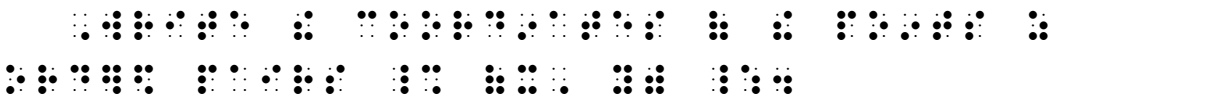
This is not an "enclosed list" because there are no commas and because it contains a word. A numeric indicator is required before the numeral 7. The word AND has mathematical significance and so is brailled in Nemeth Code without contractions. (The words AND, OR, and NOT may be used as mathematical operators in the fields of probability and logic.)

5.15.2 Nonuse of the English Letter Indicator in an "Enclosed List": In an "enclosed list", the English letter indicator is not used with any English letter or combination of letters in regular type.

Example 5.15-7 (a, 2x, … , b, ab)



Example 5.15-8 Write the coordinates of the points as ordered pairs (x, y).



The ELI is not used with a Roman numeral in regular type.

Example 5.15-9 (i, ii, iii, iv)



A letter from another alphabet must retain the appropriate alphabetic indicator in an "enclosed list".

Example 5.15-10 (α, a, β, b)

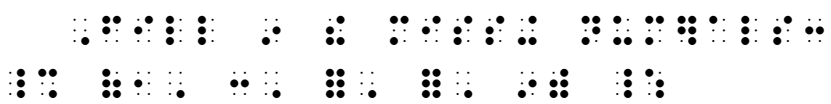


The Greek letters alpha and beta are in this enclosed list.

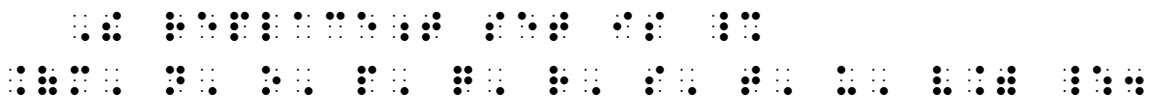
Note: There are situations where a numeric indicator or an English letter indicator is required in an enclosed list. More information will be in the lessons on functions and shapes.

5.15.3 FORMAT: Keep Together: Items in an enclosed list must not be divided between braille lines if the entire list will fit on a single braille line.

Example 5.15-11 Fill in the missing numerals: $(1, 3, ?, ?, 9)$

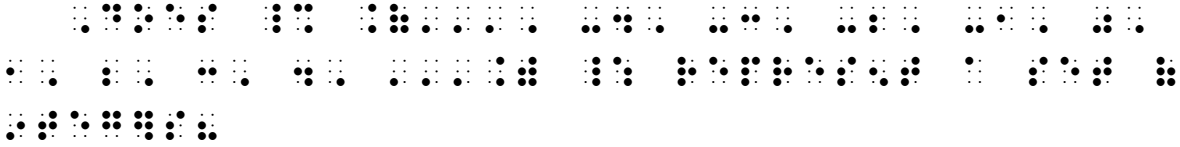


Example 5.15-12 The replacement set is $\{m, n, o, p, q, r, s, t, u, v\}$.



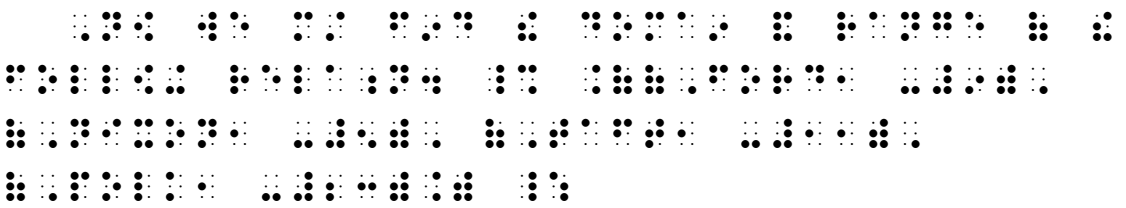
5.15.3.a Division Between Lines: If the enclosed list will not fit on a single braille line, use as much of the current line as possible and begin a runover line after a comma. When the items in an enclosed list must be divided between braille lines, neither the numeric indicator nor the English letter indicator is used before the runover on the new line.

Example 5.15-13 Does $\{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$ represent a set of integers?



Although the next example is not an "enclosed list" according to the Nemeth Code (it contains words), rules regarding division between braille lines are followed.

Example 5.15-14 Now we must find the domain and range of the following relation:
{(Ford, –9), (Nixon, –5), (Taft, –11), (Polk, –23)}

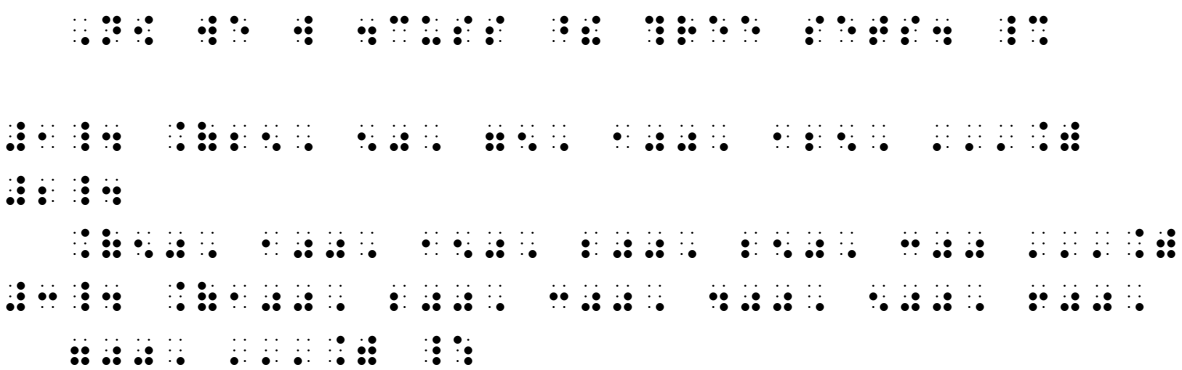


Even though "(Polk," will fit on line 3, the pair enclosed in parentheses "(Polk, –23)" is kept together on one braille line.

Remember, if a math expression will fit on one line, do not divide it. See how this rule applies in an itemized format.

Example 5.15-15 Now we will discuss these three sets.

1. {25, 50, 75, 100, 125, ...}
2. {50, 100, 150, 200, 250, 300 ...}
3. {100, 200, 300, 400, 500, 600, 700, ...}



Item 1 requires no division.
Item 2: When the enclosed list is brought down to the next line (starting in the runover cell, cell 3) no division is needed. This keeps the enclosed list all together on one line.
In item 3, division is unavoidable.

Instructions: First determine if each item is or is not an "enclosed list". Write YES if the item is an "enclosed list" and NO if it is not. Then transcribe the entire practice using Nemeth Code throughout.

PRACTICE 5G

{a, b, c, d}

(−1, −2, −3)

(h ft, k in)

(ab, cd, ef)

1, i, −1, −i

(1, i, 2, ii)

(1st, 2nd, 3rd)

(A, A', B, B', C)

{___, .3, .5, .7, ___}

(1 + h, 2 + k, 0)

(x = 1, 2, ..., 10)

(a, b]

(1 2 3)

[0, 1]

(u, v; x, y)

{(Denver, 19), (Utah, 27), (Minnesota, 24), (San Antonio, 28)}

(a, b, ...)

(x + 1, x + 2, ?, ?, x + 5)

<−1, 0]

(2, 4, 6, ___, 10)

(0, a, 1, b, 2)

{1's, 2's, 3's}

MORE ABOUT ENGLISH LETTERS

5.16 English Letters and Grouping Symbols: In **Lesson 4** you learned that the ELI is not needed when a "single letter" or a letter grouping that corresponds to a shortform is enclosed between mathematical grouping symbols.

⋙ (c) ⠐⠉⠐ *compare to* "c" ⠐⠐⠉⠐

⋙ (alm) ⠐⠇⠍⠐ *compare to* "alm" ⠐⠐⠇⠍⠐

However, when a "single letter" or a letter grouping that corresponds to a shortform is in direct contact with only its opening or only its closing grouping sign, and the letter is not an item in an "enclosed list" as defined above, rules regarding the English letter indicator are applied as though the grouping sign was not present.

Example 5.16-1 $(k = 1, 2, \ldots, n)$.

⠐⠣ ⠰⠅ ⠐⠶ ⠼⠁ ⠼⠃ ⠠⠠⠠ ⠼⠝ ⠰⠜⠲

Without the opening parenthesis, the letter k would not need an ELI because it is followed by an equals sign. Without the closing parenthesis, the letter n would need an ELI because it is preceded by a space and followed by punctuation.

Example 5.16-2 Consider the set {m and n}.

⠠⠉⠕⠝⠎⠊⠙⠑⠗ ⠮ ⠰⠍ ⠯ ⠰⠝⠘⠷ ⠰⠍ ⠯ ⠰⠝⠘⠾

Without the opening brace, the letter m would need an ELI because it is preceded and followed by a space. Without the closing brace, the letter n would need an ELI because it is preceded by a space and followed by punctuation. Set notation is mathematical and so a switch to Nemeth Code is required. The word "and" is part of the math.

Example 5.16-3 If two events are mutually exclusive we write P(A AND B) = 0 where P(A AND B) means "the probability of A and B occurring at the same time".

⠠⠏⠐⠣⠠⠁ ⠠⠁⠝⠙ ⠠⠃⠐⠜ ⠊⠎ ⠉⠁⠇⠇⠑⠙ ⠞⠓⠑ ⠠⠠⠠

The same rule applies to a Roman numeral that is in direct contact with only an opening or closing grouping sign. The ELI is used or is not used as though the grouping sign was absent. The following example illustrates Roman numerals used as identifiers, assuming uninterrupted mathematical context.

⋙ i)

⋙ iv)

⋙ v)

If the grouping sign includes a prime or other modifying symbol, the ELI is not used with the single English letter that touches the (modified) grouping symbol.

⋙ t]'

Example 5.16-4 t]' and v]' have unique meaning.

The same rule applies to Roman numerals – if the grouping sign carries a prime sign, the Roman numeral is now considered to be part of an unspaced mathematical expression even if the context is not mathematical.

Example 5.16-5 Read sections X) and X)′.

Switch Decision: To maintain consistency, both section numbers are brailled inside the switches.

5.17 English Letters and Endings: When a "single letter" or a letter grouping that corresponds to a shortform has a plural, possessive or ordinal ending, in mathematical context the ELI rules of the Nemeth Code are applied as though such endings were not present. The following examples illustrate proper use of the ELI, assuming mathematical context. Note that the expressions are punctuated mathematically. The presence of a plural, possessive, or ordinal ending does not change the fact that the punctuation mode is mathematical.

Plural:

⋙ ps, qs, rs

Think: p, q, r – ELI is required

⋙ Xs, Ys, Zs

Think: X, Y, Z – ELI is required

Possessive: *Reminders: A punctuation indicator is required before an apostrophe; otherwise dot 3 is read as a prime sign. The ELI is not used with the letter "s" when it is part of the apostrophe-s combination.*

⋙ p's, q's, r's

Think: p, q, r – ELI is required

⋙ X's, Y's, Z's ⠠⠭⠄⠎ ⠠⠽⠄⠎ ⠠⠵⠄⠎

Think: X, Y, Z – ELI is required

Ordinal:

⋙ nth, 2nth ⠝⠹ ⠼⠃⠝⠹

Think: n – ELI is required; 2n – ELI is <u>not</u> required

Remember: Letter combinations require a switch to Nemeth Code.

⋙ ABs and GHs ⠠⠁⠠⠃⠎ ⠠⠛⠠⠓

Think: AB GH – Capitalize each letter individually, no ELI.

⋙ AB's and GH's ⠠⠁⠠⠃⠄⠎ ⠠⠛⠠⠓

Think: AB GH – Capitalize each letter individually, no ELI.

⋙ ab's and gh's ⠁⠃⠄⠎ ⠛⠓

Think: ab gh – "ab" requires ELI because it is the same as a shortform; "gh" does not.

⋙ abth and jkth ⠁⠃⠹ ⠚⠅⠹

Think: ab jk – Only the letter combination that is the same as a shortform requires an ELI.

Instructions: Stay in Nemeth Code to transcribe items C) and E).

PRACTICE 5H

A) Find all ABs, CDs, and EFs; draw XYZs.
B) Find all AB's, CD's, and EF's; draw XYZ's.
C) (1st, 2nd, ... nth, ... 49th)
D) Does $|a| \times |b| = |ab|$?
E) If Q, then {[NOT-P] OR P}.

MORE ABOUT ABBREVIATIONS

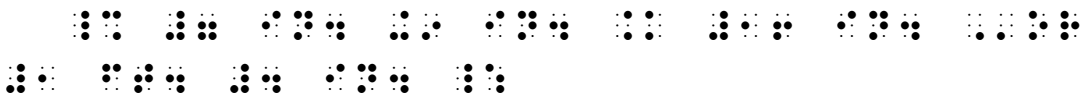
Abbreviation Reminders

- Abbreviations are not mathematical expressions although they may be part of a mathematical expression.
- A space comes between an abbreviation and its related value, even if no space is shown in print. This rule applies even in UEB context.
- An abbreviation and its related value must not be divided between braille lines. This rule applies even in UEB context.
- Between a two-word abbreviation, follow the same spacing as used in print and do not divide the abbreviation between lines.
- Abbreviations are punctuated in literary mode even in mathematical context.

5.18 More Spacing Rules

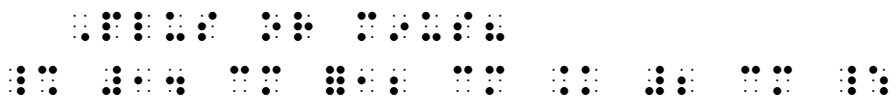
5.18.1: Spacing of Abbreviations With Operation Signs: A space is required between an abbreviation and a sign of operation.

Example 5.18-1 7 in. + 9 in. = 16 in. or 1 ft. 4 in.

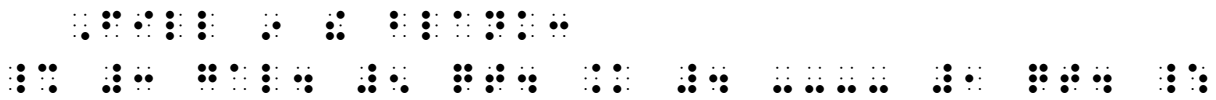


5.18.2 Spacing of Omission Symbols: If a sign of omission is used to represent an abbreviation, the omission symbol is spaced as the abbreviation which it replaces. This spacing rule is crucial to provide clarity in the braille transcription. Spacing in the print copy often does not follow this design and must be disregarded when applying spacing to the braille transcription.

Example 5.18-2 Plus or minus? 14 cm _?_ 12 cm = 2 cm

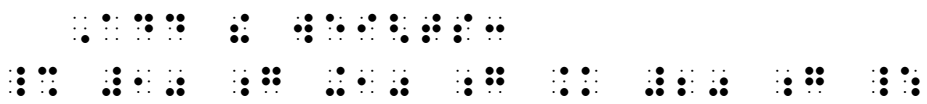


Example 5.18-3 Fill in the blank: 3gal.5qt. = 4___1qt.



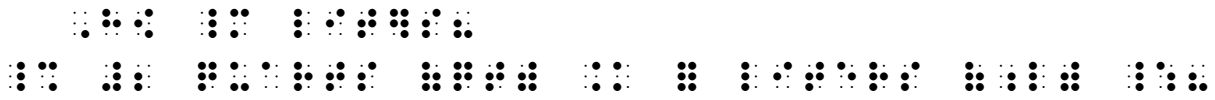
5.19 Single-Letter Abbreviations: A single-letter abbreviation from the English alphabet that does not have a related period must always begin with an English letter indicator. Rules for the nonuse of the ELI with mathematical "single letters" do not apply to abbreviations.

Example 5.19-1 Add the weights: 10 g + 10 g = 20 g



Even the "g" that is immediately followed by a comparison sign requires an ELI because "g" is a single-letter abbreviation without a related period.

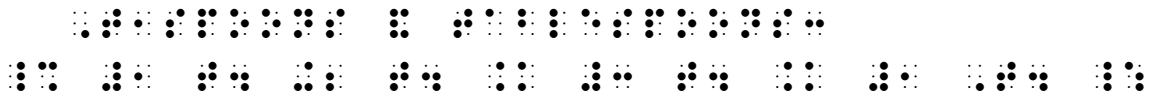
Example 5.19-2 How many liters? 2 quarts (qt) = ? liters (l)?



Even the "l" that is enclosed within parentheses requires an ELI because "l" is a single-letter abbreviation without a related period.

A single-letter abbreviation from the English alphabet that has a related period does not use an ELI.

Example 5.19-3 Teaspoons and tablespoons: 1 t. + 2 t. = 3 t. = 1 T.



Reminder: Abbreviations are punctuated in literary mode even in mathematical context.

5.20 Abbreviations Whose Letters Correspond to a Shortform: When an abbreviation whose letters correspond to a shortform does not have a related period, an ELI is required. Rules for the nonuse of the ELI with mathematical letter groupings that correspond to a shortform do not apply to abbreviations.

Example 5.20-1 1 lt-yr = $9.461e + 12$ km



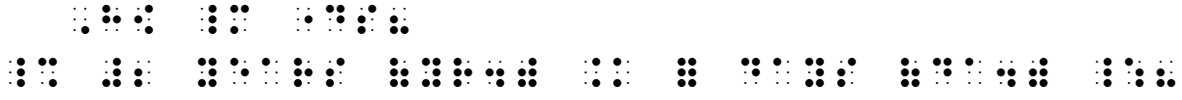
Even though "yr" is immediately followed by a comparison sign, an ELI is required because "yr" is an abbreviation whose letters corresponds to a shortform and there is no related period. The letter combinations "lt" and "km" do not correspond to a shortform so no ELI is needed. (The letter "e" is a constant and follows the rules of a "single letter".)

An ELI is not needed when an abbreviation includes a related period, even when the letters correspond to a shortform.

Example 5.20-2 1 yr. = 12 mo.



Example 5.20-3 How many days? 2 years (yr.) = ? days (da.)?



5.21 Context Clues: Look for context clues when an abbreviation ends a sentence. When in doubt about the function of a period at the end of a sentence, assume that the period applies to the abbreviation as well. Compare these three examples.

Example 5.21-1 Fact: 8 oz. = 1 c.



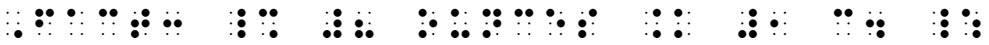
Because the abbreviation "oz." has a related period, treat the period after "c." in the same manner. The related period is brailled before the Nemeth Code terminator. No ELI is needed for the letter "c" because the abbreviation has a related period.

Example 5.21-2 Fact: 8 oz = 1 c.



Because the abbreviation "oz" does not have a period, treat the period after "c" as an end-of-sentence period only. The period is brailled outside of the switch. An ELI is needed for the letter "c" because the abbreviation does not have a related period.

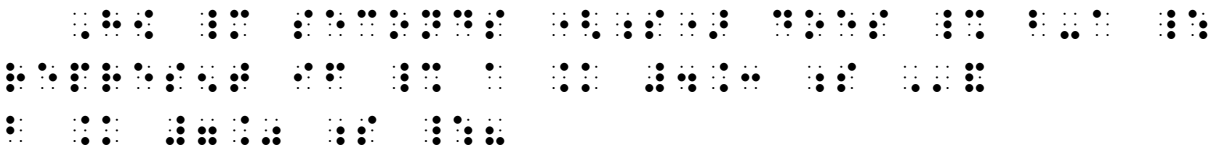
Example 5.21-3 Fact: 8 ounces = 1 c.



Within this short example, there are no context clues to determine if this period applies to the single-letter abbreviation "c". When in doubt, assume that the period <u>does</u> apply to the abbreviation. The period is brailled inside the switch. No ELI is needed.

In the next example, s is the abbreviation and *a* and *b* are the variables. The single-letter abbreviation requires an ELI; the variables do not.

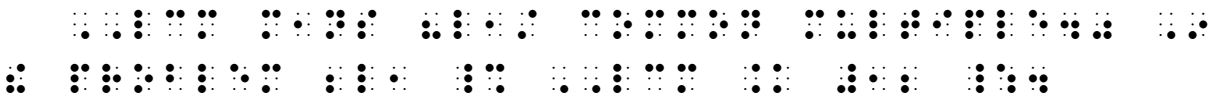
Example 5.21-4 How many seconds (s) does $b - a$ represent if $a = 4.3$ s and $b = 7.0$ s?



*Reminder: A variable is usually printed in italics; an abbreviation is printed in normal typeface. Review **<u>5.1.2</u>** "Abbreviation or Variable?"*

5.22 Fully Capitalized Acronyms: An abbreviation consisting of more than one capital letter is capitalized as a unit using the double capitalization indicator of the Nemeth Code.

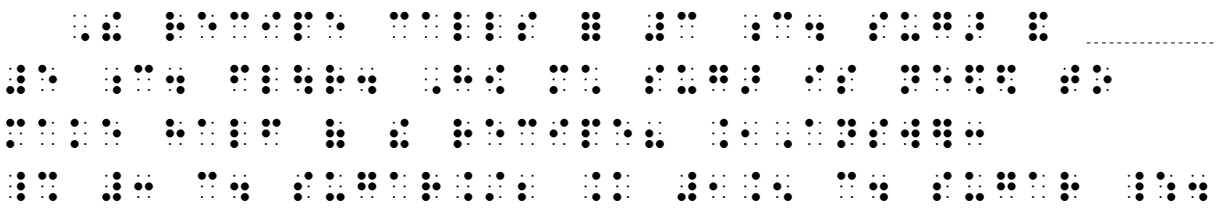
Example 5.22-1 LCM means "least common multiple." In the problem below, LCM = 12.



On line 2, the three-letter abbreviation (acronym) is part of the math.

5.23 UEB vs. Nemeth Code: Apply UEB rules in literary mode; apply Nemeth Code abbreviation rules in Nemeth mode.

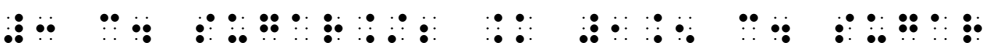
Example 5.23-1 The recipe calls for 3 c. sugar and 5 c. flour. How much sugar is needed to make half of the recipe? *Answer:* 3 c. sugar ÷ 2 = 1.5 c. sugar.



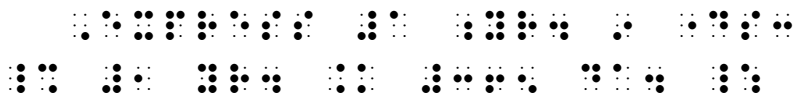
COMPARE: A UEB Grade 1 indicator is required for the abbreviation c. *in the literary portion of the example. Between the Nemeth Code switches, however, an abbreviation with its period does not need an English letter indicator. Recall that the abbreviation must appear on the same braille line as the number it labels, in either code.*

Take Another Look: In the equation, the word "sugar" is part of the math problem and so is brailled inside the switches, without contractions.

⋙ 3 c. sugar ÷ 2 = 1.5 c. sugar



Example 5.23-2 Express 1 yr. in days: 1 yr. = 365 da.



COMPARE: A UEB Grade 1 indicator is required for the abbreviation yr. in the literary portion of the example. Between the Nemeth Code switches, however, an abbreviation with its period does not need an English letter indicator.

5.24 Initiating Nemeth Code Before Itemized Material, Following a Heading: As seen in **Lesson 3**, in order to ensure that identifiers each begin in the same cell, the opening Nemeth Code indicator is placed at the end of the text that precedes the listed items. When an itemized set of problems immediately follows a heading, the following layouts are recommended.

5.24.1 Centered Heading: When itemized material immediately follows a centered heading, place the opening Nemeth Code indicator alone on the line immediately before the first identifier. A switch indicator alone on a line does not replace a necessary blank line.

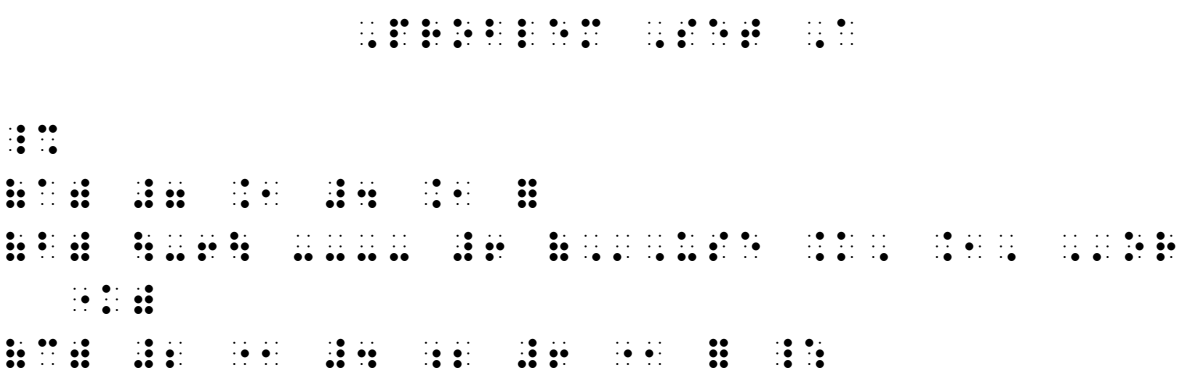
Example 5.24-1A

Problem Set A

(a) $7 > 4 > -?-$

(b) $|-6|$ __ 6 (Use $=$, $>$, or $<$)

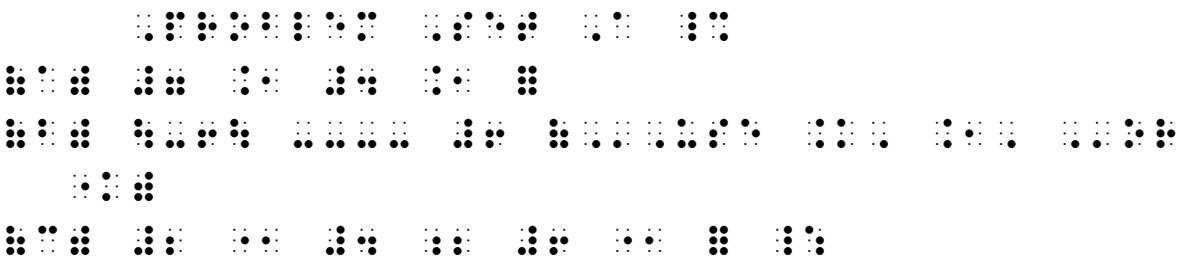
(c) $2 : 4 :: 6 : ?$



Line 2 is the blank line which is required following a centered heading.

5.24.2 Cell-5 and Cell-7 Heading: When itemized material immediately follows a cell-5 or a cell-7 heading, place the opening Nemeth Code indicator after the last word in the heading. If there is no room on the line, the indicator will fall in the runover cell of the heading.

Example 5.24-1B



The opening Nemeth Code indicator is brailled after the last word in the cell-5 heading.

Instructions: Review the "keep together" format rule for abbreviations and a preceding or following numeral to which it applies, presented in **Lesson 4**.

PRACTICE 51

A. 1 m = 100 cm

B. 3 yrs = 365 × 3 days

C. Draw three triangles using the given side lengths: (i) 1.5 cm, 5 cm, and 4.5 cm (ii) 4.5 cm, 5 cm, and 7.5 cm (iii) 1.5 cm, 4.5 cm, and 7 cm.

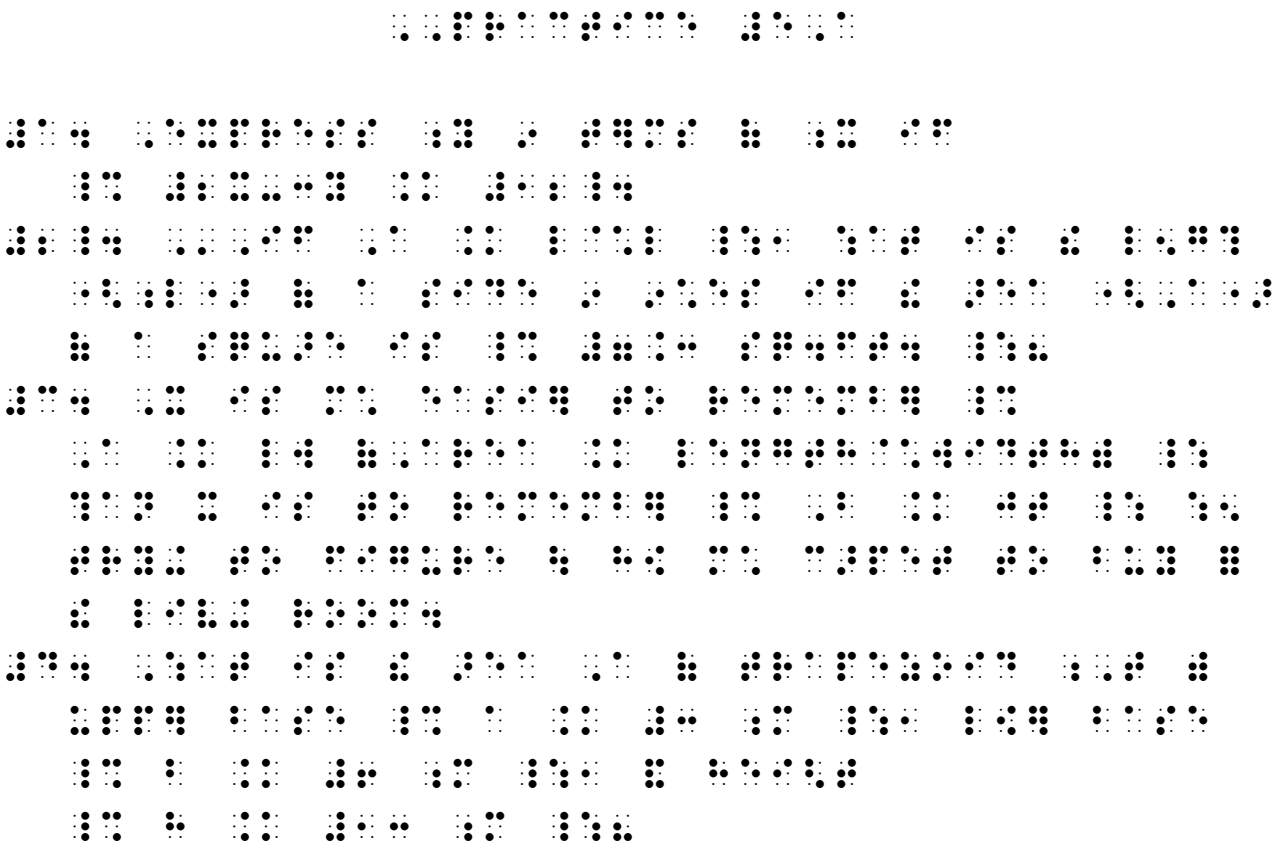
D. 1 square mile converted to acres: 1 sq mi = 640 ac

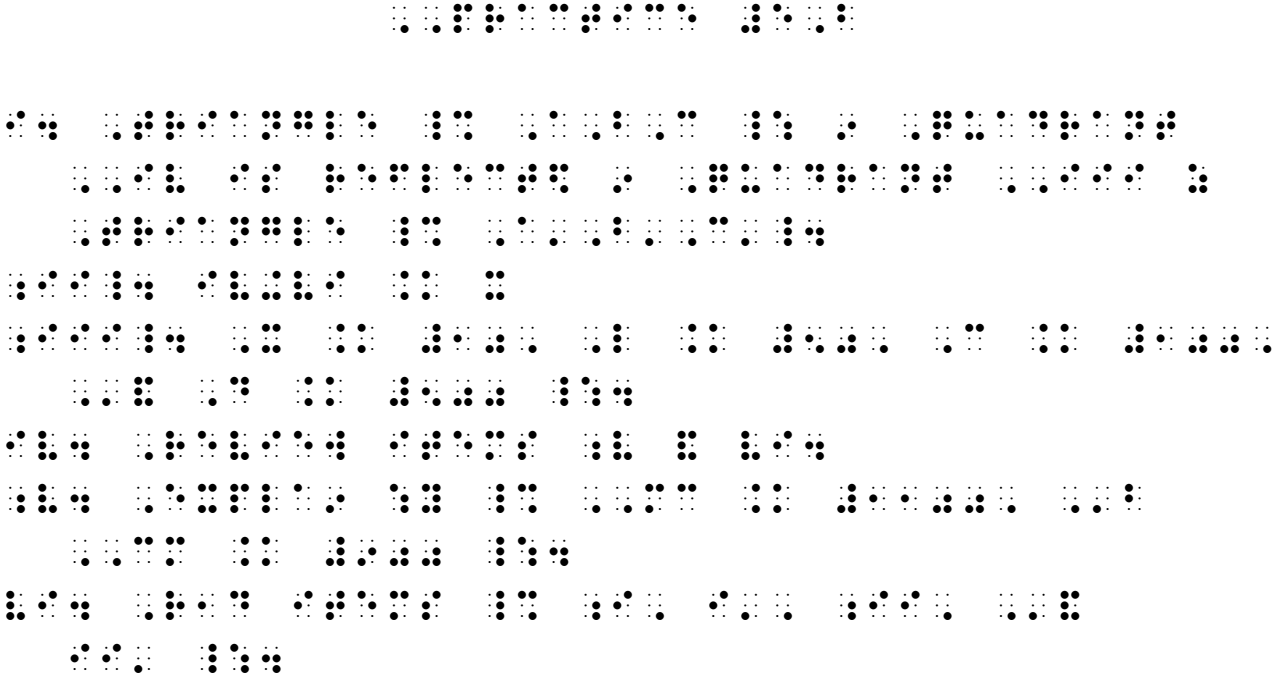
E. 5 in + 25 in = 30 in

F. Fill in the missing information in the Lifetime Value Formula using SAC (Subscriber Acquisition Costs): LTV = ___–SAC

For further practice, see Appendix A—Reading Practice.

ANSWERS TO PRACTICE MATERIAL





PRACTICE 5F

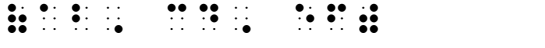
⠠⠠⠠

PRACTICE 5G

YES 

YES 

NO 

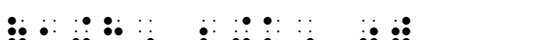
YES 

NO 

YES 

NO 

YES 

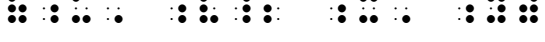
YES 

YES 

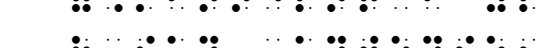
NO 

YES 

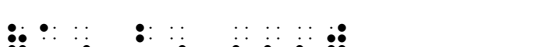
NO 

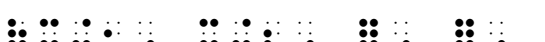
YES 

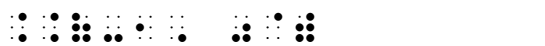
NO 

NO 



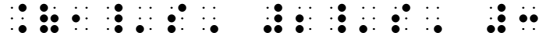


YES 

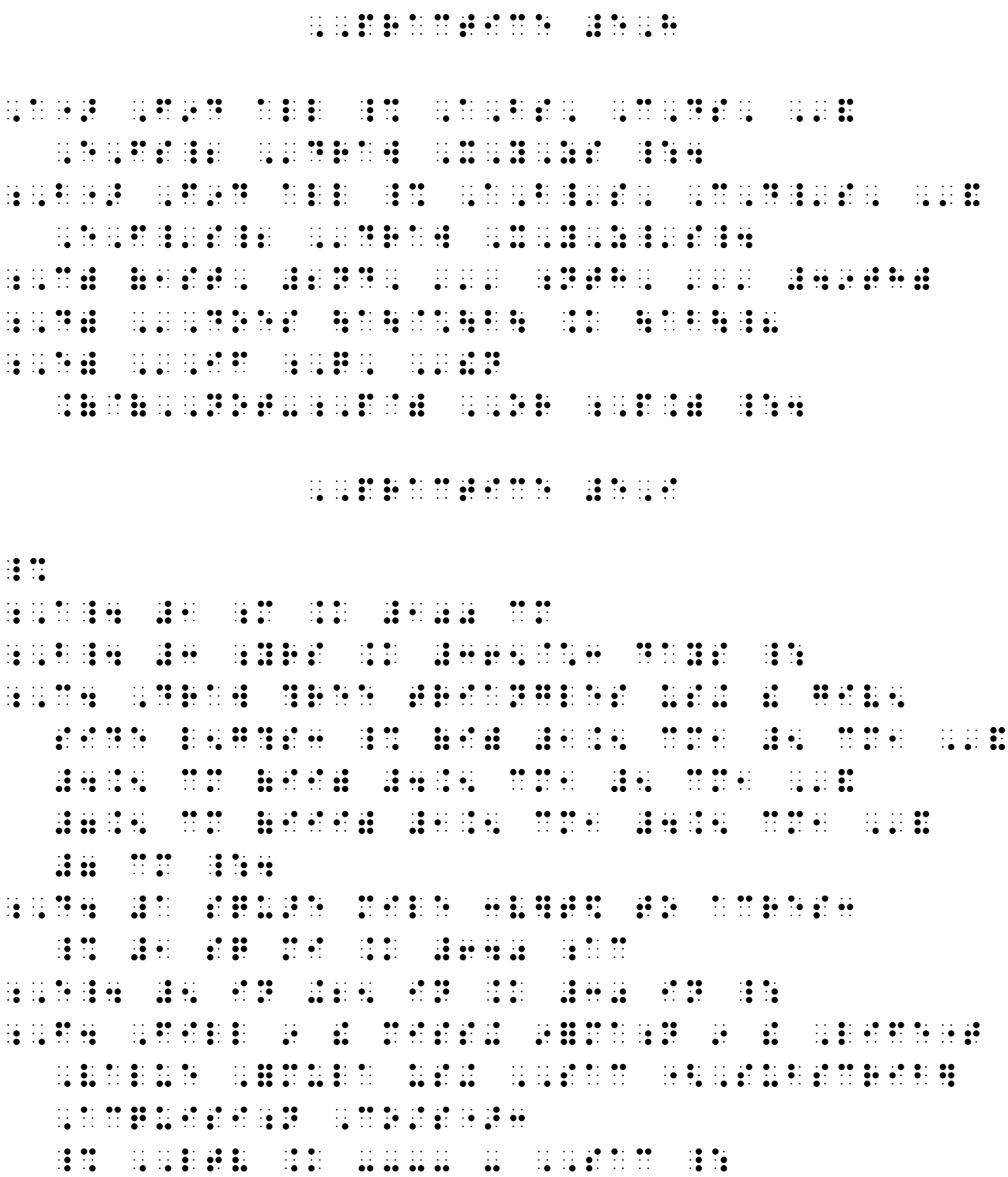
YES 

YES 

YES 

YES 

NO



EXERCISE 5

Exercise 5 will be available when this course is finished being written and is no longer "Provisional".

Proceed to Lesson 6.

LESSON 6

Read about this PROVISIONAL EDITION in the front matter to this book.
Check the NFB website periodically for updates to this lesson.

- *SIGNS OF OPERATION, cont.*
- *SIGNS OF COMPARISON, cont.*

FORMAT
Simple Tables
Instructions

This lesson introduces many more symbols that you will come across in all areas of mathematics. Look carefully at the print symbol as many look similar to each other. Understand the context – some symbols are used as signs of operation as well as signs of comparison. The function of the symbol must be determined in order to apply proper spacing rules.

SIGNS OF OPERATION, cont.

6.1 Review of Signs of Operation: In braille, no space is left before or after a sign of operation unless it is preceded or followed by a sign of comparison, an ellipsis, a dash, an unrelated word, or an abbreviation. Signs of operation are mathematical symbols and are punctuated accordingly. The following signs of operation have already been introduced.


+ Plus


− Minus


× Multiplication Cross


• Multiplication Dot


÷ Division

6.2 Signs of Operation Using Plus and Minus: The following signs are a combination of the plus and minus signs, written either side-by-side or one atop another.

"Plus or Minus"	±	
"Minus or Plus"	∓	
Plus followed by Minus	+ −	
Minus followed by Plus	− +	
Minus followed by Minus	− −	

Note that in the first two symbols, the upper sign is brailled first, followed immediately by the lower sign.

⋙ 38 ± 7 

⋙ 38 ∓ 7 

In the side-by-side symbols, a multipurpose indicator (dot 5) is inserted between the unspaced symbols. The multipurpose indicator indicates that the symbols are printed horizontally, not vertically.

Multipurpose Indicator	

⋙ $10 + -5$ 

⋙ $10 - +5$ 

⋙ $10 - -5$ 

Review the rules in **1.8.2** regarding the use/nonuse of the numeric indicator with positive and negative numbers.

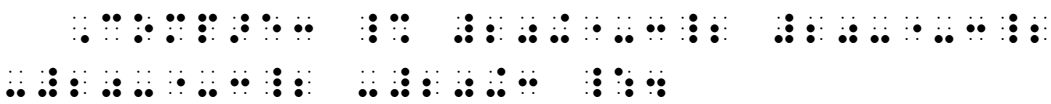
Example 6.2-1 ± 5 means $+5$ and -5.



Example 6.2-2 Can $3 \pm 1 = +4$ *and* $+2$?

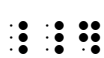
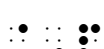
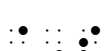


Example 6.2-3 Compare: $20 + -3;\ 20 - -3; -20 - -3;\ -20 + 3$.



CHALLENGE: If you come across the notation for "plus positive 3", as in $-20 ++3$, how would you braille $++$? Using the logic shown in the other side-by-side operation signs,  is the correct transcription. Because this symbol is not shown in the Nemeth Code, a transcriber's note explaining its print form ("plus followed by plus") would be required.

6.3 Signs of Operation That Look Like Literary Symbols: Some mathematical symbols use characters also seen in literary writing. When the following symbols are used in mathematical context (as a mathematical sign) the symbols shown below must be used.

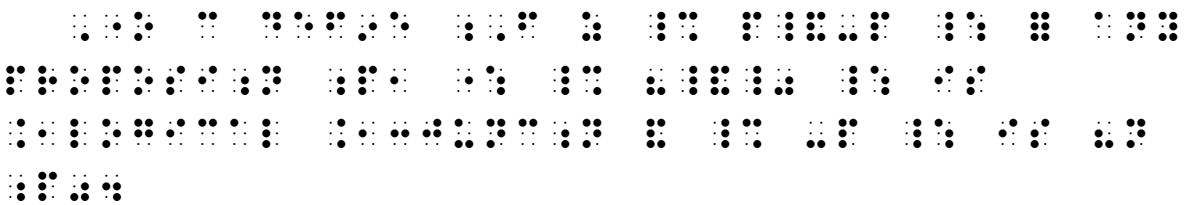
Ampersand	&	
Asterisk	*	
Crosshatch	#	
Dagger, Single	†	
Dagger, Double	‡	
Paragraph Mark	¶	
Section Mark	§	

When these symbols are used in literary context they have no mathematical meaning, therefore the symbols and rules of UEB apply. When these symbols are used as mathematical operation signs, the rules of the Nemeth Code apply and spacing rules for operation signs must be followed. *Note:* As mathematical symbols, they may also occur in the superscript position. This usage will be covered in a later lesson.

6.3.1 Ampersand: The ampersand as an operation symbol is commonly encountered in the study of logic, where it means "and". The symbol may be referred to as "logical conjunction".

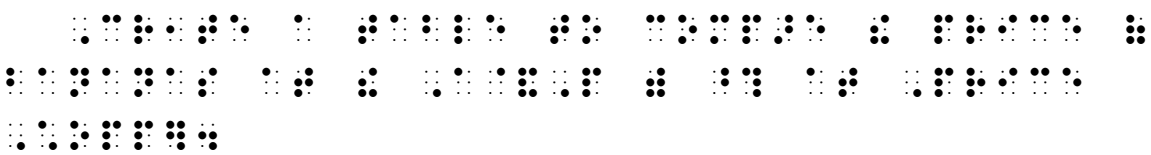
⋙ &

Example 6.3-1 One can define F as p & $-p$ for any proposition p, where "&" is *logical conjunction* and $-p$ is "not p".



When the ampersand is used as a literary device, follow the rules of UEB.

Example 6.3-2 Create a table to compare the price of bananas at the A&P with those at Price Chopper.



6.3.2 Asterisk: The asterisk as an operation symbol is commonly encountered in material about calculators where the symbol represents multiplication. When a numeral follows an asterisk, the numeric indicator is used even though the numeral is not preceded by a space.

⋙ *

⋙ $3 * 9 = 27$

⋙ $.3 * .9 = 0.27$

When an asterisk is used as a footnote symbol, follow the rules of UEB.

The BANA Nemeth Code Technical Committee is discussing details regarding the asterisk used as a reference symbol in mathematical context. This section will be completed after decisions are made.

6.3.3 Crosshatch: When the crosshatch represents a symbol of operation, the Nemeth Code symbol is used.

⋙ #

Example 6.3-3 What operations can the # symbol signify in $x \# y = y \# x$?

Example 6.3-4 $A\#B$ is the connected sum of the manifolds A and B.

When a numeral or a decimal point and a numeral follows a crosshatch, the numeric indicator is brailled even though the numeral is not preceded by a space.

⋙ $1 \# .2 = .2 \# 1$

When the crosshatch is used in literary context such as "Problem #1" or as a hashtag in a social media reference or URL, the UEB symbol may be used. When the crosshatch denotes pounds (weight), the Nemeth Code symbol is used.

Example 6.3-5 Notation Shortcut #4: "23 pounds" can be written "23#".

6.3.4 Dagger and Double Dagger: The dagger and double dagger may be used as operation symbols in binary operations.

⋙ †

⋙ ‡

⋙ A † B = B † A

⋙ A ‡ B = B ‡ A

When a dagger or a double dagger is used as a footnote symbol, follow the rules of UEB.

The BANA Nemeth Code Technical Committee is discussing details regarding daggers used as a reference symbol in mathematical context. This section will be completed after decisions are made.

6.3.5 Paragraph Mark: In mathematical context, the Nemeth Code symbol is used. A numeric indicator is required before a numeral following a paragraph mark.

⋙ ¶

⋙ A ¶ B

⋙ 3 ¶ 4 = 4 ¶ 3

When the paragraph mark is used as a literary device, follow the rules of UEB.

6.3.6 Section Mark: In mathematical context, the Nemeth Code symbol is used. A numeric indicator is required before a numeral following a section mark.

⋙ §

⋙ A § B

⋙ 3 § 4 = 4 § 3

When the section mark is used as a literary device, follow the rules of UEB.

Instructions: Review the spacing rules for operation signs before transcribing the practice. Transcribe this entire list in Nemeth Code.

PRACTICE 6A

$4 \pm 1,\ 400 \pm 10,\ 6 \mp 1,\ 600 \mp 10$

$\mu \pm 1.645\ \sigma$

$50 - +5 = 45$

$50 + -5 = ?$

$-3 - -3 = 0$

A&B = B&A

$a * (b * c) = (a * b) * c$

$(1 + 2) * (3 + 4) = 3 * 7$

#A = #B

.5#.7 = .7#.5

[(p † p) † (q † q)]

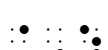
s ¶ t = u ¶ v

1 ¶ 3 = 4 ¶ 3

m § y = y § m = y

5 § 6 = 6 § 5 = 6

6.4 Signs of Operation Unique to Mathematics:

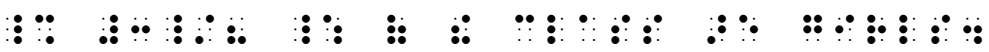
Slash	/	
Back Slash	\	
Dot	•	
Hollow Dot	∘	
Intersection ("cap")	∩	
Logical Product	∧	
Logical Sum	∨	
Minus with Dot Over	∸	
Tilde, Simple	~	
Tilde, Extended	≈	
Union ("cup")	∪	
Vertical Bar	|	
Vertical Bar Negated	∤	

Examples of each symbol are shown below. Note that, as with the other operation signs you have learned, these signs are unspaced from related mathematical terms regardless of the spacing shown in print.

6.4.1 Slash: The "forward slash" slants from lower-left to upper-right. When the slash means "divided by" (or "over"), as in a linear fraction, the slash is a mathematical symbol of operation and a switch to Nemeth Code is required. (In a linear fraction, the numbers are printed in normal size and each number rests on the baseline of writing, in line with the text.) No space is left between the slash and any numeral to which it applies.

⋙ / 

Example 6.4-1 3/8 of the class are girls.



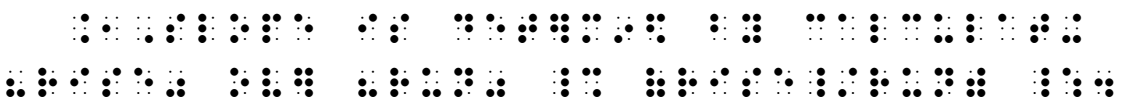
In Nemeth Code, the numeric indicator is not needed for a numeral immediately following a slash.

In a rate or ratio expressed in words, the slash means "per" (or "over"). This, too, requires a switch to Nemeth Code. No space is left between the slash and any word, part of a word, or abbreviation to which it applies.

Example 6.4-2 1 watt = 1 joule/sec.



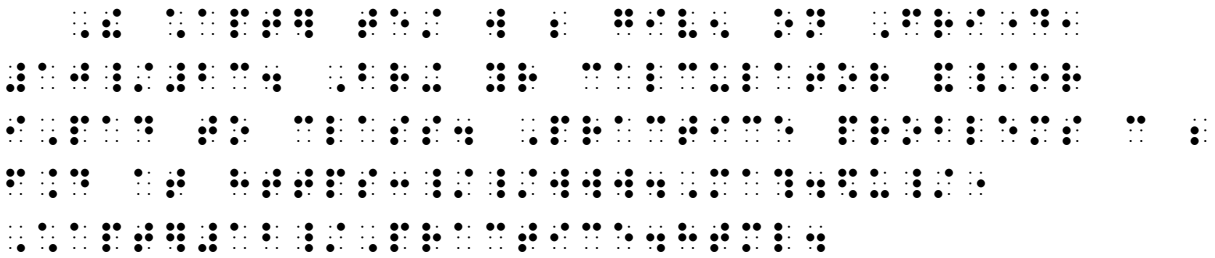
Example 6.4-3 *Slope* is determined by calculating "rise" over "run" (rise/run).



The slash used as a fraction line in non-linear fractions will be covered in a later lesson.

6.4.1.a UEB rules for the solidus (forward slash) are followed in non-mathematical context.

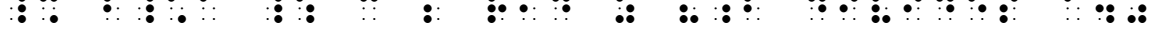
Example 6.4-4 The chapter test will be given on Friday, 10/23. Bring your calculator and/or iPad to class. Practice problems can be found at https://www.Math.edu/Chapter12/Practice.html.



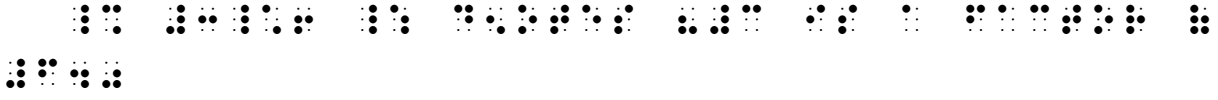
6.4.2 Back Slash: The back slash slants upper-left to lower-right. In the context of operation signs, the back slash means "divides" or "is a factor of".



Example 6.4-5 b \ a can be read as "b divides a."



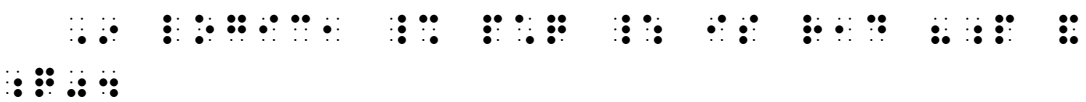
Example 6.4-6 3 \ 6 denotes "3 is a factor of 6."



6.4.3 Dot: You have seen the dot used as a multiplication sign. It may also be used to denote "and" as an operation sign in the study of logic. In either case, the symbol is brailled without a space.

⋙ ⋅ ⠨⠄

Example 6.4-7 In logic, $p \cdot q$ is read "p and q".



6.4.4 Hollow Dot: The hollow dot may be used as a sign of operation.

⋙ ∘ ⠨⠴

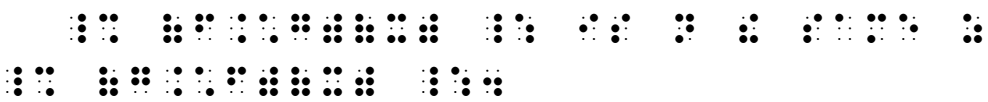
Example 6.4-8 $a \circ (b \circ c) = (a \circ b) \circ c$



The hollow dot is often seen in function notation. Be sure to braille the hollow dot symbol – this is not a letter "o".

⋙ $(f \circ g)$ ⠐⠣⠋⠴⠛⠐⠜

Example 6.4-9 $(f \circ g)(x)$ is not the same as $(g \circ f)(x)$.



The hollow dot used as a raised sign representing degrees of temperature or angle measurement will be discussed in a later lesson.

6.4.5 Intersection: This operation symbol is also called a "cap".

⋙ ∩ ⠨⠩

⋙ $A \cap B = B \cap A$ ⠠⠁⠨⠩⠠⠃⠀⠶⠀⠠⠃⠨⠩⠠⠁

6.4.6 Logical Product: In the study of logic, this operation sign means "and" or "meet".

⋙ ∧ ⠨⠬

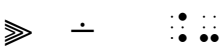
⋙ $p \wedge q \wedge r$ ⠏⠨⠬⠟⠨⠬⠗

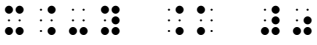
6.4.7 Logical Sum: In the study of logic, this operation sign means "or" or "join".

⋙ ∨ ⠨⠳

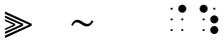
⋙ $p \vee q \vee r$ ⠏⠨⠳⠟⠨⠳⠗

6.4.8 Minus With Dot Over: This operation sign means "proper difference".

⋙ $\dot{-}$ 

⋙ $x \dot{-} y = 0$ 

6.4.9 Tilde: In logic, the tilde may be used as a sign of operation meaning "not".

⋙ $\sim$ 

⋙ $\sim p$ 

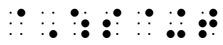
⋙ $\sim (\sim p)$ 

When two symbols for the tilde follow one another, a multipurpose indicator (dot 5) is inserted between them to indicate that they are written horizontally.

⋙ $\sim\sim\ p \vee q$ 

The extended tilde is one symbol with two "bumps".

⋙ $\sim\!\!\sim$ 

⋙ $\sim\!\!\sim s \vee t$ 

When the tilde symbol is used to replace the word "approximately", "about", or "around" it is brailled as an unspaced symbol.

Example 6.4-10 The test will begin in ~15 seconds.



The tilde is also used as a sign of comparison. Read the context to determine its meaning. See **6.7.14**.

6.4.10 Union: This operation symbol is also called a "cup".

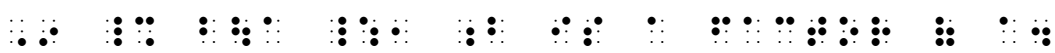
⋙ $\cup$

⋙ $A \cup B = B \cup A$

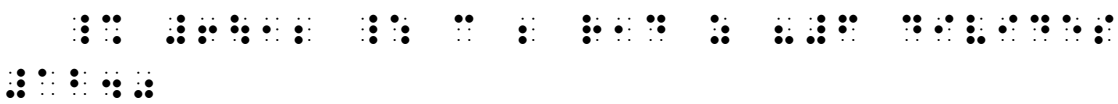
6.4.11 Vertical Bar: When the vertical bar means "is a factor" or "divides", it is functioning as an operation sign.

⋙ | ⠳

Example 6.4-11 In $b|a$, b is a factor of a.



Example 6.4-12 6 | 12 can be read as "6 divides 12."



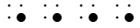
The vertical bar is also used as a sign of grouping and as a sign of comparison. Read the context to determine its meaning. See **3.9** and **6.7.16**.

6.4.12 Negated Vertical Bar: This symbol means "does not divide".

⋙ ∤ ⠸⠳

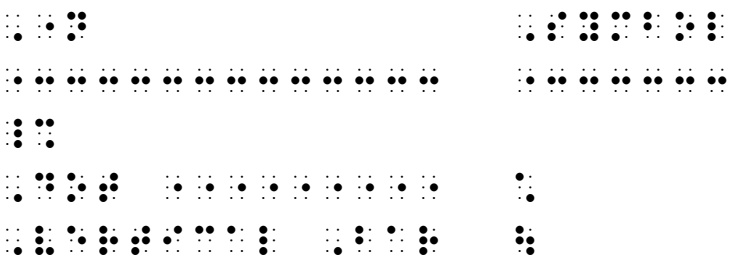
⋙ $5 \nmid n$ ⠼⠢⠸⠳⠝

6.5 Introduction to Table Format: Guidelines for the layout of tables are given in *Braille Formats.* Study or review *Braille Formats* regarding the definition of a table, margins used, column separation lines, space between columns, use of guide dots, and considerations when a table is too wide to fit on the braille page.

Column Separation Line (varies in width)	
Guide Dots (a minumum of two)	

When table entries consist of technical material, the entire body of the table is brailled in Nemeth Code, including any words. One opening Nemeth Code indicator precedes row 1 and one Nemeth Code terminator is placed at the end of the table. Words within the table are brailled without contractions, and the single-word switch indicator is not used. Further details regarding tables in Nemeth Code will be covered in **Lessons 15 and 17**. The following example illustrates the layout you will use in Practice 6B, which begins like this:

Name	Symbol
Dot	•
Vertical Bar	|



(further entries)



Line 1: The column headings are not mathematical. Contractions are used.
Line 2: Column separation lines are inserted according to Braille Formats *guidelines.*
Line 3: The opening Nemeth indicator is placed in cell 1.
Line 4: The first row is brailled in Nemeth Code. Guide dots are inserted according to Braille Formats *guidelines. Symbols in column 2 do not use the UEB dot locator.*
Line 5: Words are uncontracted.
Lines 6+ Column 1 will need to be wider once you determine the widest table entry.
Last line: Terminate Nemeth Code on the line after the completion of the table, in cell 1.

Instructions: Practice brailling these operation signs in table format. See instructions in **6.5**, above.

PRACTICE 6B

Name	Symbol
Dot	•
Vertical Bar	|
Negated Vertical Bar	⫽
Logical Product	∧
Simple Tilde	~
Logical Sum	∨
Extended Tilde	≁
Backslash	\
Slash	/
Hollow Dot	∘
Intersection	∩
Union	∪
Minus with Dot Over	∸

6.6 Review of Signs of Comparison: A space must be left before and after a sign of comparison. However, no space is left between a sign of comparison and a sign of grouping, a braille indicator, or a punctuation mark which is related to it. The following signs of comparison have already been introduced.


= Equals

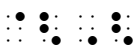

> Greater Than


< Less Than


:: Proportion

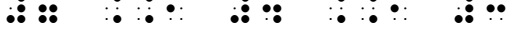

: Ratio

6.7 More Signs of Comparison:

Greater Than with Curved Sides	≻	
Less Than with Curved Sides	≺	
Arc, Concave Upward	⌒	
Arc, Concave Downward	⌣	
Equivalence	≎	
Identity	≡	
Inclusion	⊂	
Membership	ε or Ɛ or ∈	
Parallel to	‖	
Perpendicular to	⊥	
Relation	R	
Reverse Inclusion	⊃	
Reverse Membership	∋	
Tilde, Simple	~	
Tilde, Extended	≈	
Variation	∝	
Vertical Bar	|	

Examples of each symbol are shown below. Note that, as with the other comparison signs you have learned, these signs are preceded and followed by a space regardless of the spacing shown in print.

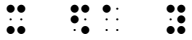
6.7.1 Greater Than With Curved Sides ("is greater than")

⋙ $7 \succ 4 \succ 3$ 

6.7.2 Less Than With Curved Sides ("is less than")

⋙ $5 \prec 9 \prec 11$

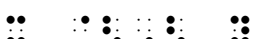
6.7.3 Arc, Concave Upward

⋙ $x \overset{\frown}{\ } y$ 

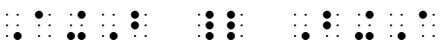
6.7.4 Arc, Concave Downward

⋙ $x \overset{\smile}{\ } y$ 

6.7.5 Equivalence ("is equivalent to")

⋙ $x \Bumpeq y$ 

6.7.6 Identity (Triple Bar): This symbol is used in several different contexts. Most commonly it means "is identical with" or "is congruent to". The transcriber brailles the same symbol regardless of its meaning. Do not confuse the triple bar with the Greek letter Xi or the triple bond in Chemistry.

⋙ $A + B \equiv B + A$ 

6.7.7 Inclusion: This symbol is generally used for sets and their elements, meaning "is contained in" or "is a subset of".

⋙ $A \subset D$ 

6.7.8 Membership: This symbol is generally used for sets and their elements, meaning "is an element of" or "belongs to". It must not be mistaken for the Greek uncapitalized epsilon even though it may be referred to as such.

⋙ $5 \in B$ 

6.7.9 Parallel To ("is parallel to")

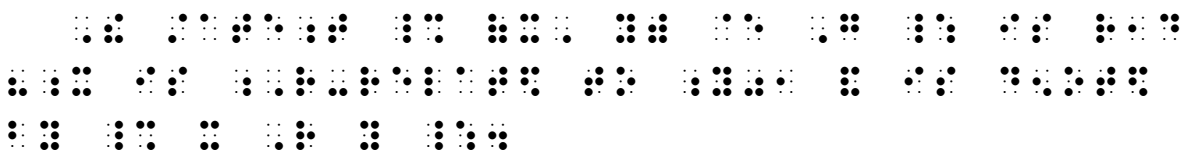
⋙ $AB \parallel CD$ 

6.7.10 Perpendicular To ("is perpendicular to")

⋙ $PQ \perp RS$ 

6.7.11 Relation: When a letter R is used between two expressions to show relation ("is related to"), the letter is treated as a sign of comparison with a space before and after it regardless of print spacing. Read the surrounding narrative to realize the meaning of the letter "R" in order to transcribe it correctly.

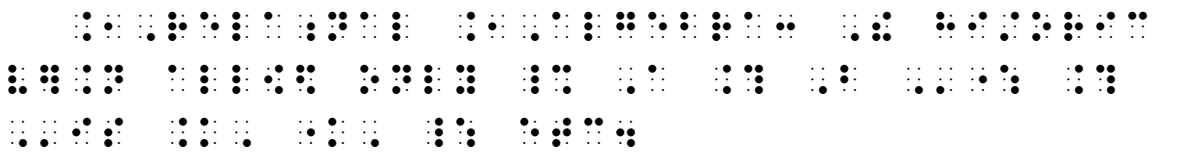
Example 6.7-1 The statement $(x, y) \in G$ is read "x is R-related to y", and is denoted by xRy.



The comparison sign (R) is preceded and followed by a space in braille.

Other letters or signs may also be used to show relation.

Example 6.7-2 *Relational Algebra:* The historic version allowed only $A\theta B$ where θ is $=, <$, etc.



6.7.12 Reverse Inclusion: This symbol may mean "contains" or, in logic, "implies".

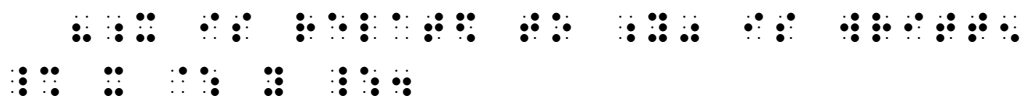
⫸ $D \supset A$

6.7.13 Reverse Membership: This symbol means "contains the element".

⫸ $B \ni 5$ 

6.7.14 Tilde as a Sign of Comparison: When used as a comparison sign (often encountered in set theory and relations) the tilde means "is related to" or "is similar to". The simple tilde has one peak.

Example 6.7-3 "x is related to y" is written $x \sim y$.



The extended tilde has more than one peak.

⫸ $x \sim\!\sim\, y$ 

The tilde is also used as a sign of operation. Read the context to determine its meaning. See section **6.4.9**.

6.7.15 Variation ("varies as")

⋙ $x \propto y$

6.7.16 Vertical Bar: When used as a sign of comparison, the vertical bar means "such that" or "given". As a comparison sign, it usually occurs in an expression within braces used for set notation. The print copy may or may not show this as a spaced symbol; in braille, comparison signs must be preceded and followed by a space.

⋙ $\{x \in E|\Phi(x)\}$

Example 6.7-4 $\{(x,y)|x+y<6\}$ means "The set of points (x,y) such that $x+y<6$."

Example 6.7-5 P(A|B) means "The probability of Event B **given** Event A."

A vertical bar may also appear in other situations as a sign of comparison. Read the context to determine its meaning. Recall that this symbol is also used as a sign of grouping and as a sign of operation. Apply proper spacing according to its function. See **3.9** and **6.4.11**.

6.7.16.a Colon Meaning "Such That": A colon may also be used to mean "such that" in set notation. Regardless of the spacing shown in print, the colon is brailled unspaced from the letter, preceded by a punctuation indicator. A space follows the colon in braille.

Example 6.7-6 $\{x : x>0\}$ means "The set of numbers x such that $x>0$."

Instructions: Review the spacing rules for comparison signs before transcribing the practice. Assume all tildes and vertical bars are comparison signs in these examples.

PRACTICE 6C

$A \ni x$

$\{x \in A \mid x \backsim a\}$

$j \eqsim k$

$r \propto s$

$f(x) \equiv D(x) \cdot q(x)$

$-12 \prec -4 \prec 0$

$.9 \succ .5$

$m \smile l$

$l \frown m$

$(A \cup E) \subset (F \cup B)$

$Q \supset Z$

$PQR \sim P'Q'R'$

$AB \parallel \mathrm{MN}$

$CD \perp OP$

$EF \parallel GH \perp QR$

$\{m \mid 3(m-6) = -9\}$

$\{x : x \text{ has the property } T\}$

6.8 Signs of Comparison Compounded Vertically: When two or more simple signs of comparison are arranged one under the other, the combination becomes a single comparison sign compounded vertically. The symbol for the uppermost sign is written first, immediately followed by and unspaced from the symbol for the lower sign. Comparison signs compounded vertically not shown in the lists below are transcribed in accordance with this principle.

6.8.1 Greater Than Or Equal To: The "equal to" sign may be printed as an equals sign or as a single line – either a horizontal bar or an oblique line. Note that both the horizontal bar and the oblique line are represented by the same braille symbol, ⠸⠶

Bar Over Greater Than	$\geqslant$ or $\geqslant$	⠸⠶ ⠸⠕
Equals Sign Over Greater Than	$\geqq$ or $\geqq$	⠸⠶⠶ ⠸⠶⠂
Greater Than with Bar Under	$\geq$ or $\geqslant$	⠸⠕ ⠸⠶
Greater Than with Equals Sign Under	$\geqq$ or $\geqq$	⠸⠕ ⠸⠶⠶

⠸⠶ $a \geqslant b$ ⠰ ⠸⠶⠕⠸⠕ ⠰

⠸⠶ $a \geqq b$ ⠰ ⠸⠶⠶⠸⠕⠂ ⠰

⠸⠶ $x \geq y$ ⠸⠸ ⠸⠕⠸⠶ ⠸⠸

⠸⠶ $|x| \geqq 0$ ⠸⠿⠸⠿ ⠸⠕⠸⠕ ⠼⠚

6.8.2 Inclusion ("is a subset of")

Bar Over Inclusion	$\subseteqslant$	⠸⠶ ⠸⠶⠂
Equals Sign Over Inclusion	$\subseteqq$	⠸⠶⠶ ⠸⠶⠂
Inclusion with Bar Under	$\subseteq$	⠸⠶ ⠸⠕
Inclusion with Equals Sign Under	$\subseteqq$	⠸⠶ ⠸⠶⠶

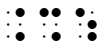
⠸⠶ $C \subseteqslant B'$ ⠰ ⠸⠶⠶⠸⠕⠂ ⠰

⠸⠶ $C \subseteqq B'$ ⠰ ⠸⠶⠶⠸⠶⠂ ⠰

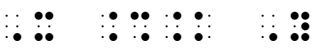
⠸⠶ $(D \cap E) \subseteq (E \times E)$ ⠸⠷ ⠸⠶⠶⠸⠶⠂⠸⠷ ⠸⠶⠕⠸⠶ ⠸⠷ ⠸⠶⠶⠸⠶⠂⠸⠶⠂⠸⠷

⠸⠶ $(D \cap E) \subseteqq (E \times E)$ ⠸⠷ ⠸⠶⠶⠸⠶⠂⠸⠷ ⠸⠶⠕⠸⠕ ⠸⠷ ⠸⠶⠶⠸⠶⠂⠸⠶⠂⠸⠷

6.8.3 Intersection: The intersection sign is a sign of comparison when modified by a bar or equals sign below it. It is also called a "cap".

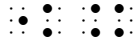
Intersection with Bar Under	$\underline{\cap}$	
Intersection with Equals Sign Under	$\underline{\underline{\cap}}$	

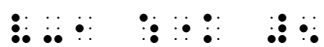
⋗ $X \underline{\cap} Y$ 

⋗ $X \underline{\underline{\cap}} Y$ 

An unmodified intersection sign is a sign of operation. See **6.4.5**.

6.8.4 Less Than Or Equal To

Bar Over Less Than	$\overline{<}$ or $\leqslant$	
Equals Sign Over Less Than	$\overline{\overline{<}}$ or $\leqslant$	
Less Than with Bar Under	$\underline{\leqslant}$ or $\leqslant$	
Less Than with Equals Sign Under	$\underline{\underline{\leqslant}}$ or $\leqslant$	

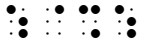
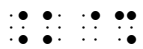
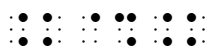
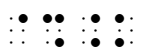
⋗ $v - 1 \overline{<} 5$ 

⋗ $v - 1 \leqslant 5$ 

⋗ $6 \leq x \leq 9$ 

⋗ $6 \leqslant x \leqslant 9$ 

6.8.5 Logical Product: The logical product sign is a sign of comparison meaning "meet" when modified by a bar or equals sign above or below it.

Bar Over Logical Product	$\overline{\wedge}$	
Bar Over and Bar Under Logical Product	$\underline{\overline{\wedge}}$	
Bar Over and Equals Sign Under Logical Product	$\underline{\overline{\wedge}}$	
Logical Product with Bar Under	$\underline{\wedge}$	
Equals Sign Over Logical Product	$\overline{\overline{\wedge}}$	
Equals Sign Over and Bar Under Logical Product	$\underline{\overline{\overline{\wedge}}}$	
Equals Sign Over and Equals Sign Under Logical Product	$\underline{\underline{\overline{\overline{\wedge}}}}$	
Logical Product with Equals Sign Under	$\underline{\underline{\wedge}}$	

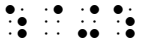
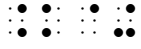
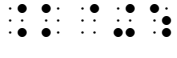
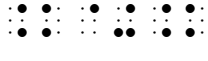
⋙ $ABD\ \overline{\wedge}\ A'B'D'$ 

⋙ $\{A\}\ \underline{\underline{\wedge}}\ K$ 

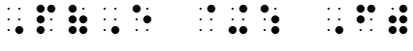
⋙ $p\ \underline{\wedge}\ q$ 

An unmodified logical product sign is a sign of operation. See **6.4.6**.

6.8.6 Logical Sum: The logical sum sign is a sign of comparison meaning "join" when modified by a bar or equals sign above or below it.

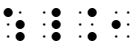
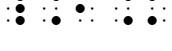
Bar Over Logical Sum	$\overline{\vee}$	
Bar Over and Bar Under Logical Sum	$\overline{\underline{\vee}}$	
Bar Over and Equals Sign Under Logical Sum	$\overline{\underline{\underline{\vee}}}$	
Logical Sum with Bar Under	$\underline{\vee}$	
Equals Sign Over Logical Sum	$\overline{\overline{\vee}}$	
Equals Sign Over and Bar Under Logical Sum	$\underline{\overline{\overline{\vee}}}$	
Equals Sign Over and Equals Sign Under Logical Sum	$\underline{\underline{\overline{\overline{\vee}}}}$	
Logical Sum with Equals Sign Under	$\underline{\underline{\vee}}$	

⋙ $ABC \ \bar{\vee} \ A'B'C'$ 

⋙ $P(E \underline{\vee} F)$ 

An unmodified logical sum sign is a sign of operation. See **6.4.7**.

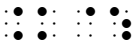
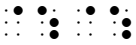
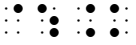
6.8.7 Reverse Inclusion

Bar Over Reverse Inclusion	$\overline{\supset}$	
Equals Sign Over Reverse Inclusion	$\overline{\overline{\supset}}$	
Reverse Inclusion with Bar Under	$\underline{\supseteq}$	
Reverse Inclusion with Equals Sign Under	$\underline{\underline{\supseteq}}$	

⋙ $B \overline{\supset} A$ 

⋙ $D \underline{\underline{\supseteq}} C$ 

6.8.8 Tilde:

Bar Over Single Tilde	≁	
Equals Sign Over Single Tilde	≋	
Double Tilde	≈	
Bar Over Double Tilde	≉	
Equals Sign Over Double Tilde	≊	
Single Tilde with Bar Under	≂	
Single Tilde with Equals Sign Under	≅	
Double Tilde with Bar Under	≆	
Double Tilde with Equals Sign Under	≇	

⋙ $3.14159 \approx 3.1416$ 

⋙ $ABC \cong DEF$ 

6.8.9 Union: The union sign is a sign of comparison when modified by a bar or equals sign above or below it. It may also be referred to as a "cup".

Bar Under Union	$\underline{\cup}$	
Equals Sign Under Union	$\underline{\underline{\cup}}$	

⋙ $A \underline{\cup} B$ 

An unmodified union sign is a sign of operation. See **6.4.10**.

Instructions: Change the two-column format to a nested list by starting each phrase in cell 1, with each math example starting on a new line in cell 3. Braille "Signs of Comparison Compounded Vertically" as a cell-5 heading.

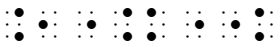
PRACTICE 6D

Signs of Comparison Compounded Vertically

Greater Than Or Equal To	$ab \eqgtr de$
	$\|y\| \geq 0$
With Inclusion	$C' \subset F'$
	$(B \cap E) \subseteqq (E \times E)$
With Intersection	$X \underline{\cap} Y$
	$X \underline{\underline{\cap}} Y$
Less Than Or Equal To	$q - 7 \leqslant 5z$
	$-6 \leq x \leq -1$
With Logical Product	$QRS \overline{\wedge} Q'R'S'$
	$y \underline{\wedge} z$
With Logical Sum	$ABC \overline{\vee} A'B'C'$
	$M(E \underline{\vee} H)$
With Reverse Inclusion	$D \supset C$
	$B \supseteqq A$
With Tilde	$3.14159 \approx 3.1416$
	$ABC \cong DEF$
With Union ("Cup")	$A \underline{\underline{\cup}} B$

6.9 Signs of Comparison Compounded Horizontally: When two or more signs of comparison are arranged side by side, the combination becomes a single comparison sign compounded horizontally. A multipurpose indicator (dot 5) is inserted between the unspaced symbols to indicate that they are printed horizontally, not vertically. Comparison signs compounded horizontally not shown in the lists below are transcribed in accordance with this principle.

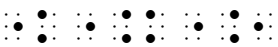
6.9.1 Greater Than ...

Followed by Less Than	$><$	
Followed by Equals Followed by Less Than	$>=<$	

⋙ $n><1$ 

⋙ $n>=<1$ 

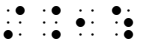
6.9.2 Less Than ...

Followed by Greater Than	$<>$	
Followed by Equals Followed by Greater Than	$<=>$	

⋙ $n<>1$ 

⋙ $n<=>1$ 

6.10 Negated Signs of Comparison: In print, a sign of comparison may be negated by a vertical or a slanted line drawn through it. The print negation symbol may be slanted in either direction. In braille, ⠠ represents any of the print negation lines. ⠠ is placed immediately before the sign of comparison being negated. Some examples are shown below. Negated signs of comparison not illustrated below are transcribed according to the same principle.

Negated Equals Sign	$\neq$ or $\nexists$	
Negated Parallel To	$\nparallel$	
Negated Perpendicular To	$\not\perp$	
Negated Greater Than or Equal To	$\not\geq$	
Negated Membership	$\notin$ or $\nin$	

⋙ $4 \times 13 \neq 14$ 

⋙ $CD \nparallel EF \not\perp GH$ 

⋙ $4 \not\geq 7$ 

⋙ $9 \notin D$ 

Instructions: Use the principles learned in this section to construct symbols that are not shown in the examples.

PRACTICE 6E

(1) $x \notin A$ means "x is not an element of A".

(2) By typing <= the symbol ≤ will appear. By typing /< the symbol ≮ will appear.

(3) $A \nsubseteq B$ means that at least one element of A is not an element of B.

(4) WXY $\backsim$ VXW

(5) Since $L \nparallel M$ and $M \nparallel N$, does it follow that $L \nparallel N$?

(6) The domain is all $x \neq -4, 0, 5$.

FORMAT: INSTRUCTIONS

6.11 Margins for Instructions Preceding Itemized Material (5-3): When a group of numbered or lettered problems is preceded by instructions, the instructions begin in cell 5, with runovers in cell 3. One line is left blank above instructions unless the instructions follow a cell-5 or a cell-7 heading. Instructions may begin on line 1 of the braille page if no running head is in use. The related itemized material follows on the next line unless the material itself requires a blank line before it.

Example 6.11-1 (The dashed line indicates a page turn in the print copy.)

Problem Set 7F Tell whether the following ratios are equivalent.

1. $3 : 2 = 75 : 50$
2. $6 : 4 = 15 : 30$

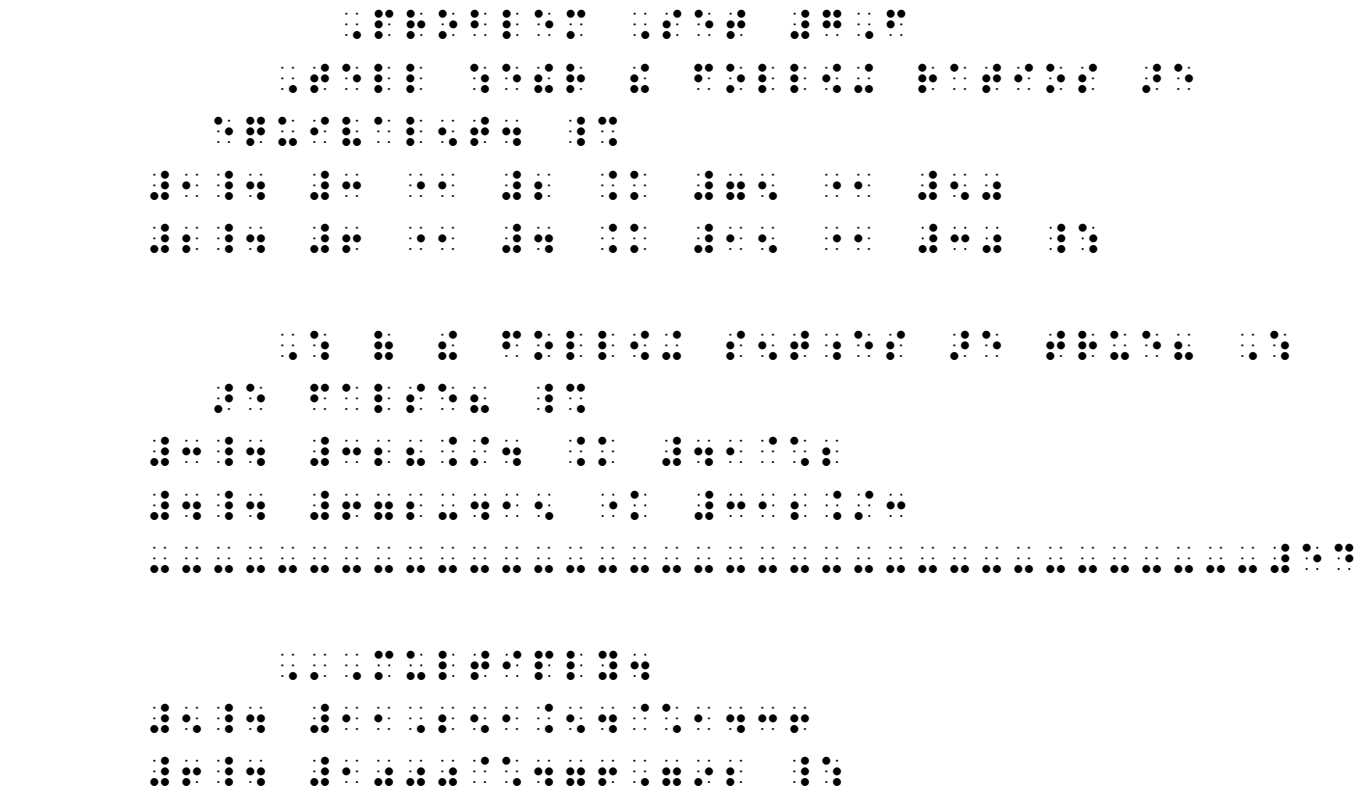
Which of the following sentences are true? Which are false?

3. $328 \div 4 = 41 \times 2$
4. $672 - 415 < 312 \div 3$

———————————————————————————————————————

Multiply. 54

5. $11{,}251.54 \times 1436$
6. $1000 \times 476{,}792$



In the example above, line 1 is a cell-7 heading. The instructions are lines 2-3, 7-8, and 13.

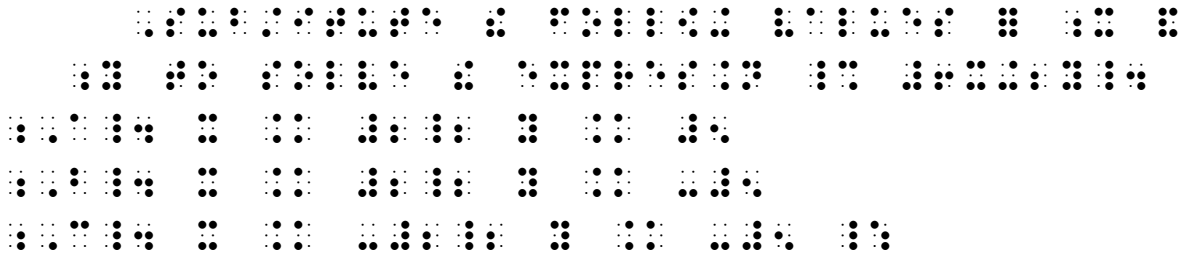
If page layout constraints force the instructions to be divided between braille pages, at least one line of the instructions must be on the same braille page as the related problems which follow.

6.11.1 Code Switching and Instructions: If exercise directions ("instructions") end with an expression in Nemeth Code and the subsequent math problem starts with Nemeth Code, Nemeth Code may be left in effect between the end of the directions and the start of the problem.

Example 6.11-2

Substitute the following values for x and y to solve the expression $6x + 2y$.

A. $x = 2; y = 5$ B. $x = 2; y = -5$ C. $x = -2; y = -5$



Reminder: When the print copy arranges itemized material side-by-side across the page and there are no subdivisions, the braille format is changed so that all identifiers start in cell 1.

6.12 Not "Instructions": (5-3) instruction format applies only to instructions that are followed by a set of numbered or lettered problems. If the material following the instructions is not numbered or lettered, if the instructions themselves are numbered or lettered, or if the narrative is not giving explicit directions that apply to the following itemized material, then other established formats are followed as illustrated below.

6.12.1 Margins for Instructions Preceding Unitemized Material (3-1): If there is no numbered or lettered exercise material following instructions, the text is treated as narrative material and is transcribed as a (3-1) paragraph. The (5-5) style of "directions" given in *Braille Formats* does not apply in a transcription that contains Nemeth Code.

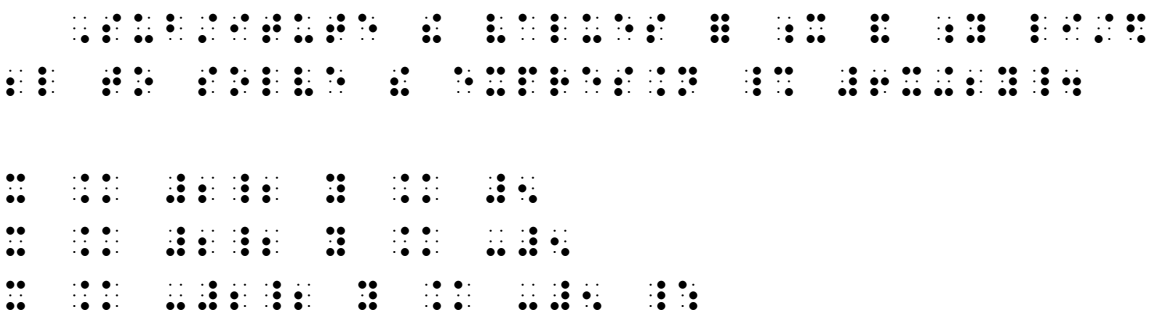
Example 6.12-1

Substitute the values for x and y listed below to solve the expression $6x + 2y$.

$x = 2; y = 5$

$x = 2; y = -5$

$x = -2; y = -5$

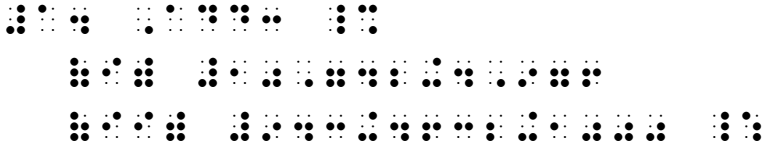


6.12.2 Itemized Instructions: Instructions that are preceded by a number or letter are formatted according to the Nemeth Code margin rules for itemized material.

Example 6.12-2

1. Add:

(i) $10{,}742 + 4{,}976$ (ii) $943 + 4632 + 1000$

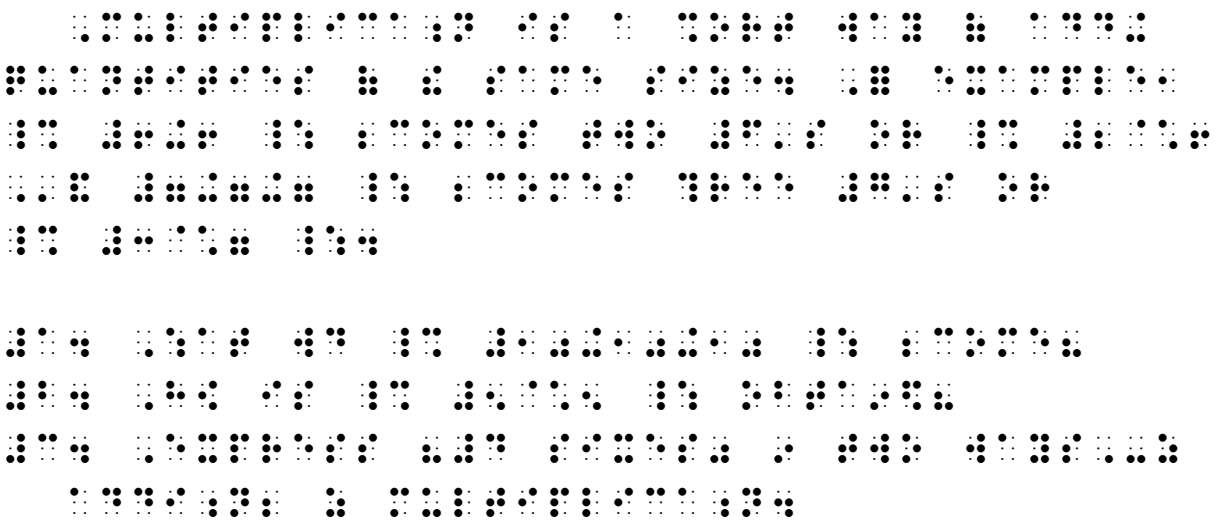


6.12.3 Margins for Narrative Preceding Itemized Material (3-1): Only explicit instructions are regarded as "instruction format" (5-3) in Nemeth Code. If the narrative is purely explanatory, regular paragraphing is applied and a blank line precedes the itemized material.

Example 6.12-3

Multiplication is a short way of adding quantities of the same size. For example, 6 + 6 becomes two 6's or 2 × 6 and 7 + 7 + 7 becomes three 7's or 3 × 7.

1. What would 10 + 10 + 10 become?
2. How is 5 × 5 obtained?
3. Express "4 sixes" in two ways—as addition; as multiplication.



Instructions: Braille "Signs of Comparison" and "Adding and Subtracting Integers" as cell-5 headings.

PRACTICE 6F

Signs of Comparison

These examples illustrate the basic spacing rules for comparison signs learned in this unit.

(1) $5 \prec 9 \prec 11$

(2) $11.7 \succ 1.17$

(3) $550 : 11 :: \ ? : 12$

Adding and Subtracting Integers

Find the sum or difference as indicated by the signs.

1) $-6 + -5 =$ ____

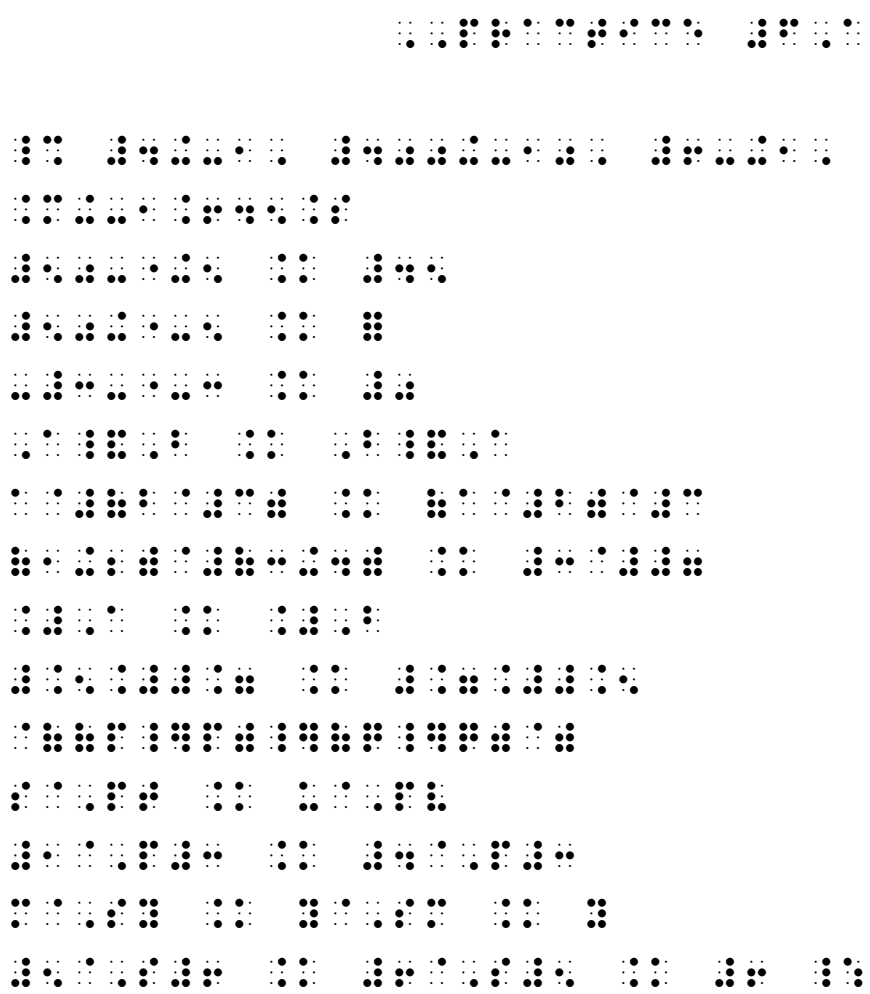
2) $5 + -19 =$ ____

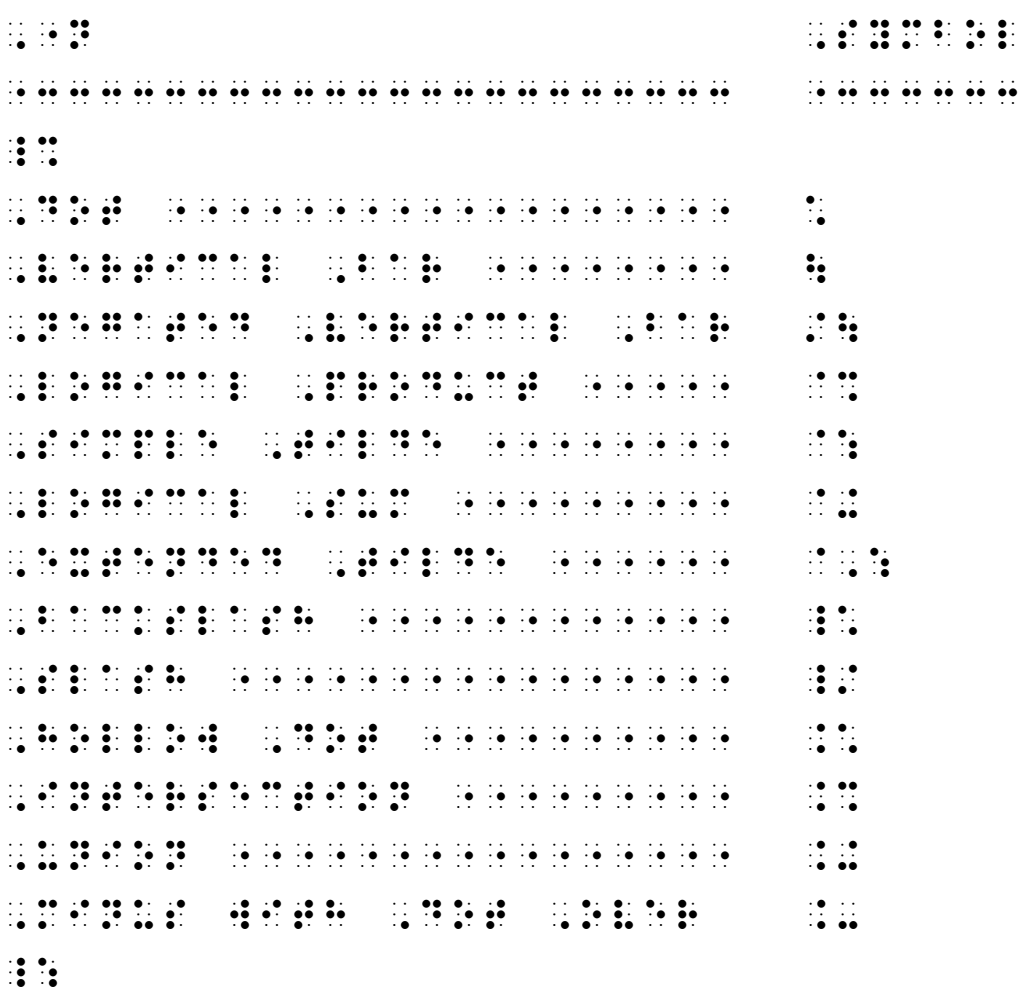
3) $-7 - -13 =$ ____

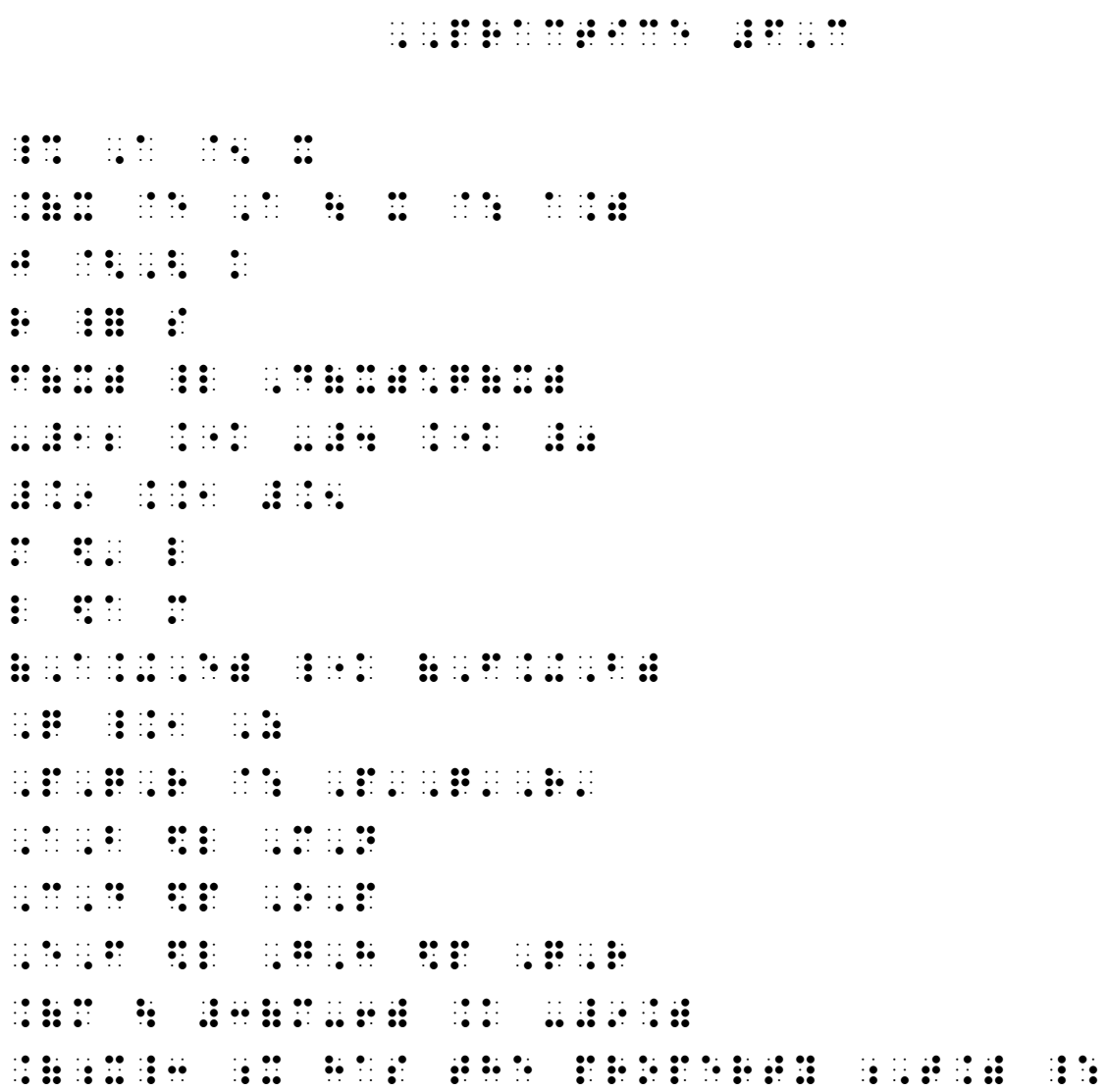
4) $29 - -24 =$ ____

For further practice, see Appendix A—Reading Practice.

ANSWERS TO PRACTICE MATERIAL







The words are part of the math and so are brailled uncontracted without switching out of Nemeth Code. Each single letter requires an ELI because each is preceded and followed by a space and/or punctuation (or begins a new line). The presence of a single grouping symbol is ignored when determining whether an ELI is needed.

⠠⠠⠉⠓⠁⠏⠞⠑⠗ ⠋⠊⠧⠑ ⠠⠠⠞⠗⠁⠝⠎⠏⠕⠗⠞⠁⠞⠊⠕⠝ ⠠⠠⠎⠑⠗⠧⠊⠉⠑⠎
⠠⠠⠠⠁⠙⠍⠊⠝⠊⠎⠞⠗⠁⠞⠊⠕⠝

⠠⠠⠠⠁⠇⠇ ⠠⠠⠠⠁⠗⠞⠊⠉⠇⠑⠎ ⠠⠠⠠⠁⠗⠞⠊⠉⠇⠑⠎ ⠠⠠⠠⠁⠗⠞⠊⠉⠇⠑⠎ ⠠⠠⠠⠁⠗⠞⠊⠉⠇⠑⠎ ⠠⠠⠠⠁⠗⠞⠊⠉⠇⠑⠎ ⠠⠠⠠⠁⠗⠞⠊⠉⠇⠑⠎

EXERCISE 6

Exercise 6 will be available when this course is finished being written and is no longer "Provisional".

Proceed to Lesson 7.

LESSON 7

Read about this PROVISIONAL EDITION in the front matter to this book.
Check the NFB website periodically for updates to this lesson.

- *TYPEFORMS*
  - *LETTERS AND NUMERALS*
  - *THE FOUR MATHEMATICAL TYPEFORM INDICATORS*
  - *BOLDFACED MATHEMATICAL SYMBOLS*
  - *COMPOUND EXPRESSIONS*
  - *Typeform Summary*
- *UNSPACED NUMBER/LETTER COMBINATIONS*
- *REVISITING THE NUMERIC INDICATOR*

FORMAT
Itemized Material with Subdivisions
Tabular Form
Margins for Exercise Sets

TYPEFORMS

7.1 Introduction to Typeforms: The typeform indicators of UEB are used in the narrative text. When it is necessary to indicate emphasis or distinction inside the Nemeth Code switches, the typeform indicators of the Nemeth Code are used. The Nemeth Code has indicators and rules for applying four special typeforms to individual letters, to numerals, and to symbols, as well as indicators and rules for applying typeform to words.

LETTERS AND NUMERALS

7.2 Determining Significance of a Variant Typeform: Typeform is retained in a mathematical expression only if it conveys special mathematical meaning or distinction. Variant typeform which has no mathematical significance is not shown in braille. A few typical print examples for you to consider are shown below.

- If the author has specifically distinguished between two meanings of the same letter, the distinctive typeform is significant.

  Example: R denotes the set of rational numbers and $\mathcal{R}$ denotes the set of real numbers.

  The second letter R must maintain its distinction in the braille transcription.

- It is common practice to print the letters of all formulas throughout a book in italicized type. Such letters are brailled without typeform indicators.

  Examples: The variables *x*, *y*, and *z* are real numbers.
  π is used to determine the circumference of a circle: $2\pi r$.

  The italic typeform is not mathematically significant when the letters in all formulas throughout the book are printed in italics.

- A variant typeform is often used for the sole purpose of attracting the reader's attention. This is particularly common at the lower grade levels. Such variant typeforms are not represented in the braille transcription.

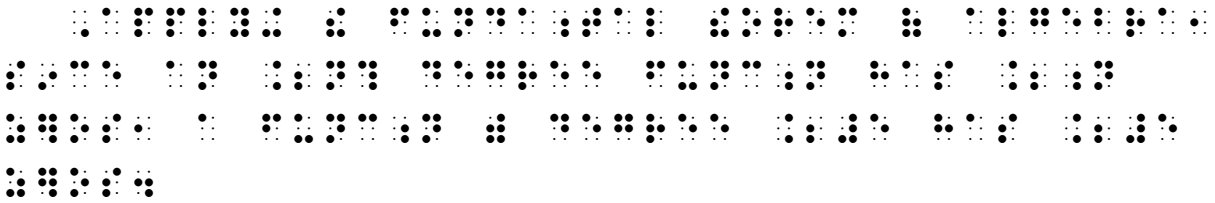
Example: Let n be the smaller number, and 13+n be the larger number.

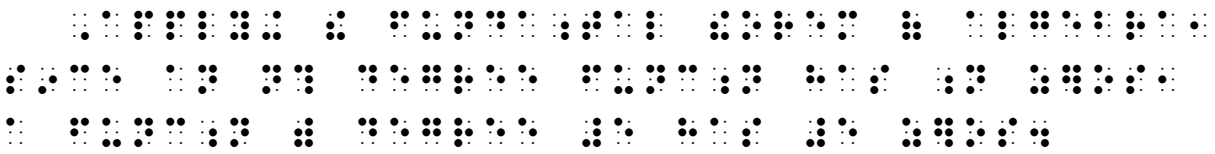
The boldface is not mathematically significant—its purpose is to attract the reader's attention. No typeform indicators are used in the braille transcription.

Food For Thought: The decision whether to retain a variant typeform can be difficult. The next example shows two italicized letters and two italicized numbers. The italics in print are the author's way of making a connection between the two *n*s and the two *5*s. The italics used in the example below is not "mathematically significant" but the transcriber must determine if the non-regular typeform is needed for "emphasis or distinction" according to UEB. Does the italic typeform aid in the understanding of the material or is it functioning simply as a visual device?

The example is brailled two ways. The first transcription retains the italics and the second does not.

Example 7.2-1 Applying the fundamental theorem of algebra, since an *n*th degree function has *n* zeros, a function with degree *5* has *5* zeros.





Either transcription could be supported as being the correct one. The transcriber's responsibility is to apply the decision consistently throughout a document.

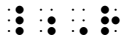
THE FOUR MATHEMATICAL TYPEFORM INDICATORS

7.3 Mathematical Typeforms and Their Indicators: Specific provision is made in the Nemeth Code for the transcription of four print typeforms: boldface, italic, script, and sanserif type. The various typeforms may be applied to the letters of the English, German, Greek, Hebrew, and Russian alphabets as well as to numerals, mathematical symbols, and words or phrases in mathematical context.

7.4 Boldface, Italic, Script, and Sanserif Type for Letters and Numerals: Compare characters to the surrounding text to determine whether the print style differs. If it is determined that the typeform is mathematically significant, one of the following Nemeth Code indicators is used.

Boldface Typeform Indicator	
Italic Typeform Indicator	
Sanserif Typeform Indicator	
Script Typeform Indicator	

7.5 Typeform Indicators with One Letter: When the distinction has mathematical significance, a switch to Nemeth Code is required. A typeform indicator must always be followed by an alphabetic indicator. Here is the English letter R in these four styles.

⋙ R  *(boldface letter R)*

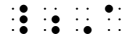
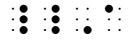
⋙ *R*  *(italic letter R)*

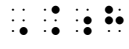
⋙ R  *(sanserif letter R)*

⋙ ℜ  *(script letter R)*

Note: Sanserif style is recognized by the lack of small lines or serifs at the ends of the letter parts. Only the English (Roman) alphabet has a sanserif style of type.

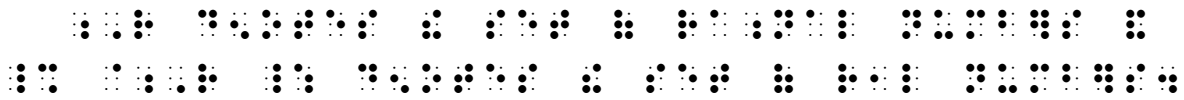
Review the five alphabetic indicators of the Nemeth Code in **Lessons 4 and 5**. Here are isolated examples of capitalized and uncapitalized letters in the four typeforms.

boldface English lowercase a	a	
boldface English capital a	A	
boldface Greek lowercase alpha	𝛂	
boldface German lowercase ah	𝖆	
boldface German capital ah	𝕬	
boldface Russian lowercase ah	а	
italic English lowercase a	*a*	
italic English capital a	*A*	

italic German lowercase ah	$\mathit{\mathfrak{a}}$	
sanserif English lowercase h	h	
sanserif English capital h	H	
script English lowercase a	$\mathcal{a}$	
script English capital a	$\mathcal{A}$	

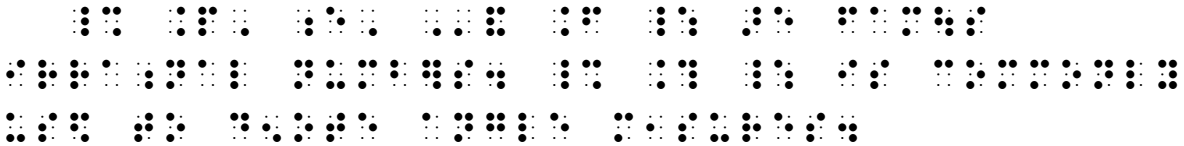
In the next example, the script typeform of the second English letter R is retained in braille for distinction. Although there is a script typeform indicator in UEB, the letter is mathematically significant and so a switch to Nemeth Code is required.

Example 7.5-1 R denotes the set of rational numbers and $\mathcal{R}$ denotes the set of real numbers.



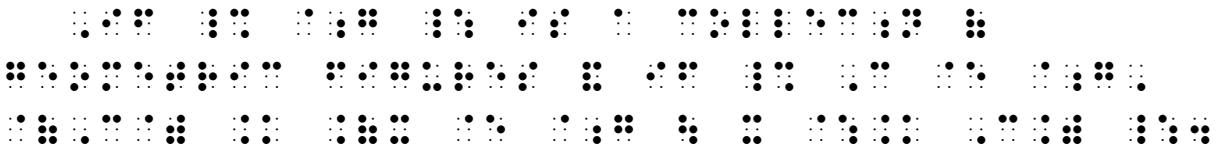
In the next example, pi, e, phi, and theta are printed in italics. Recall that the italic typeform is not mathematically significant when all mathematical letters in the book are printed in italics. No italic indicators are used in the transcription of this example.

Example 7.5-2 π, e, and ϕ are famous irrational numbers. θ is commonly used to denote angle measures.



In the next example, the script typeform gives the English letter g a distinct mathematical identity, and so the script typeform is retained in the braille transcription. The italicized letters in the formula are not italicized in braille, according to the general guidelines regarding italicized mathematical letters.

Example 7.5-3 If $\mathcal{g}$ is a collection of geometric figures and if $C \in \mathcal{g}$, $[C] = \{x \in \mathcal{g} \mid x \cong C\}$.



If a bold letter is also printed in italics, determine if the italic typeform is significant as well as the bold. In the next example, the bold typeform is retained in braille because it has mathematical significance, but since all variables and constants in the book are printed in italics, the italic typeform is disregarded in braille.

Example 7.5-4 $J = \Delta p = m\Delta v$



7.5.1 Recognition of Script Type in Other Alphabets: The alphabet tables in the Nemeth codebook show the print letter for script font styles in each category: English (Roman), German, Greek, Hebrew, and Russian. Use the tables to identify such letters. Then apply the appropriate indicator when it is determined to be mathematically significant. Follow the typeform indicator with the correct alphabetic indicator. Here are some isolated examples.

script German Deh 

script Greek Psi 

script Hebrew aleph 

script Russian yah 

Instructions: Braille only the forty letters, using the typeform indicated before each set: boldface*, script, or sanserif. Do not braille the descriptions or the names—just braille four letters on each line, with one blank cell between each of the four letters.

Because these letters are out of context, the alphabet and the individual letter name is given. Refer to the alphabet lists in the Nemeth codebook for the German, Hebrew, and Russian letters in order to select the correct braille character. Capital letter names are capitalized in the description. The first line in the practice is shown below to get you started.



**Images have been imported for many of these letters. Please disregard the uneven baseline, spacing, size variances, and dark type in the sections that are not labeled "boldface". Use the boldface typeform indicator only for the letters in the first section.*

PRACTICE 7A

Use <u>BOLDFACE</u> typeform for these ...

 ... English (Roman) letters (a, Bee, cee, Dee)

 ... German letters (fao, Yaht, ypsilon, Tseh)

 ... Russian letters (Cheh, zeh, gheh, Zheh)

 ... Greek letters (rho, Psi, Phi, chi)

Use <u>SCRIPT</u> typeform for these... [none are bold]

 ... German letters (Gheh, peh, Beh, tset)

 ... Greek letters (omega, lambda, Sigma, Eta)

 ... English (Roman) letters

 ... Hebrew letters (zayin, ayin, gimel, koph)

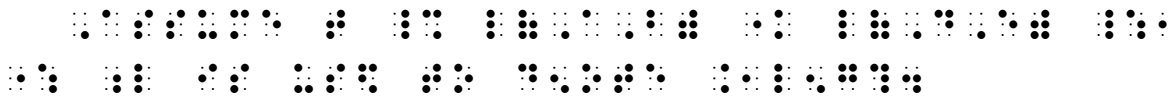
 ... Russian letters: eh, Yeh, shah, Yah

Use <u>SANSERIF</u> typeform for these ...

K R h p ... English (Roman) letters (Kay, Ar, aitch, pee)

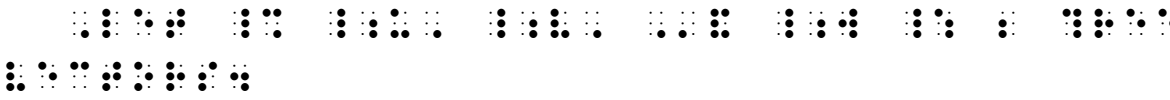
7.5.2 Script Letter of Special Interest: Print publishers often use the script form of the lowercase English letter "ell" simply to differentiate it visually from the numeral 1 (one). Since the letter and the numeral cannot be confused in braille, there is no reason to retain the script font.

Example 7.5-5 Assume that $\ell(AB) < \ell(DE)$, where ℓ is used to denote *length*.



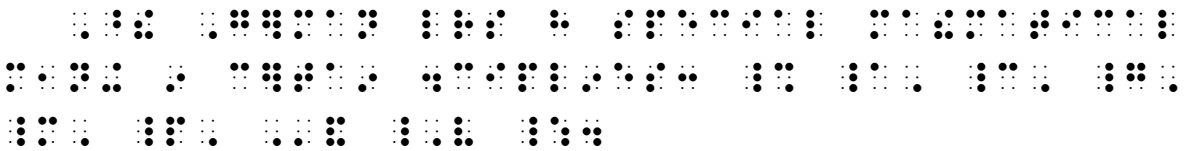
7.5.3 Boldface Letters of Special Interest—Vectors: Boldface type used to identify letters as vectors must be preserved in the braille transcription. A switch to Nemeth Code is required for the single bold letter in the narrative—UEB typeform indicators are not used in mathematical context.

Example 7.5-6 Let u, v, and w be three vectors.



7.5.4 Boldface Letters of Special Interest—German Letters: The letters of the German "fraktur" alphabet may appear to be printed in boldface, but if <u>all</u> German letters in the document are dark, bold typeform is not applied. *(In this example, assume that <u>all</u> German letters in the document are printed in a dark font.)*

Example 7.5-7 These German letters have special mathematical meaning in certain disciplines: 𝖆, 𝖈, 𝖌, 𝖒, 𝖕, and 𝕭.



Reminder: ⠰ functions as the German letter indicator when immediately followed by one of the 26 letters of the German alphabet. For ⠰ to mean "boldface letter" the typeform indicator must be immediately followed by an alphabetic indicator.

7.6 Typeform Indicators with One Numeral: The appropriate typeform indicator of the Nemeth Code is used when it is determined that the nonregular type is mathematically significant. The numeric indicator is required between a typeform indicator and a numeral or decimal point. Here are isolated examples of a numeral in the four typeforms.

boldface 4	4	⠸⠼⠙	ordinary plus, boldface 8	
italic 4	*4*	⠨⠼⠙	+8	⠬⠼⠓
* italic .4	*.4*	⠨⠼⠲⠙	ordinary minus, boldface 8	
script 4	4	⠈⠼⠙	−8	⠤⠼⠓

**The (46) preceding the numeric indicator is the italic typeform indicator; the (46) following the numeric indicator is the decimal point.*

7.6.1 Boldface Zero: An example of boldface type that has mathematical significance is the boldface zero as the "null vector". The boldface typeform is retained in braille. A switch to Nemeth Code is required because the boldface typeform will be retained when this special zero appears in a mathematical expression.

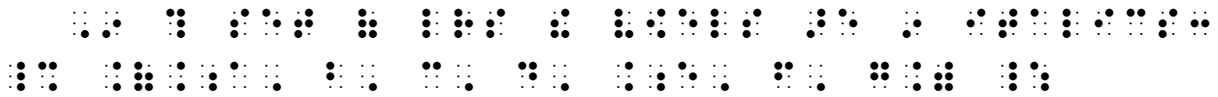
Example 7.6-1 0 is the **null vector**.



7.7 Nonregular Typeform next to Signs of Grouping: Because a letter or numeral in nonregular type must include a numeric or alphabetic indicator, rules regarding such letters or numerals when in contact with grouping signs are different from rules regarding letters or numerals in regular type.

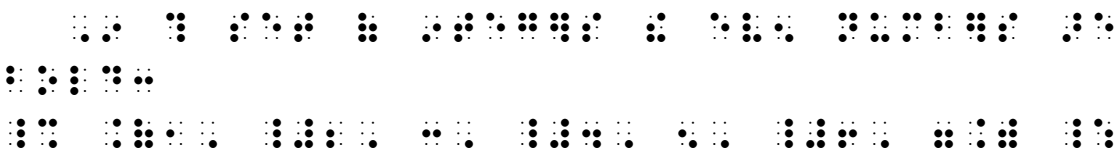
7.7.1 Nonregular Typeform in an Enclosed List: When letters or numerals in an enclosed list are printed in nonregular typeform and that typeform is retained in braille, each typeform indicator is followed by an appropriate letter indicator or numeric indicator. ("Enclosed list" is defined in **Lesson 5**.)

Example 7.7-1 In this set of letters the vowels are in italics: {*a*, b, c, d, *e*, f, g}



In the enclosed list, the English letters in regular type do not require an English letter indicator.

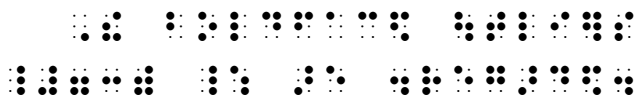
Example 7.7-2 In this set of integers the even numbers are bold: {1, 2, 3, 4, 5, 6, 7}



In the enclosed list, the numerals in regular type do not require a numeric indicator.

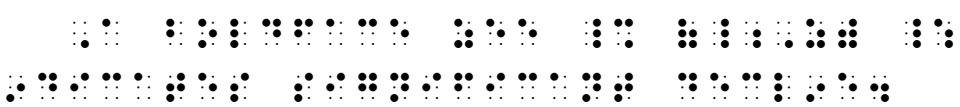
7.7.2 Nonregular Typeform in Contact with a Grouping Symbol: A numeric or alphabetic indicator is required for numbers or letters in nonregular type even when touching or enclosed between grouping symbols.

Example 7.7-3 The boldfaced outliers (**9** and **73**) are disregarded.



A numeral in <u>regular</u> type would not need a numeric indicator when following a sign of grouping.

Example 7.7-4 A boldface zee (**Z**) indicates significant decline.



An English letter in <u>regular</u> type would not need an alphabetic indicator when enclosed between signs of grouping.

PRACTICE 7B

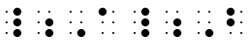
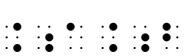
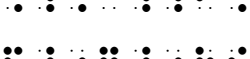
i. The perimeter of a rectangle is obtained by adding the measurements of the sides—two lengths and two widths—expressed as $P = 2\ell + 2w$. What is P if $\ell =$ 5.5 mi and $w = 3.2$ mi ?

ii. This expression represents Nate's sock drawer after five days of wear and laundry. Numbers in italics indicate pairs of blue socks; numbers in bold indicate pairs of red socks. How many pairs of red socks are in Nate's drawer after five days? *4*+**1**−*1*+**3**−**1**−*1*−**1**+*2*−**1**

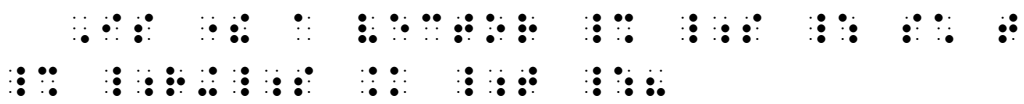
iii. For vectors (a, b, c) can it be said that $a + (b + c) = (a + b) + c$?

Typeform Indicators with More Than One Letter or Numeral

7.8 More Than One Letter: The effect of a typeform indicator extends only to the letter which immediately follows it. Thus, in a sequence of unspaced letters, a typeform indicator must be used before each letter that is not in regular type. Here are some isolated examples.

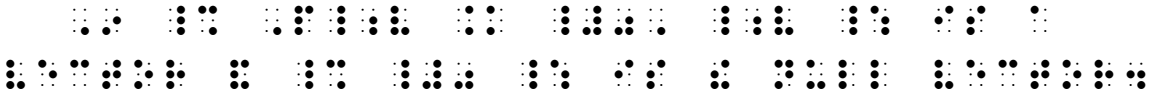
AB		(boldface English A and B)
ab		(boldface English a and b)
αβ		(boldface Greek alpha and beta)
ab		(italic English a and b)
𝒜ℬ		(script English A and B)
*A***b***Cd*		(italic English A, boldface b, italic C, script d)
p**qr**s		(regular English p, boldface q and r, regular s)
x**iyj**		(regular English x, boldface i, regular y, boldface j)
α**a**		(regular Greek alpha, boldface English a)
𝖧H		(sanserif English H, regular English H)

Example 7.8-1 Is there a vector s such that r + s = t?



Each letter is individually boldfaced; an ELI is required following each typeform indicator.

Example 7.8-2 In P**v** = 0, **v** is a vector and 0 is the null vector.



*In P**v**, the letter P does not require an ELI because it is in regular type and is not followed by a space or a punctuation mark. The boldface letter v requires an ELI because an alphabetic indicator must follow a typeform indicator.*

7.9 More Than One Numeral: The effect of a typeform indicator with numerals extends until there is a change in type. Thus, when numerals contain digits in more than one typeform, the appropriate typeform indicator and the numeric indicator must be used before each change in type. When the change is to regular type, only the numeric indicator is used. Here are some isolated examples.

123 ⠸⠼⠁⠃⠉ (all three numerals are boldface)

4*5***6** ⠸⠼⠙⠨⠑⠼⠋ (boldface 4, italic 5, regular 6)

4567 ⠸⠼⠙⠑⠼⠛ (boldface 4 and 5, regular 6 and 7)

12**34** ⠼⠁⠃⠸⠼⠉⠙ (regular 1 and 2, boldface 3 and 4)

100 + **2**00 = **3**00

⠸⠼⠁⠴⠼⠚⠚ ⠬ ⠸⠼⠃⠴⠼⠚⠚ ⠨⠅ ⠸⠼⠉⠴⠼⠚⠚

(boldface 1, 2, and 3; all zeros in regular type)

28-**571** ⠨⠼⠃⠓⠤⠸⠼⠑⠛⠁ (italic 28, hyphen, boldface 571)

47-653 ⠨⠼⠙⠛⠤⠼⠋⠑⠉ (italic 47, hyphen, regular 653)

Other Details

7.10 Underlining and Other Typeforms: There is no underline indicator in the Nemeth Code. Underlining of letters, numbers, and mathematical symbols is presented in the lesson about modifiers. Typeforms for which there are no provisions in the Nemeth Code may use one of the four typeform indicators that is not used elsewhere in the document. A transcriber's note should explain the substitution. For example, "double struck" (also called "blackboard bold") is a typeface style sometimes used for certain symbols in mathematical texts. Here is an earlier example shown with this font.

Example 7.10-1 R denotes the set of rational numbers and ℝ denotes the set of real numbers.

⠠⠗ ⠙⠑⠝⠕⠞⠑⠎ ⠮ ⠎⠑⠞ ⠕⠋ ⠗⠁⠞⠊⠕⠝⠁⠇ ⠝⠥⠍⠃⠻⠎
⠯ ⠠⠁⠝⠙ ⠠⠘⠗ ⠙⠑⠝⠕⠞⠑⠎ ⠮ ⠎⠑⠞ ⠕⠋ ⠗⠑⠁⠇
⠝⠥⠍⠃⠻⠎⠲

A transcriber's note will explain the substitution: "Double-struck letters are indicated in braille with the script typeform indicator."
The UEB "underlined" indicator is used in the narrative text.

The *Sha* (Ш from the Cyrillic alphabet) usually keeps company with bold and double-struck letters, as seen in the next example.

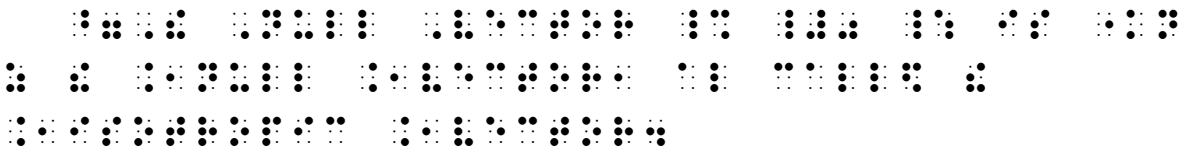
Example 7.10-2 $Ш(\boldsymbol{E}/\mathbb{Q})[\boldsymbol{p}]$ is trivial for $\boldsymbol{p} \neq 2, 3, 5, 7$.

⠸⠘⠠⠷⠠⠠⠑⠸⠌⠠⠟⠸⠜⠸⠘⠠⠷⠠⠏⠸⠜ ⠸⠱ ⠸⠣
⠸⠘⠠⠷⠠⠠⠑⠸⠌⠠⠟⠸⠜⠸⠘⠠⠷⠠⠏⠸⠜
⠸⠣ ⠸⠘⠠⠷⠠⠏ ⠸⠣⠂⠸⠣ ⠸⠘⠠⠷⠠⠑ ⠸⠣⠂⠸⠣ ⠸⠘⠠⠷⠠⠟ ⠸⠣⠂⠸⠣ ⠸⠘⠠⠷⠠⠝ ⠸⠣⠂⠸⠣ ⠸⠘⠠⠷⠠⠺ ⠸⠣⠂⠸⠣

7.11 Termination of a UEB Typeform Passage: A switch from UEB to Nemeth Code terminates the effect of a UEB typeform indicator. A UEB typeform terminator is not needed before an opening Nemeth Code indicator.

The next example shows a boldfaced paragraph heading followed by the special boldfaced mathematical numeral zero.

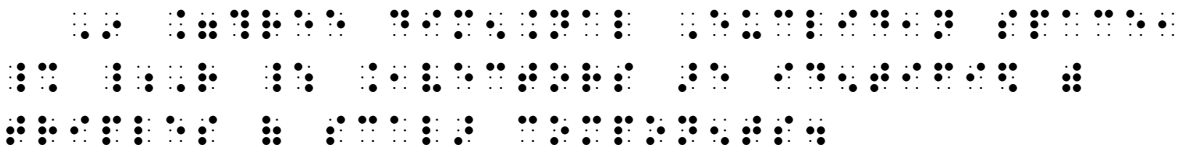
Example 7.11-1 **The Null Vector** $\mathbf{0}$ is known as the *null vector,* also called the *isotropic vector*.



A UEB boldface terminator is not needed because the phrase is immediately followed by an opening Nemeth Code indicator.

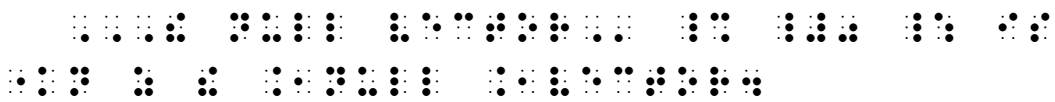
If the nonregular typeform continues after the termination of the mathematical portion, a UEB typeform indicator must be re-entered.

Example 7.11-2 In *three dimensional Euclidean space,* $\mathbf{R}$ *vectors* are identified with triples of scalar components.



7.11.1 Capitalization: Capitalization is not a typeform and must be explicitly terminated.

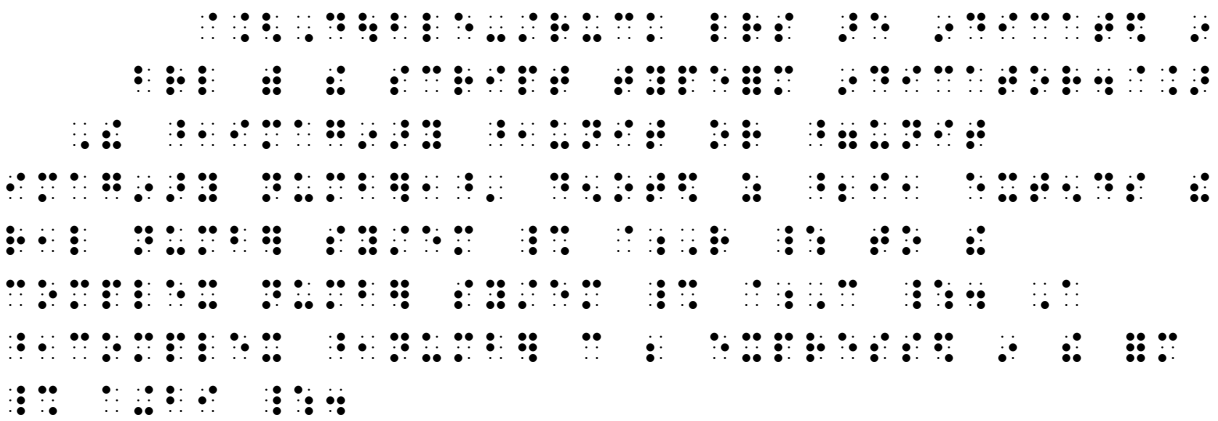
Example 7.11-3 **THE NULL VECTOR** $\mathbf{0}$ is known as the *null vector*.



A capitals terminator is required at the end of this paragraph heading. (The boldface is disregarded in a fully-capitalized paragraph heading according to Braille Formats.)

7.12 Context Clues: The boldfaced words in the next example indicate to the reader that they are defined in a glossary. But what about the boldfaced letter "i"? Search the surrounding text to determine whether the letter "i" retains the bold typeform within a mathematical expression. If it does, the boldface is mathematical and a switch to Nemeth Code is required. However, you can see in the expression $a + bi$ that the imaginary unit is not printed in bold. Therefore. the first mention of $\boldsymbol{i}$ is brailled in UEB, using the UEB boldface symbol indicator. The italic typeform is disregarded in both cases.

Example 7.12-1 The **imaginary unit** or **unit imaginary number**, denoted as $\boldsymbol{i}$, extends the real number system $\mathbb{R}$ to the complex number system $\mathbb{C}$. A **complex number** can be expressed in the form $a + bi$.



Instructions: Begin this practice with a transcriber's note explaining your treatment of the double-struck letters.

PRACTICE 7C

1. The 1-D coordinate system is denoted by $\mathbb{R}$.
2. If the boldface number signifies a withdrawal from your account, can you explain why $250 + 250 = 0$?
3. $c(\mathrm{a}, \mathrm{b}) = (c\mathrm{a}, \mathrm{b})$ as well as $(\mathrm{a}, c\mathrm{b})$. a and b are **vectors**. Define ab.
4. **Two Number Sets:** $\mathbb{N}$ denotes the set of *natural numbers* – that is, the set of nonnegative integers $\{0, 1, 2, \ldots\}$. The set of <u>all</u> integers is denoted by $\mathbb{Z}$.

BOLDFACED MATHEMATICAL SYMBOLS

7.13 Dots 456: In the following cases, ⠸ is considered to be an actual part of the symbol and must not be considered to be a boldface typeform indicator. Do not use ⠸ with any other sign other than those shown in this section.

7.13.1 Signs of Operation in Boldface Type: The signs of operation listed below are to be used to show boldface type only when the distinction between the regular and the boldface forms of the same sign has mathematical significance. The surrounding text should be scrutinized to determine if this is the case. Each symbol consists of ⠸ followed by the appropriate sign of operation. ⠸ *must not be used with any other sign of operation.*

Boldface Plus	**+**	⠸⠬
Boldface Minus	**−**	⠸⠤
Boldface Plus Followed by Boldface Minus	**+ −**	⠸⠬⠸⠤
Boldface Plus Followed by Regular Minus	**+** −	⠸⠬⠤
Regular Plus Followed by Boldface Minus	+ **−**	⠬⠸⠤
Boldface Minus Followed by Boldface Plus	**− +**	⠸⠤⠸⠬
Boldface Minus Followed by Regular Plus	**−** +	⠸⠤⠬
Regular Minus Followed by Boldface Plus	− **+**	⠤⠸⠬

Out of context, the mathematical significance of the boldface in the three examples below is debatable. They are included here for instructional purposes.

Example 7.13-1 a **+** b = b **+** a but a **−** b ≠ b **−** a

⠸⠬ ⠀⠸⠬ ⠀⠸⠬ ⠀⠸⠬ ⠀⠸⠤ ⠀⠸⠤ ⠀⠸⠤

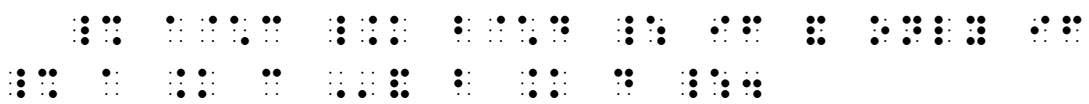
Example 7.13-2 Compare: a **+−**b, b **−+** a

⠸⠬⠸⠤ ⠀⠸⠬ ⠀⠸⠤⠸⠬ ⠀⠸⠤⠸⠬

7.13.2 Equals Sign in Boldface Type: When it is necessary to show that an equals sign is printed in boldface type, ⠸ is placed before the equals symbol in braille. Boldface equals signs are used only when the distinction between the regular and boldface forms of the same sign has mathematical significance. The surrounding text should be scrutinized to determine if this is the case.

Boldface Equals	**=**	⠸⠶

Example 7.13-3 $a \times c = b \times d$ if and only if $a = c$ and $b = d$.



7.13.3 Grouping Signs in Boldface Type: When square brackets or vertical bars are printed in mathematically-significant boldface, ⠸ is used before the grouping symbol.

Boldface Opening Bracket	**[**	⠸⠣
Boldface Closing Bracket	**]**	⠸⠜
Boldface Vertical Bar	**|**	⠸⠳
Boldface Double Vertical Bar	**‖**	⠸⠳⠸⠳

Double boldface vertical bars are usually read as "the norm of."

⠠⠠ **‖** f **‖** ⠸⠳ ⠋ ⠸⠳

Boldface brackets are often used to designate the "integer function".

⠠⠠ **[** x **]** ⠸⠣ ⠭ ⠸⠜

Instructions: Practice brailling the isolated symbols according to Nemeth Code. Follow the print format of a 3-column list. After the centered heading, place the opening Nemeth Code indicator in cell 1 on line 3. Begin the list on the next line (lines 4-6). Follow the list with the Nemeth Code terminator in cell 1 (line 7). Braille the two itemized sentences starting on line 9.

PRACTICE 7D

=	**+ −**	**− +**
+	**+** −	**−** +
−	+ **−**	− **+**

A. Find the greatest integer function where **[**z**]** is the greatest integer ≥ z .

B. ... where **‖ ‖** is the norm of Y, written **‖**Y**‖**.

COMPOUND EXPRESSIONS

7.14 Definition of "Compound Expression": A compound expression consists of a numeral or a letter joined to a word or an abbreviation by a hyphen. Here are some examples of compound expressions.

x-axis	(letter-word)
3-dimensional	(numeral-word)
n-mi.	(letter-abbreviation)
35-ft	(numeral-abbreviation)
Special-K	(word-letter)
Uranium-235	(word-numeral)
DSL-w	(abbreviation-letter)
DC-10	(abbreviation-numeral)

The compound expression may be brailled in UEB unless the expression itself is in mathematical context or if the numeral or letter requires a switch. In such cases, the entire compound expression is brailled inside the switches.

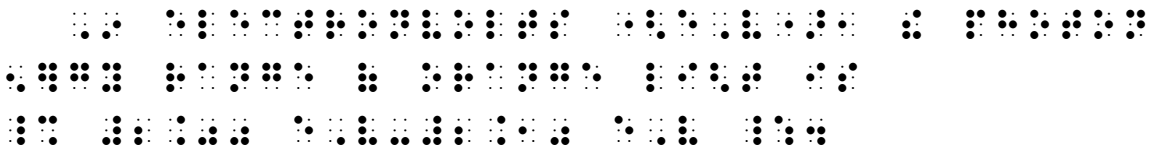
7.14.1 Extent of Capitalization: The effect of the double capitalization indicator of the Nemeth Code is terminated by a space or by any symbol other than a letter.

⋙ DSL–β 

The hyphen terminates the effect of the double capitalization indicator. The beta is lowercase.

7.14.2 Hyphen Followed by a Number: A numeric indicator is required when a numeral is connected to a preceding word or abbreviation by a hyphen.

Example 7.14-1 In electronvolts (eV), the photon energy range of orange light is 2.00 eV-2.10 eV.



Note that the abbreviation is not spaced away from the hyphen. You can be sure this is a hyphen and not a minus sign if you can substitute the word "to" for the hyphen.

7.15 Typeform Indicators in Compound Expressions: If one or both components of a compound expression are printed in a nonregular typeform, the typeform is retained in the braille transcription only if it is required for emphasis or distinction, or if the typeform has mathematical significance. The compound expression may be brailled in UEB unless the expression itself is in mathematical context or if the numeral or letter requires a switch. In such cases, the entire compound expression is brailled inside the switches.

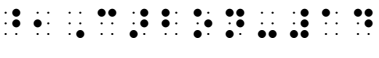
Context will determine your decision to switch codes. If the compound expression is brailled in Nemeth Code, the following indicators are used.

Italic Typeform Indicator	
Boldface Typeform Indicator	

The compound expression "Carbon-14" is used below to illustrate several combinations of typeforms. These examples are purely illustrative. The decision to follow UEB or to switch to Nemeth Code as well as the decision to retain the nonregular typeform can only be determined in context.

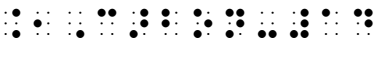
- The effect of the typeform indicator continues through the hyphen.

  Carbon-14 (entirely bold)

  UEB: 

  NC: 

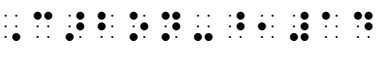
  Carbon-14 (entirely italics)

  UEB: 

  NC: 

- If there is a change from regular to nonregular type after the hyphen, the appropriate typeform indicator is used after the hyphen.

  Carbon-**14** (word is normal; number is bold)

  UEB: 

  NC: 

- If there is a change from one typeform to a different typeform after the hyphen, the appropriate typeform indicator is used before each part of the expression. In UEB, a typeform terminator is brailled before the hyphen. In Nemeth Code, the typeform of the word terminates when a new typeform indicator follows the hyphen. No explicit terminator is needed.

  Carbon-**14** (word is in italics; number is bold)

  UEB: 

  NC: 

- If there is a change from nonregular to regular type after the hyphen, the typeform must be terminated before the hyphen.

  Carbon-14 (word is bold; number is not)

  UEB: 

The BANA Nemeth Code Technical Committee is discussing details regarding termination of typeform in a compound expression. This section will be completed after decisions are made.

Instructions: Braille the following compound expressions in Nemeth Code, applying the typeforms shown.

PRACTICE 7E

x-*axis*

35-ft

DC-*10*

Bismuth-210

Typeform Summary

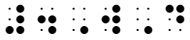
The typeform indicators presented in this lesson apply to letters, numerals, certain math symbols, and compound expressions. Indicators are provided for letters printed in boldface, italics, sanserif, and script. Boldface and italic indicators can be applied to numerals or to either or both components of a compound expression. The boldface indicator can be applied only to certain specific math symbols.

Typeform indicators for words and phrases in mathematical context will be discussed in a later lesson.

Food for Thought You may have noticed that the italic typeform indicator is the same symbol as the decimal point, the Greek letter indicator, as well as the first cell in several math symbols; and that the boldface indicator is the same symbol as the punctuation indicator, the German letter indicator, and the first cell in several symbols and indicators. The dots' functions are determined in context.

UNSPACED NUMBER/LETTER COMBINATIONS

7.16 Number/Letter Combinations Without Hyphens: Letters that occur in unspaced number/letter combinations are not considered to be abbreviations in the Nemeth Code. If a letter/number combination appears in mathematical context, each capital letter is preceded by the single capitalization indicator. Letter/number combinations are punctuated mathematically if the punctuation falls inside the Nemeth Code switches.

⋙ 4WD 

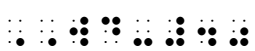
⋙ 1074FE 

If a number follows a letter, a multipurpose indicator (dot 5) precedes the number. A numeric indicator is not used.

⋙ MP3 

⋙ WD40 

Compare: Note that the presence of a hyphen turns WD40 into a compound expression and a numeric indicator is used.

⋙ WD-40 

REVISITING THE NUMERIC INDICATOR

7.17 Summary of the Numeric Indicator: It may be helpful to summarize the use and nonuse of the Nemeth Code numeric indicator studied so far. The word "numeral" includes a number that begins with a decimal point or a number that begins with a minus sign as well as a simple numeral. These rules also apply to non-decimal digits such as T and E. Assume mathematical context in example (c).

A numeric indicator <u>is needed</u> ...

... when a numeral begins a braille line or is preceded by a space. (a)

... when a numeral immediately follows a punctuation mark. (b)

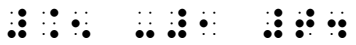
... when a hyphen connects a word or an abbreviation to a numeral, or when a hyphen connects a punctuation mark to a numeral. (c)

... when a numeral follows a mathematical asterisk, crosshatch, paragraph mark, or section mark. (d)

... when a numeral is in non-regular type. (e)

... when there is change back to regular type from non-regular type within the same numeral. (f)

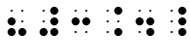
(a) .5 −1 t4e



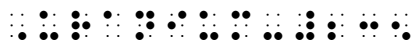
0.333 ... 3 ...



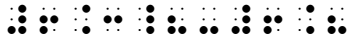
(b) "3.4"



(c) Uranium-235



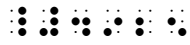
6.3?-6.8



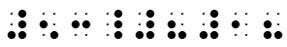
(d) $3 * 9 = 27$



(e) 4925



(f) 53818



A numeric indicator is not needed ...

... when a numeral is unspaced from and follows a mathematical character. (g)

... when a numeral is unspaced from and follows an opening grouping sign. (h)

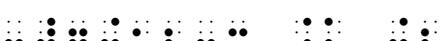
... when a numeral follows a hyphen if the hyphen follows a numeral, a letter, or other mathematical expression. (i)

... when a numeral in regular type is part of an "enclosed list". (j)

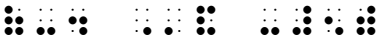
... when a single numeral is partitioned into segments with spaces. (k)

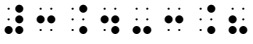
... when a numeral is unspaced from and follows a slash. (l)

... when a numeral is within an unspaced number/letter combination. (m)

(g) $-7 + 12 - 3 = +2$ 

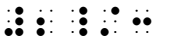
$12.98 

(h) (–4 and –5) 

(i) 3.4-3.8 

(j) {.5, .7, .9, 1.1} 

(k) 987 656 000 

(l) 2/3 

(m) 7NT45Z 

This is not an all-inclusive list. More rules will be presented in later lessons.

Instructions: Use Nemeth Code exclusively for this list of out-of-context number/letter combinations and compound expressions. These are hyphens, not subtraction signs.

PRACTICE 7F

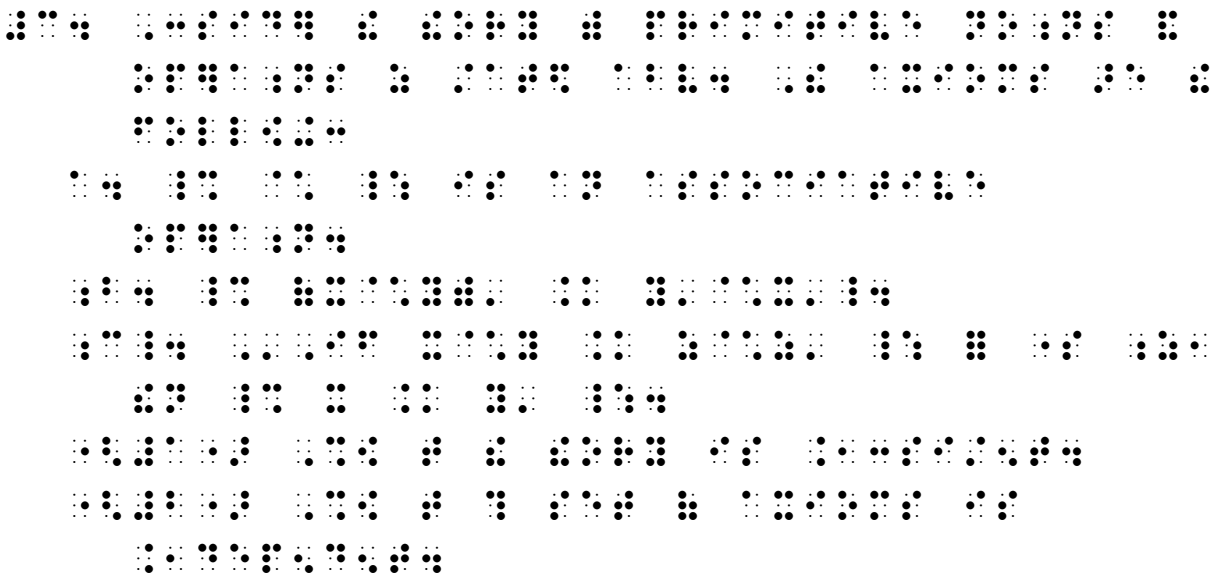
DC-7	Q85p7	23496AC
β-class	712-1-5-AC	n-ary
A-511	DCV-AZ	not-α
764B	10-to-1	

7.18 Margins for Itemized Material with Subdivisions (1-5; 3-5): To transcribe itemized material with lettered or numbered subdivisions, begin the main item designation in cell 1. Place any runovers in cell 5. Cell 3 is reserved for the beginning of each lettered or numbered subdivision. Any runovers to subdivisions also begin in cell 5. In Nemeth Code, "subdivision" applies to all itemized sublevels regardless of the print indentation layout. In other words, for an exercise with any number of subentry levels, use margins 1-5 for the first level, and 3-5 for all sublevels.

The first example shows two indentation levels in the print copy. In braille, all subdivisions begin in cell 3. All runovers are in cell 5.

Example 7.18-1

3. Consider the theory with primitive notions and operations as stated above. The axioms are the following:
   a. $\times$ is an associative operation.
   b. $(x \times y)' = y' \times x'$.
   c. If $x \times y = z \times z'$ for some z, then $x = y'$.
      (1) Show that the theory is *consistent*.
      (2) Show that this set of axioms is *dependent*.

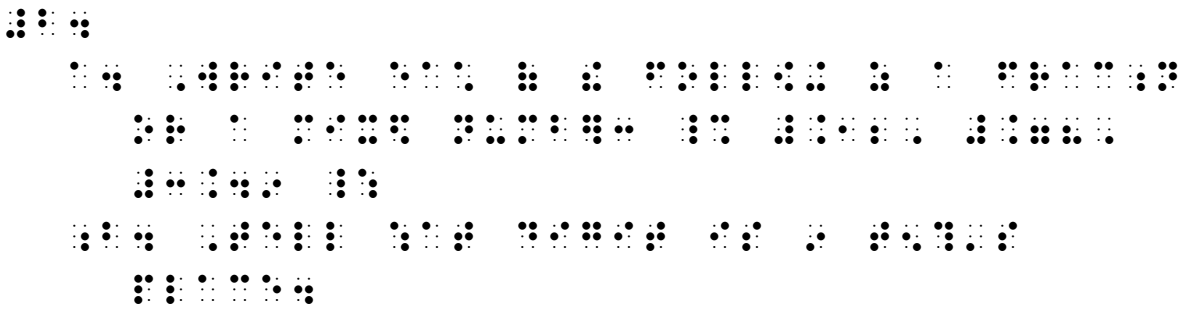


The next example shows no text following the main item number, with subdivision **a.** beginning on the same line. In braille, subdivisions must begin in cell 3 on a new line.

Example 7.18-2

2. a. Write each of the following as a fraction or a mixed number: .12, .78, 3.49

   b. Tell what digit is in tenth's place.



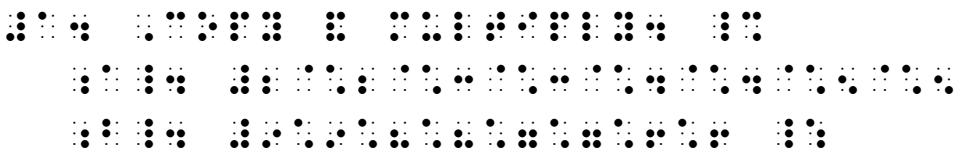
In the next example, the subdivisions are printed without indentation. In braille, subdivisions must begin in cell 3 on a new line.

Example 7.18-3

1. Copy and multiply.

a. $2 \times 2 \times 3 \times 3 \times 4 \times 4 \times 5 \times 5$

b. $9 \cdot 9 \cdot 8 \cdot 8 \cdot 7 \cdot 7 \cdot 6 \cdot 6$



Reminder: The identifiers of each subdivision should begin in the same cell (cell 3). When code switching occurs, place the opening Nemeth Code indicator at the end of the line of text that precedes the identifier.

7.18.1 Paragraphs Within Itemized Material with Subdivisions (7-5): If a main item or a subdivision has more than one paragraph, each new paragraph begins in cell 7, and its runovers begin in cell 5. The margins and spacing in the print copy help the transcriber determine the start of a new paragraph within the itemized material.

In the next example, extra spacing between lines in print distinguishes new paragraphs. In the braille transcription, indentation of the first line of each new paragraph organizes the narrative. Within the item and subitem format there are no blank lines between paragraphs or subdivisions.

5. Does the method of creating whole-number x-terms work with decimals? Consider this example.

   A jacket is marked 15% off. The sale cost is \$36.31. Expressed as an equation, $0.85x = 36.21$.

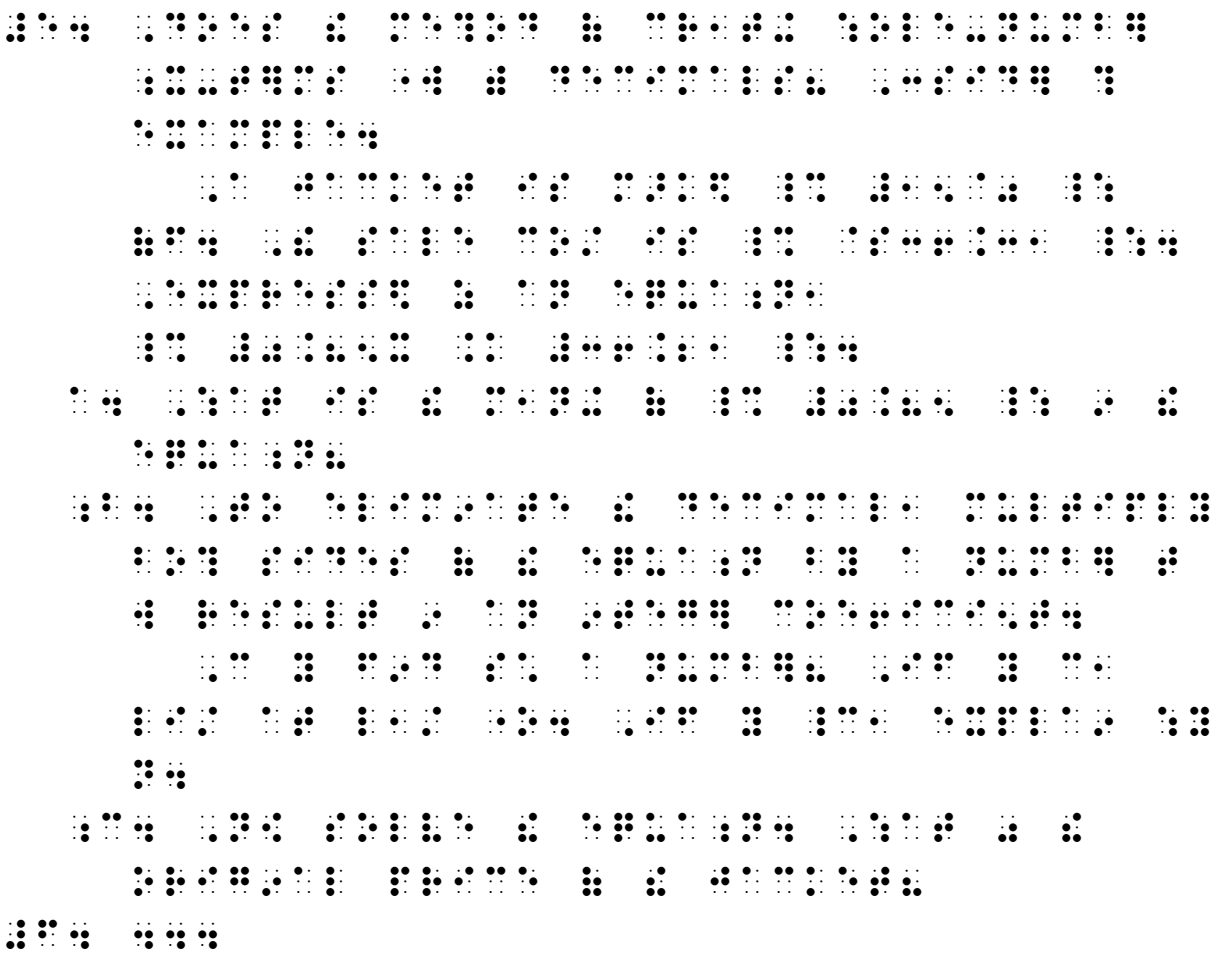
   a. What is the meaning of 0.85 in the equation?

   b. To eliminate the decimal, multiply both sides of the equation by a number that will result in an integer coefficient.

      Can you find such a number? If you can, list at least one. If you cannot, explain why not.

   c. Now solve the equation. What was the original price of the jacket?

6. ...



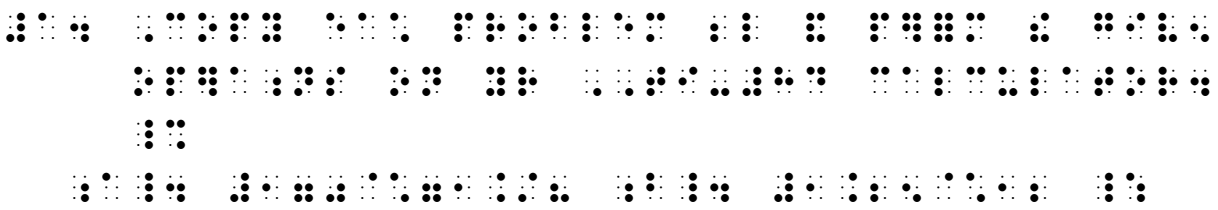
7.18.2 Side-by-Side Option: It is permissible to place the subdivisions of itemized material side by side across a page if, and only if, all the subdivisions can be accommodated on one braille line. This is an optional format style and applies *only to subdivisions*. (This restriction does not apply to spatial material, which will be studied later in this course.)

Example 7.18-5

1. Copy each problem below and perform the given operations on your TI-84 calculator.

    a. $170 \times 71 \div 8$

    b. 1.25×12



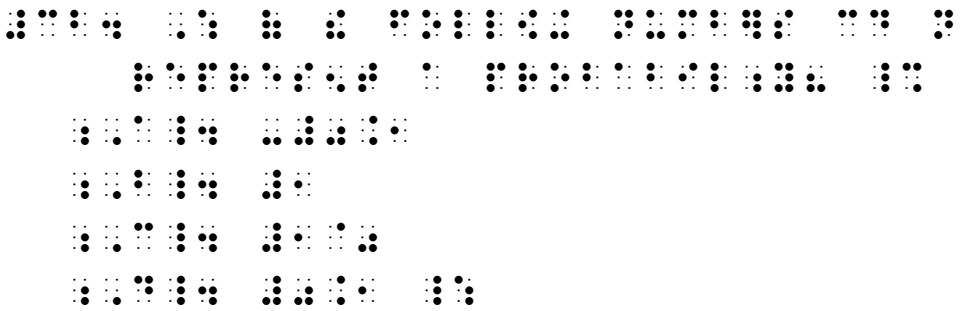
The opening Nemeth Code indicator falls in the runover cell (cell 5) of the main entry item. The identifier of subitem "a" starts in cell 3 on the next line.

In the next example, although the four subdivisions will fit across the same line in the print copy, in braille they will not. Each subdivision must begin in cell 3 on a new line.

Example 7.18-6

32. Which of the following numbers could not represent a probability?

A. -0.1 **B.** 1 **C.** 1% **D.** 0.1



Notice that, although the numeral "1" (subitem B) can be brailled in either code, code-switching clutter is avoided by staying in Nemeth Code.

PRACTICE 7G

23. Simplify and solve each equation below for *x*. Show your work and check your answer.

    a. $24 = 3x + 3$ b. $2(x - 6) = x - 14$

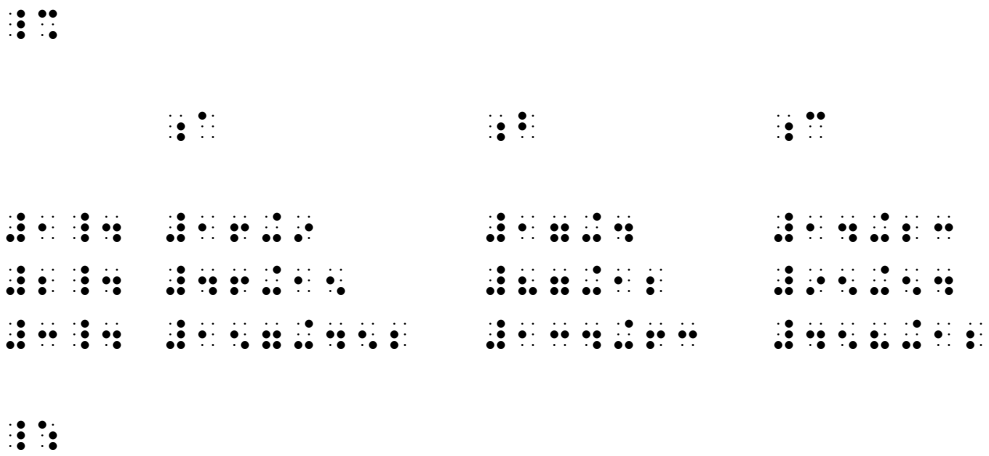
    c. $6 + 2.5x = 21$ d. $2(x + 4.5) = 32$

7.18.3 Tabular Form: When itemized material is arranged in tabular form so that items are numbered at the margin and subdivisions are aligned beneath lettered column headings, the material should be transcribed in one of the following ways, depending upon whether all of the columns can be accommodated across the braille page.

7.18.3.a When to Retain Column Format: If all the columns can be accommodated across the braille page, the print columnar arrangement is followed. Each problem number begins in cell 1. The letter identifying each column is aligned with the first cell of the related column. A blank line is left above and below the lettered column headings. Two blank cells separate the columns.

Example 7.18-7

	a	b	c
1.	$16 + 9$	$17 + 4$	$14 + 23$
2.	$46 + 15$	$87 + 12$	$95 + 54$
3.	$157 + 452$	$134 + 63$	$458 + 12$

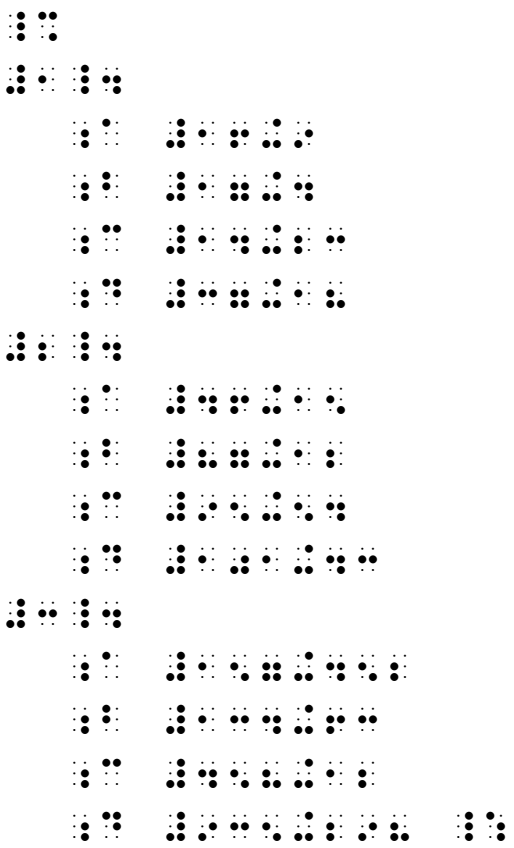


The BANA Nemeth Code Technical Committee is discussing placement of the switch indicators in the tabular example, above.

7.18.3.b When Not To Retain Column Format: If all the columns cannot be accommodated across the braille page, each subdivision in each problem must be lettered individually, and the format in **7.18** must be followed.

Example 7.18-8

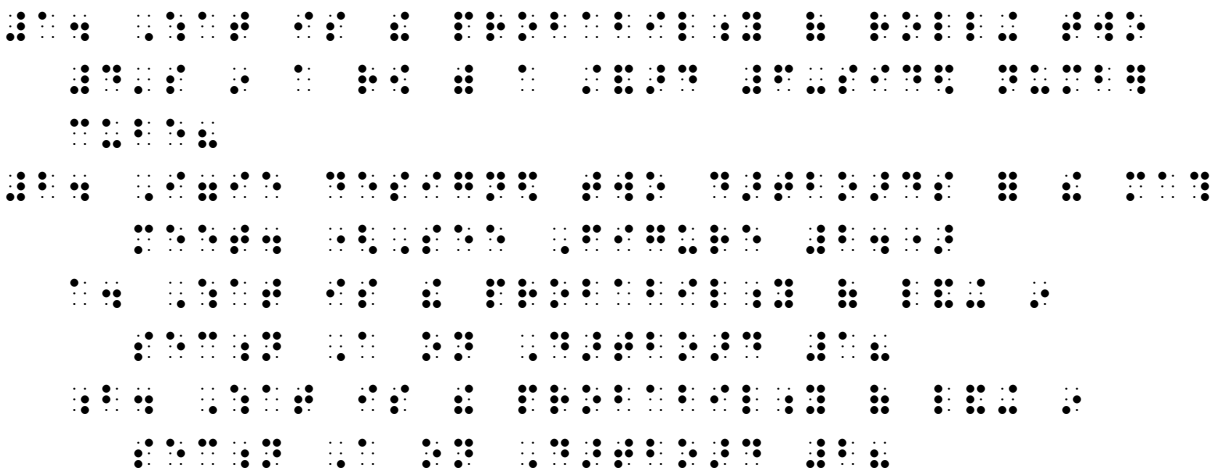
	a	b	c	d
1.	$16 + 9$	$17 + 4$	$14 + 23$	$37 + 18$
2.	$46 + 15$	$87 + 12$	$95 + 54$	$101 + 43$
3.	$157 + 452$	$134 + 63$	$458 + 12$	$935 + 298$



7.19 Margins for Exercise Sets: Runover margins for itemized material are determined individually for each question. That is, a problem with no subentries will be (1-3); the next problem may have subentries and so will be (1-5; 3-5), etc.

Example 7.19-1

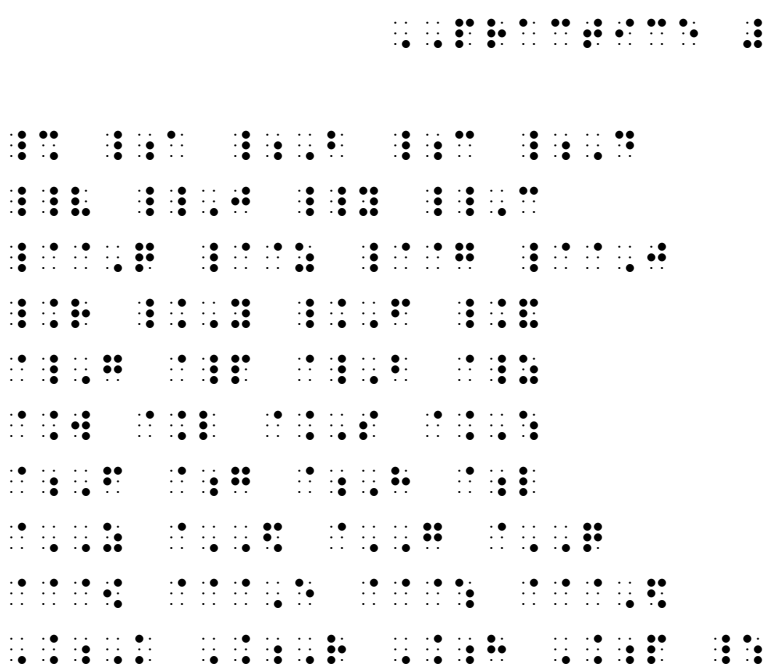
1. What is the probability of rolling two 4's in a row with a standard 6-sided number cube?
2. Iggie designed two dartboards for the math meet. (See Figure 2.)
   a. What is the probability of landing in section A on Dartboard 1?
   b. What is the probability of landing in section A on Dartboard 2?

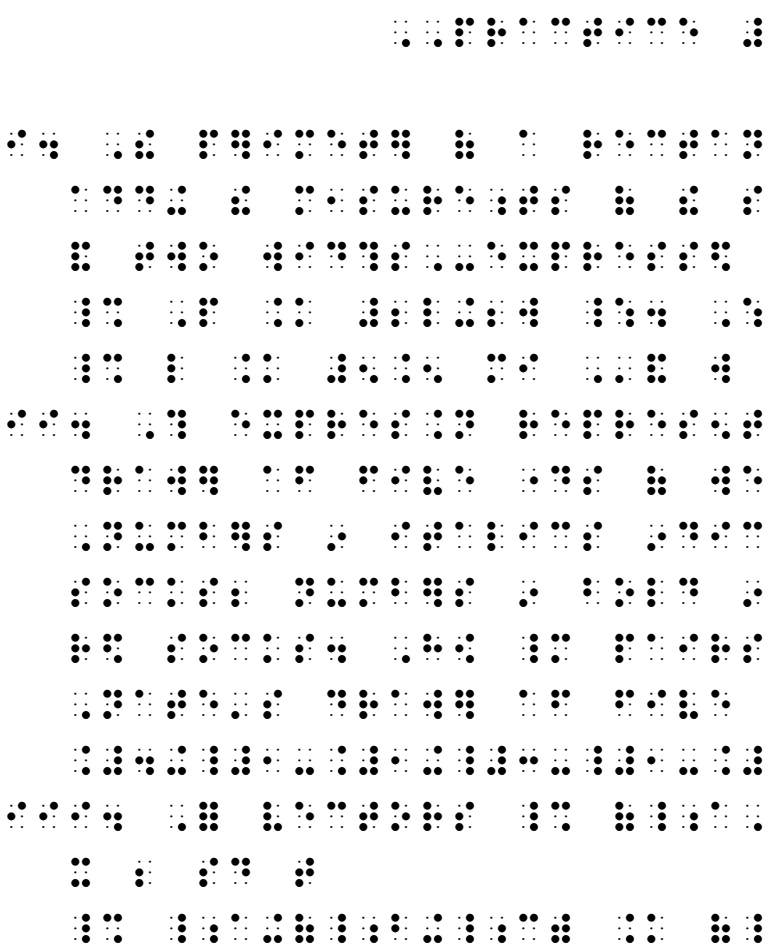


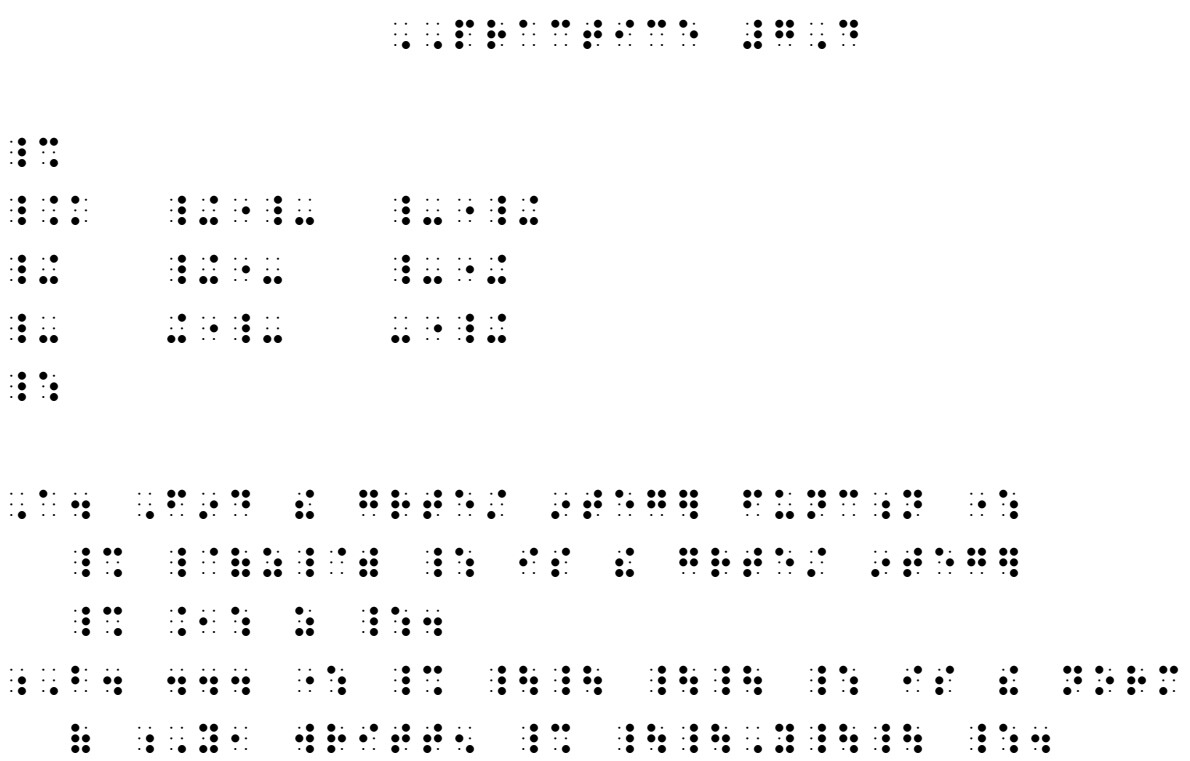
*For the remainder of the course, Nemeth Code format summaries can be found in **Appendix C**.*

For further practice, see Appendix A—Reading Practice.

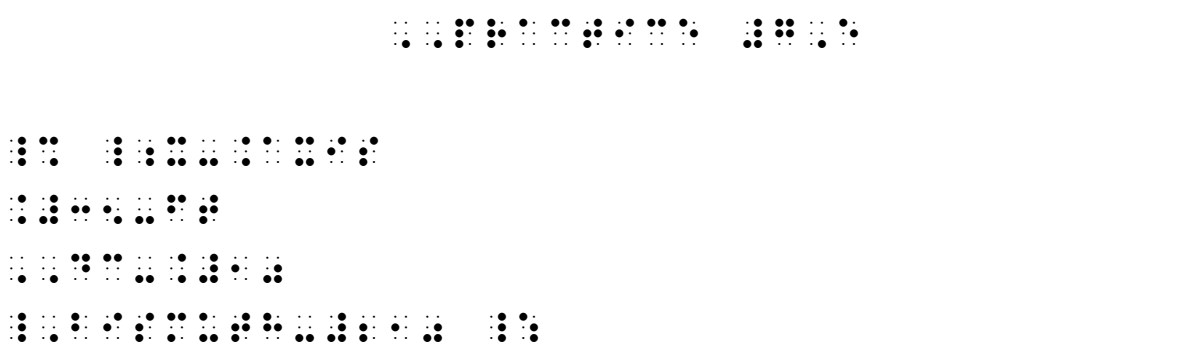
ANSWERS TO PRACTICE MATERIAL

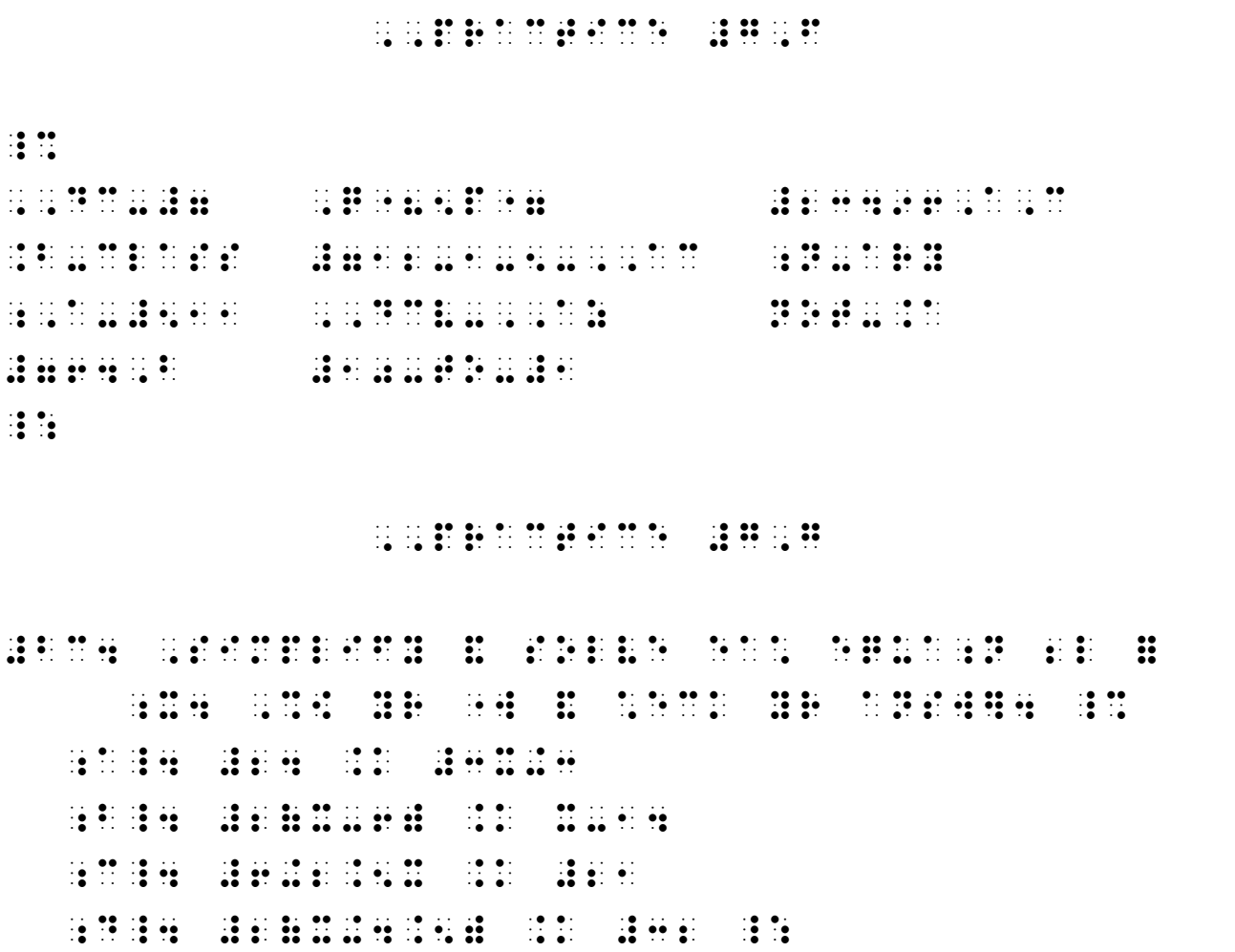






Review spacing with signs of grouping in 3.15 and 3.15.1.





EXERCISE 7

Exercise 7 will be available when this course is finished being written and is no longer "Provisional".

Proceed to Lesson 8.

LESSON 8

Read about this PROVISIONAL EDITION in the front matter to this book.
Check the NFB website periodically for updates to this lesson.

- *LEVEL INDICATORS*
  - *Superscripts*
  - *Baseline Indicator*
  - *Subscripts*
  - *More about Superscripts and Subscripts*
- *GROUPING SIGNS, cont.*

FORMAT
DISPLAYED MATHEMATICAL EXPRESSIONS

LEVEL INDICATORS

8.1 Definition: A mathematical expression can contain symbols placed above or below the baseline level (the normal line of type in print). Superscripts appear above the baseline; subscripts appear below the baseline. Here are some expressions which contain superscripts and subscripts. The baseline is marked for orientation.

baseline 10^5 H_2O $^{-}4$ °C πr^2 $^{235}_{92}U$ _baseline_

Note: The font size is increased for the isolated examples in this lesson to help in determining the levels.

Superscripts

8.2 Superscript Level Indicator: In braille, indicators are used to identify the level of a superscript or subscript. The superscript level indicator is used to show that the symbols immediately following it appear on the first level above the baseline of writing.

Superscript Indicator	⠔

⋙ 10^7 ⠔⠼⠁⠚⠔⠼⠛

⋙ $y = [\,g(x)]^n$ ⠔⠽ ⠨⠅ ⠔⠣⠛⠣⠭⠜⠣⠝

The characters in the superscript are spaced according to the rules of the Nemeth Code.

⋙ $3^{-0.05T}$ ⠔⠼⠉⠤⠼⠚⠲⠚⠑⠠⠞

Remember, a numeral does not need a numeric indicator when it follows and is unspaced from a mathematical character or indicator. A letter, a shortform letter combination, or a Roman numeral does not need an English letter indicator unless it is preceded by a space or by one or more punctuation marks and followed by a space or by one or more punctuation marks.

⋙ $x\ x^3$ ⠭ ⠔⠭⠔⠼⠉

Example 8.2-1 Statements i^2 and ii^2.

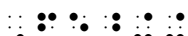


Example 8.2-2 Theorems I^2 and V^2.



8.2.1 Abbreviations with Superscripts: When a superscript applies to an abbreviation, the level indicator is unspaced from the abbreviation.

⋙ $19\ \text{in.}^3$ 

⋙ Fe^{++} 

Reminder about single-letter abbreviations: Rules for the nonuse of the ELI with mathematical "single letters" do not apply to abbreviations. A single-letter abbreviation from the English alphabet that does not have a related period must always begin with an English letter indicator.

⋙ $5.7\ \text{m}^2$ 

The English letter indicator is used for a single-letter abbreviation without a period, even when unspaced from an indicator.

PRACTICE 8A

$6x^2$	$24^?$	x^{2a}
π^2	$(y-k)^3$	8^{10}
n^m	$A^{(k+1)+m}$	$q^{b(w-1)}$
$k^{0.27}$	$e^{3{,}000}$	e^-

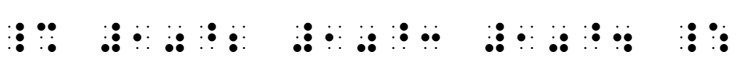
Measurements of **length** are measurements in **one dimension**. They are labeled as cm, ft, m, etc. Measurements of **area** are measurements in **two dimensions**. They are labeled as cm^2, ft^2, m^2, etc. Measurements of **volume** are measurements in **three dimensions**. They are labeled as cm^3, ft^3, m^3, etc.

The 3-D coordinate system is often denoted by $\mathbb{R}^3$. Likewise, the 2-D coordinate system is often denoted by $\mathbb{R}^2$. A general *n*-dimensional coordinate system is often denoted by $\mathbb{R}^n$.

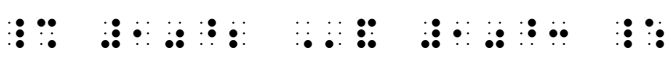
8.3 Returning to the Baseline Level: When a superscript is the last item in the expression, the following circumstances bring the reader back to the baseline level: a space, a comma followed by a space, or a punctuation indicator. "A space" includes the space before a Nemeth Code termination indicator and/or transition to a new braille line.

— A space returns the reader to the baseline.

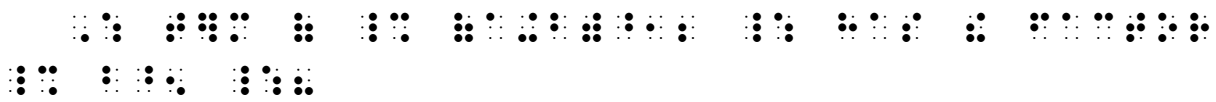
Example 8.3-1 $10^2\ 10^3\ 10^4$



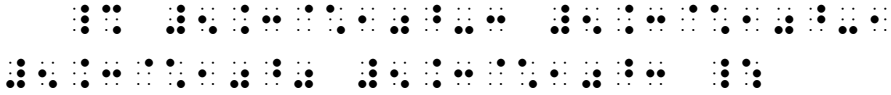
Example 8.3-2 10^2 and 10^3



Example 8.3-3 Which term of $(a+b)^{12}$ has the factor b^5 ?

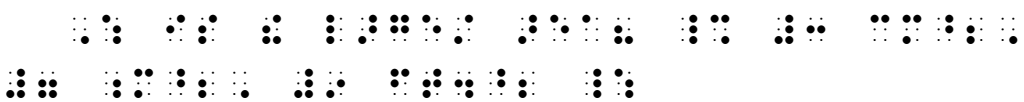


Example 8.3-4 $5.3\times10^{-3}\ \ 5.3\times10^{-1}\ \ 5.3\times10^{0}\ \ 5.3\times10^{3}$

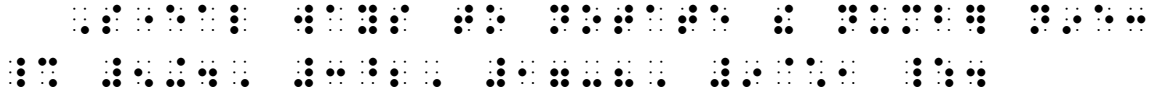


— A comma followed by a space returns the reader to the baseline.

Example 8.3-5 Which is the largest area? $3\ \text{cm}^2$, $7\ \text{m}^2$, $9\ \text{ft.}^2$

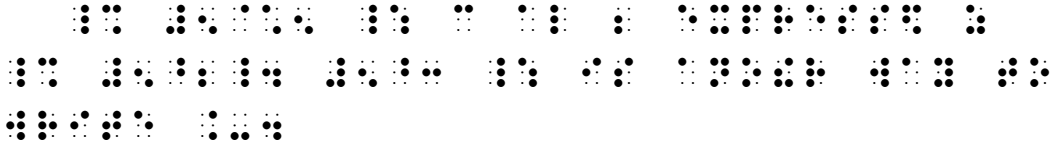


Example 8.3-6 Several ways to notate the number nine: $5+4,\ 3^2,\ 17-8,\ 9\times1.$



— A punctuation indicator returns the reader to the baseline.

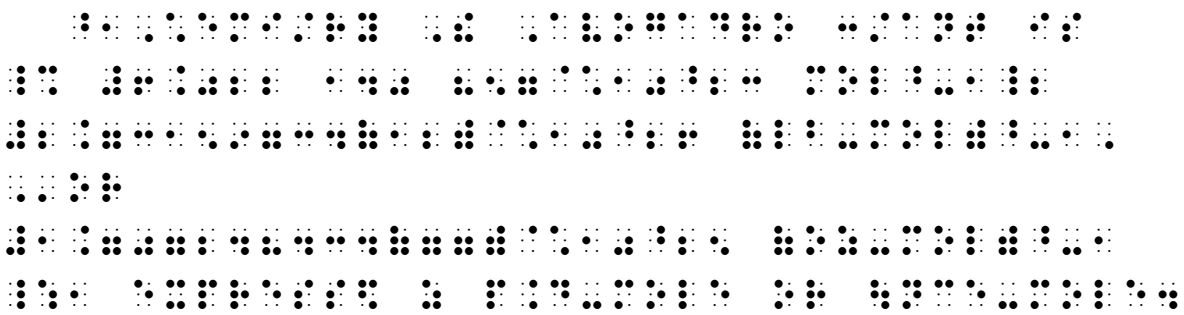
Example 8.3-7 5×5 can also be expressed as 5^2. 5^3 is another way to write ___.



Notice how the baseline level is re-established after each of the six superscripts in the next example.

Example 8.3-8

Chemistry The Avogadro constant is $6.022\ 140\ 857 \times 10^{23}\text{mol}^{-1}$; $2.73159734(12) \times 10^{26}(\text{lb-mol})^{-1}$, or $1.707248434(77) \times 10^{25}(\text{oz-mol})^{-1}$, expressed as pound-mole or ounce-mole.

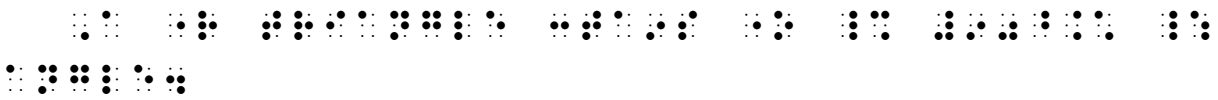


The baseline is re-established as follows: the space before each abbreviation ("mol", "(lb-mol)", and "(oz-mol)"); the punctuation indicator before the semicolon; the math comma and the following space (or, in this case, transition to a new line); the space before the Nemeth Code terminator.

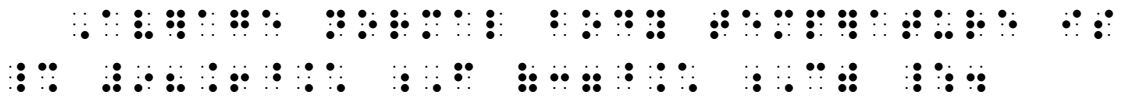
8.4 Raised Hollow Dot: The hollow dot in the superscript position may be used to represent degrees of angle or degrees of temperature. Transcription of the "degrees" symbol requires a switch to Nemeth Code.

Raised Hollow Dot representing Degrees	°	

Example 8.4-1 A right triangle contains one 90° angle.



Example 8.4-2 Average normal body temperature is 98.6° F (37° C).



"F" and "C" are spaced according to Nemeth Code rules for abbreviations.

PRACTICE 8B

1. Tell what number each of the following names: 6^2, 5^{-3}, and $(3.15)^4$.
2. Which expression denotes the area of a circle?
   a. $360°$
   b. πr^2
   c. πd
3. The symbol for "feet squared" is ft^2. $1\text{ ft} \times 1\text{ ft} = 1\text{ ft}^2$
4. 6.02×10^{23} is an approximation of "Avogadro's number".

Introduction to the Baseline Indicator

8.5 Function of the Baseline Indicator: In an unspaced expression, a return to the baseline level is brought about by the use of the baseline indicator.

Baseline Indicator ⠰

⠸⠡ $a^2 + b^2 + c^2$ ⠰⠘⠆⠰⠆⠘⠆⠰⠆⠘⠆

Example 8.5-1 Draw a 30°-60°-90° triangle.

⠰⠘⠼⠉⠚⠈⠴⠼⠋⠚⠈⠴⠼⠊⠚⠈⠴⠰⠘⠆ ⠰⠘⠼⠉⠚⠈⠴⠼⠋⠚⠈⠴⠼⠊⠚⠈⠴⠰⠆⠘⠆ ⠰⠘⠼⠉⠚⠈⠴⠼⠋⠚⠈⠴⠼⠊⠚⠈⠴⠰⠆⠘⠆ ⠰⠘⠆
⠰⠘⠼⠉⠚⠈⠴⠼⠋⠚⠈⠴⠼⠊⠚⠈⠴⠰⠆⠘⠆ ⠰⠘⠼⠉⠚⠈⠴⠼⠋⠚⠈⠴⠼⠊⠚⠈⠴⠰⠘⠆

Notice that the baseline indicator is the same symbol as the multipurpose indicator—dot 5. The indicator's function is determined in context.

8.5.1 Abbreviations with Superscripts: When returning to the baseline after an abbreviation with a superscript, no space is left before the baseline indicator.

Example 8.5-2 Adding cubic meters: $2\text{ m}^3 + 2\text{ m}^3 = 4\text{ m}^3$

⠼⠃⠰⠍⠰⠘⠆⠐⠬⠼⠃⠰⠍⠰⠘⠆⠐⠶⠼⠙⠰⠍⠰⠘⠆
⠐⠣⠐⠜ ⠐⠣⠐⠜⠰⠘⠆ ⠐⠣⠐⠜⠰⠘⠆⠐⠬⠐⠜⠰⠘⠆ ⠐⠣⠐⠜⠰⠘⠆⠐⠶⠐⠜⠰⠘⠆ ⠐⠣⠐⠜⠰⠘⠆⠐⠶⠐⠜⠰⠘⠆ ⠐⠣⠐⠜⠰⠘⠆⠐⠶⠐⠜⠰⠘⠆ ⠐⠣⠐⠜⠰⠘⠆

Reminder: The superscript indicator is unspaced from the abbreviation.

Compare the spacing in the next two examples.

⋙ 144 ft + 144 ft



A space precedes and follows the abbreviation "ft". The operation sign is unspaced from the number which follows.

⋙ 144 ft^2 + 144 ft^2



A space precedes the abbreviation "ft". The superscript is unspaced from the abbreviation. There is no space before the baseline indicator.

PRACTICE 8C

(1) $4(x-y)^3 - 2(x-y)^3$

(2) $3a^3b + 6a^6b^2 + 9a^9b^3$

(3) 98.6° F

(4) $V = 2\pi^2 Rr^2$

(5) x^n-dimensional system

(6) $6^2 \times 6^3 = 6^{2+3} = 6^5$

(7) 144 ft^2 + 144 ft^2 = ...

(8) Given that $(x+2)$ and $(x-3)$ are factors of $f(x)$, show that $a = 3$ and $b = -10$.

8.6 Higher Levels of Writing: Superscripts may carry superscripts of their own. In such cases, the superscript indicator is doubled, tripled, etc. to indicate superscripts on the second, third, or higher levels of writing.

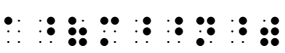
Superscript with Superscript		*(two levels above the baseline)*
Superscript with Superscript with Superscript		*(three levels above the baseline)*

⋙ n^{a^b} 

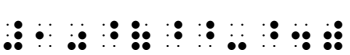
⋙ $n^{a+1^{b+1^{c+1}}}$ 

If the print copy is difficult to analyze, a straightedge and a magnifier will help to determine the levels. In general, the print font gets smaller as the level gets higher.

8.6.1 Combinations: Each level indicator is read *as it relates to the baseline level*. The effect of one level indicator is terminated by another level indicator. Keep this in mind as you read the following examples.

⋙ $a^{(m^n)}$ 

The opening and closing parentheses and the letter "m" are one level above the baseline; the letter "n" is two levels above the baseline.

⋙ $10^{(-4)}$ 

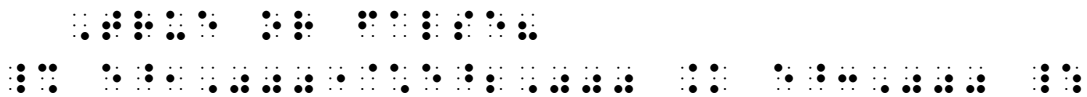
The opening and closing parentheses and the numeral "4" are one level above the baseline; the minus sign is two levels above the baseline.

Example 8.6-1 Will $E[x^2] = e^{2(a+b^2)}$?



The square brackets are on the baseline level. The second superscript is $2(a + b^2)$ *which means that the final "2" is at the super-superscript level.*

Example 8.6-2 True or false? $e^{1,000} \times e^{2,000} = e^{3,000}$



When a space brings the reader back to the baseline, a baseline indicator is not needed.

8.7 Certain Raised Signs: Some signs are printed in a raised position but are not considered to be superscripts.

8.7.1 Raised Ordinal: *Braille Formats* states that raised ordinal endings are not considered to be superscript. This guideline is also followed in a technical transcription, whether following the rules of UEB or of Nemeth Code.

Example 8.7-1 Name the 6th and 7th item in the series.



Example 8.7-2 What is the 3nth degree?



8.7.2 Prime Sign: The prime sign, introduced in **Lesson 1**, appears to be raised in print but it assumes the same level as the number or letter to which it applies.

Prime Sign	'	⠔

⋙ x'^{2} ⠭⠔⠼⠃ *The prime sign belongs with the letter x. Only the number 2 is in the superscript position.*

⋙ $A^{2''}$ ⠠⠁⠔⠔⠼⠃ *The prime signs belong with the number 2 in the superscript.*

Example 8.7-3 plane angle $\alpha = 30°2'8''$

⠰⠫⠇⠁⠝⠑⠀⠁⠝⠛⠇⠑⠀⠨⠅⠀⠼⠉⠚⠬⠼⠃⠄⠼⠓⠔⠔

The prime signs are not superscripts. They apply to numbers at the baseline of writing.

8.7.3 Apostrophe-s: In an apostrophe-s ending, the apostrophe is at the same level as the "s". Because a punctuation indicator returns the reader to the baseline, a level indicator is inserted to maintain the level of the following "s" in the next example.

⋙ $A^{m+m+m's}$ ⠠⠁⠔⠍⠬⠍⠬⠍⠄⠔⠎

Compare to this example where the "s" and its apostrophe are printed on the baseline level.

⋙ A^{m+m+m} ’s ⠠⠁⠔⠍⠬⠍⠬⠍⠄⠎

PRACTICE 8D

1. Use a calculator to find 9^{9^9}.
2. Find the rth term of $(x+y)^n$.
3. Label the x^2 ’s and x^3 ’s.
4. What is the meaning of x''^3?
5. Simplify: $(x^3-y^3)^2-(x^3+y^3)^2$.
6. $x^{y^n z}$ or $x^{y^2 z}$

Subscripts

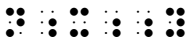
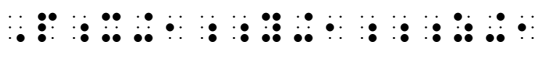
8.8 Subscript Level Indicators: *Except* as stated in **8.10** below, the subscript level indicator is used to show that the symbols immediately following it appear on the first level below the baseline of writing.

Subscript Indicator	⠰

- ⋙ f_n 
- ⋙ $a_{(k+1)}$ 

Subscripts may carry subscripts of their own. In such cases, the subscript level indicator is doubled, tripled, etc. to indicate subscripts on the second, third, or lower levels.

Subscript with Subscript	⠰⠰	*(two levels below the baseline)*
Subscript with Subscript with Subscript	⠰⠰⠰	*(three levels below the baseline)*

- ⋙ n_{x_y} 
- ⋙ $P_{x+1_{y+1_{z+1}}}$ 

Note that the subscript indicator ⠰ is the same symbol as the English letter indicator. The indicator's function is determined in context.

Recall that an English letter indicator is not used in an unspaced mathematical expression unless the letter is printed in a mathematically-significant typeform. Compare these transcriptions of the letter "i" in regular type and bold type:

- ⋙ $3i$ 

  Spoken: three i

- ⋙ 3_i 

  Spoken: three sub i

- ⋙ $3\boldsymbol{i}$ 

  Spoken: three bold i

- ⋙ $3_{\boldsymbol{i}}$ 

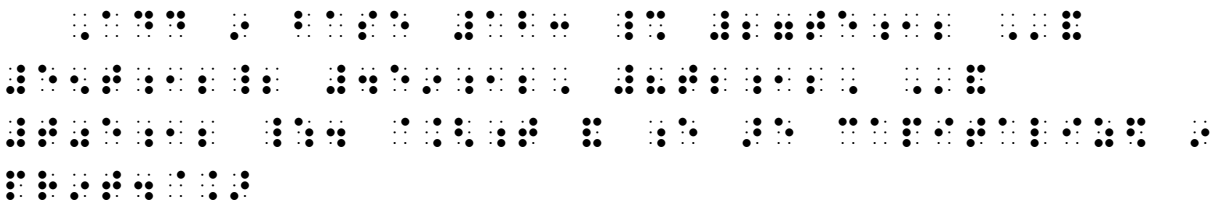
  Spoken: three sub bold i

PRACTICE 8E

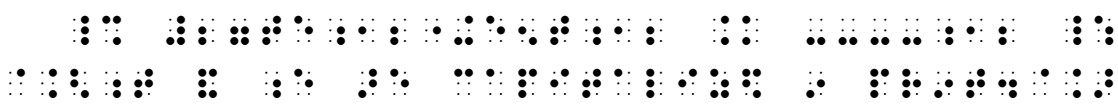
3_c	10_8	$?_3$	x_{2+k}
δ_{ij}	$10_?$	$\mathrm{Ca(OH)_2}$	y_{-2}
y_{n_k}	3_{five}	P_{3n}	a_{m1}

8.9 Returning to the Baseline Level: The same circumstances discussed with superscripts bring the reader back to the baseline level from a subscript: a space, a comma followed by a space, a punctuation indicator, or a baseline indicator. "A space" includes the space before a Nemeth Code termination indicator and/or transition to a new braille line.

Example 8.9-1 Add in base 12: $27\mathrm{TE}_{12}$ and $\mathrm{E5T}_{12}$; $4\mathrm{E9}_{12}$, $8\mathrm{T2}_{12}$, and $\mathrm{T0E}_{12}$.



Example 8.9-2 $27\mathrm{TE}_{12} + \mathrm{E5T}_{12} = ___{12}$



Note that the level indicator is unspaced from the long dash to which it applies.

Example 8.9-3 $(R_H T_H) + (R_S T_S)$



⠰ *is the subscript indicator throughout this expression. Letters H and S are subscripts.*

PRACTICE 8F

1. Name the numeral in base ten equal to:
   a. 47_8 b. 34_6 c. 1101_2
2. $\mathrm{C}_{\mathrm{hex}} = 12_{\mathrm{dec}} = 14_{\mathrm{oct}}$
3. What do we know if $P_{n_r} = (x_{n_k}, y_{n_k})$?
4. $7_8 - 4_8$

8.10 SPECIAL CASE—Nonuse of the Subscript Level Indicator: A special rule applies to certain subscripts. The subscript level indicator is <u>not</u> used when <u>all</u> of the following apply:

- the subscript is *numeric;*
- the numeral is a *right subscript to a letter*;
- the subscript is on the *first* level below the baseline of writing.

Example 8.10-1 Apply Definitions i_2 and I_2 .



Each numeral 2 is printed at the subscript level.

When a subscript does not require a subscript indicator, a return to the baseline is implied following the numeric subscript. A baseline indicator is not needed.

⋙ $\mathrm{H_2O}$ 

Letters H and O are on the baseline of writing. Numeral 2 is a subscript.

⋙ $a_1a_2 + b_1b_2 + c_1c_2$ 

All numerals are subscripts. A baseline indicator is not needed to show that the letters and the plus signs are on the baseline level because no subscript indicators are used.

⋙ $\mathrm{Cd(NO_3)_2}$ 

The subscripts are numerals 3 and 2. Everything else is on the baseline of writing, including the closing parenthesis. The second subscript requires a level indicator because it is not a right subscript to a letter.

8.10.1 Further Conditions: The definition of "numeric" and "letter" include the following:

— The numeric subscript may contain a *segmenting comma* or a *decimal point*.

⫸ $x_{1,000}$ ⠭⠂⠁⠚⠚⠚ (The numeral 1,000 is a right subscript to the letter x.)

⫸ $x_{5.3}$ ⠭⠲⠩ (The numeral 5.3 is a right subscript to the letter x.)

— The letter may carry one or more *primes*.

⫸ x'_2 ⠭⠄⠆ (The numeral 2 is a right subscript to x'.)

— The letter may be taken from *any alphabet* in *any typeform*.

⫸ $\aleph_0$ ⠈⠫⠴ (Hebrew aleph, subscript zero)

⫸ $\mathbf{i}_1$ ⠨⠊⠂ (bold English letter i, subscript one.)

— The letter may also carry a *superscript (*See **8.16**, *Simultaneous Superscripts and Subscripts*)

— The letter may be part of a *two-letter chemical abbreviation.**

⫸ Cl_4 ⠠⠉⠇⠲ (Two-letter chemical abbreviation for chlorine, subscript four.)

*A one- or two-letter "abbreviation" for a chemical compound follows spacing rules for letters. For example, Fe_2O_3 is brailled without spaces:
⠠⠋⠑⠆⠠⠕⠒

8.10.2 Restrictions: Just because a numeral is a right subscript to a letter does not mean that the special rule can be applied. The subscript level indicator must be used if any of the following conditions apply:

— If the letter is functioning as a numeral in a nondecimal numeration system a subscript indicator is required. (This example shows base 12 numeration.)

⫸ TE_{12} ⠠⠞⠑⠰⠂⠃

— If the letter is part of a word or abbreviation a subscript indicator is required.

⫸ $five_3$ ⠋⠊⠧⠑⠰⠒

— If the subscript contains any symbol other than a numeral with its comma or decimal point a subscript indicator is required.

⫸ x_{2k} ⠭⠰⠆⠅

⫸ $x_{2'}$ ⠭⠰⠆⠄

⫸ x_{-2} ⠭⠰⠤⠆

— If the subscript carries a superscript or subscript of its own a subscript indicator is required.

⋙ x_{2_n} 

— Numeric subscripts on the second or lower levels always require their appropriate subscript level indicators.

⋙ P_{n_1} 

8.10.3 Rewind: Lesson 7 looked at unspaced number/letter combinations. **7.16** illustrated that, when a number appears to the right of a letter on the baseline level, a multipurpose indicator is necessary.

⋙ R12 

Without the multipurpose indicator, the number will read as a subscript.

But if the expression is a subscript, no indicator is needed because the level in effect continues.

⋙ t_{R12} 

8.10.4 Summary: The rules regarding use of the subscript indicator are summarized as follows.

The subscript level indicator must be used with a numeral on the first level below the baseline if the subscript contains any symbol other than a numeral with its comma or decimal point, or if the subscript carries a superscript or subscript of its own. Subscripts on the second or lower levels always require their appropriate subscript level indicators. If the letter is part of a word or an abbreviation, a subscript indicator is required. If the letter is functioning as a numeral in a nondecimal numeration system, a subscript indicator is required.

A subscript level indicator is not used before a numeric subscript on the first level below the baseline of writing when the numeral is a right subscript to a letter. The numeric subscript may contain a segmenting comma or a decimal point. The letter may carry one or more primes or a superscript, may be taken from any alphabet in any typeform, and may be part of a two-letter chemical abbreviation.

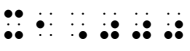
PRACTICE 8G

1) These expressions need subscript indicators in braille: y_{-2}, x_{2+k}, a_{m1}, x_{3_n}, x_{y_2}.
2) These expressions do not use a subscript indicator in braille: x_1, ax_2, CO_2, $z_{.7}$, β_2.
3) Decide whether these expressions require a subscript indicator and braille them correctly: shape_4, Q'_2, $C_6Fe_2O_{12}$, n_k, x_{2k}, $P_{r_{s_t}}$, D_{56}, $G_{9{,}999}$, $2E6B_{16}$.
4) Use $\alpha_1, \beta_1, \gamma_1$ and $\alpha_2, \beta_2, \gamma_2$ to denote the direction vectors $\mathbf{v}_1$ and $\mathbf{v}_2$.
5) *Chemistry*: While Na_2ZnCl_4 could be cooled in the normal way, $Na_2[CoCl_4]$ had to be quenched in the liquid N_2.
6) $f_1(x) = g(x) \cdot q_2(x) + f_2(x)$

8.11 Spaces Within Superscripts and Subscripts: A space usually returns the reader to the baseline. Various strategies are used to retain the level in effect when a space occurs within a superscript or a subscript.

8.11.1 Commas: As noted above, the effect of a level indicator is terminated by a comma unless the comma occurs in a long numeral (a "segmenting" comma). *Recall:* A segmenting comma maintains the level in effect,

⋙ $x_{1{,}000}$



The numeral 1,000 is a right subscript to the letter x.

but a comma followed by a space re-establishes the baseline. The return to the baseline starts at the comma. No baseline indicator is needed.

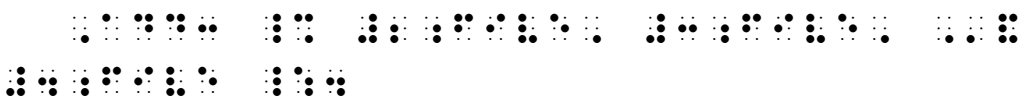
⋙ x^2, y^2, z^2



⋙ (a_1, a_{1i}, a_{2i})



Example 8.11-1 Add: 2_{five}, 3_{five}, and 4_{five}.



When a comma separates individual items <u>within</u> a superscript or subscript, the contracted comma symbol is used. This symbol represents the comma or the comma and a space. The level in effect continues through this symbol.

Contracted Comma + Optional Space	

⋙ $x^{1,2}$ 

⋙ $x_{i,j,k}$ 

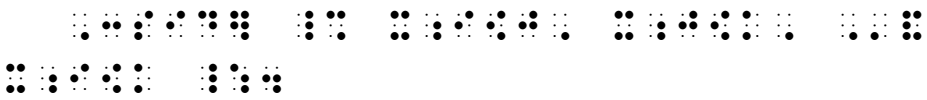
The contracted comma symbol is not to be used to replace a comma on the baseline of writing. A dot 2 or dot 6 comma is used for the comma that is part of the sentence structure.

Example 8.11-2 If ___, then $x_{i,j,k}$, as shown below.



Contracted commas are used within the subscript; a dot 2 comma is used for the comma that is part of the sentence structure (on the baseline level) outside of the Nemeth switches.

Example 8.11-3 Consider $x_{i,j}$, $x_{j,k}$, and $x_{i,k}$.



Contracted commas are used within each subscript; a dot 6 comma is used for the commas that are part of the sentence structure (on the baseline level) within the Nemeth switches.

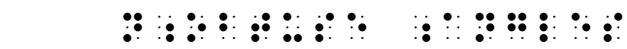
In the next example, the commas separate three terms at the subscript level, but each "n" has a subscript as well. The correct level of each comma is shown by shrewd use of level indicators.

⋙ P_{n_1,n_2,n_3}



8.11.2 Words: If a subscript or a superscript contains a space between words or abbreviations, the level must be restated after the space. The level indicator is unspaced from the following word or abbreviation.

⋙ $n_{\text{obtuse angles}}$



The subscript indicator before the word "angles" shows that the word is in the subscript position.

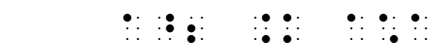
⋙ $n_{\text{st. angles}}$



The abbreviation st. (for "straight") has a related period. No punctuation indicator is used with abbreviations.

8.11.3 Comparison Signs: The space before a comparison sign returns the reader to the baseline, as illustrated in the next example.

⋙ $a^2 = a \cdot a$



The superscript is 2. The equals sign is on the baseline.

If the comparison sign is within a superscript or a subscript, the level is reinstated before the comparison sign. The indicator is unspaced from the comparison sign. *The space after a comparison symbol preserves the level that is already in effect.*

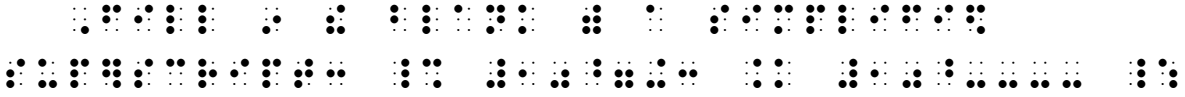
⋙ $S_{\text{u = a}}$



The subscript is u = a. The level in effect extends through the space following the comparison sign.

8.11.4 Ellipsis or Long Dash in Superscripts and Subscripts: When an ellipsis or long dash is located at a different level from the material preceding it, the appropriate level indicator is used before the ellipsis or long dash. The level indicator is unspaced from the symbol.

Example 8.11-4 Fill in the blank with a simplified superscript: $10^{7+3} = 10^{-}$



An ellipsis or a long dash within a superscript or a subscript assumes the level in effect.

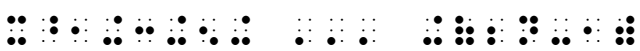
⋙ $P_{n_1, n_2, \ldots}$



The letter n, the commas, and the ellipsis are at the subscript level. Even though a contracted comma includes the following space, there must be a space before an ellipsis unless it is immediately preceded by an indicator. The subscript level extends through the space before the ellipsis.

The level extends through the required spaces before and after an ellipsis or a long dash.

⋙ $x^{1 + 3 + 5 + \ldots + (2n-1)}$



The superscript is $1 + 3 + 5 + \ldots + (2n - 1)$. *The superscript level extends through the spaces before and after the ellipsis.*

Exception: If an ellipsis or long dash in a superscript or subscript is followed by a sign of comparison or by literary text, reading returns to the baseline level without the need for a baseline indicator.

⋙ $10^{7+}— = 10^{21}$



The first superscript is 7+___. The superscript level extends through the required space before the long dash. The sign of comparison after the dash is assumed to be at the baseline level.

⋙ $10^{7+}—$ equals 10^{21}



The first superscript is 7+__ . The superscript level extends through the required space before the long dash. The word after the dash is assumed to be at the baseline level.

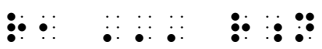
8.11.5 Ellipsis or Long Dash on the Baseline of Writing: Because the space before an ellipsis or long dash maintains the level in effect, an indicator is required to return to the baseline when the symbol is printed there. The baseline indicator takes the place of the required space.

⋙ $a^1 b^2 c^3 d^4 \ldots z^n$



There is no need to indicate a return to the baseline after a numeric subscript that does not require a subscript indicator.

⋙ $r_1 \ldots r_n$



The ellipsis is printed on the baseline. The subscripts are 1 and n.

8.11.6 Segmented Numbers: The effect of a level indicator extends through the space inserted in a numeral for the purpose of dividing it into short regular groups of digits.

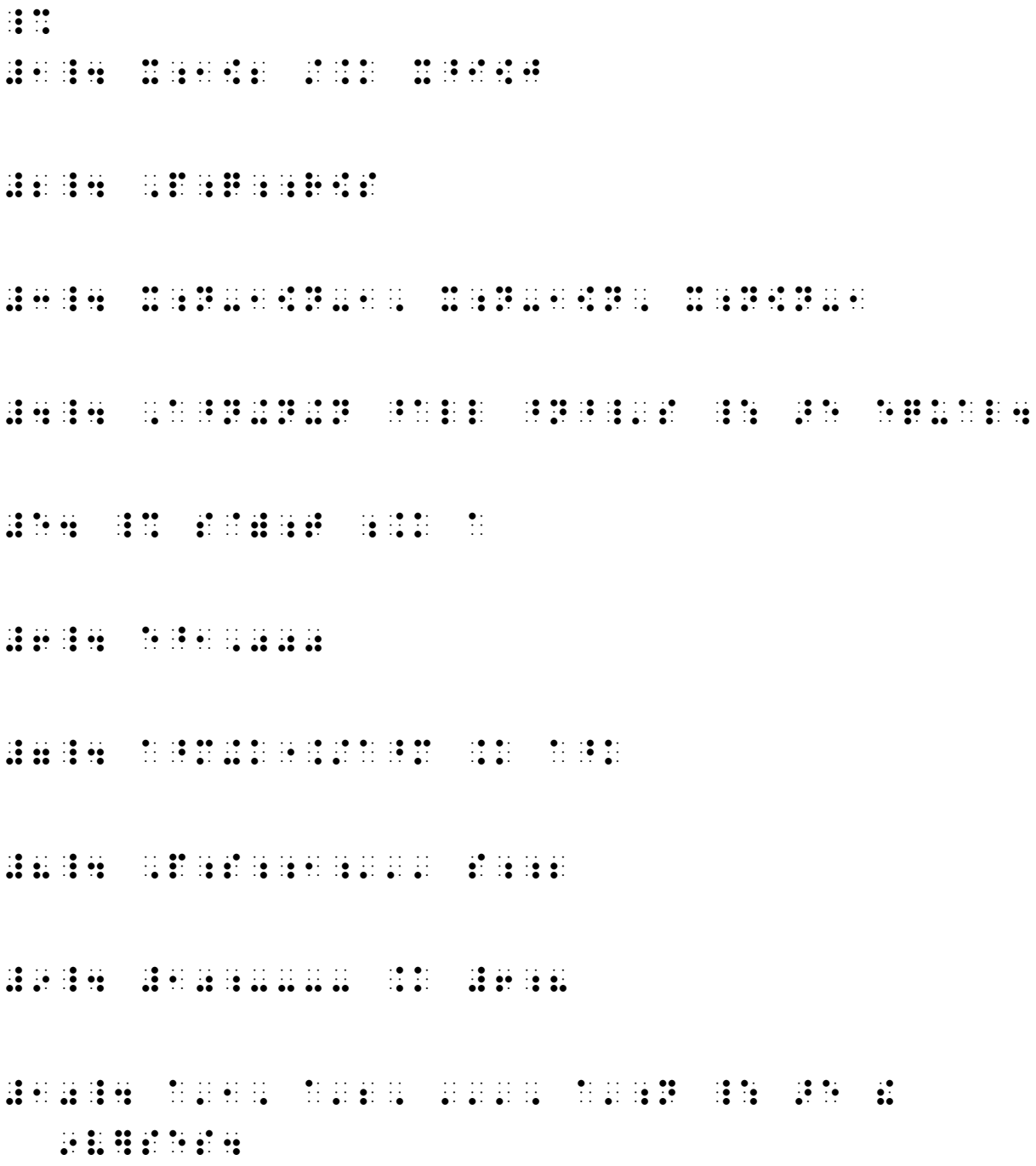
⫸ $e^{3.14159\ 26535}$

⠢⠑⠘⠼⠉⠲⠁⠙⠁⠊⠃⠀⠼⠃⠋⠉⠓⠑

The superscript is 3.14159 26535

Instructions: Translate the simulated braille by writing the mathematical characters in the blank space below each line. Show spaces and levels clearly. Compare your translation to the print copy in the answer section.

PRACTICE 8H



More about Superscripts and Subscripts

8.12 Superscript and Subscript Combinations: Combinations of subscripts to superscripts or of superscripts to subscripts require level indicators composed of two or more braille symbols. Keeping in mind that level indicators relate to the baseline, transcribing the following types of expressions is an exercise in logical thinking.

Superscripts with Subscripts

Superscript With Subscript	⠔⠢

⋙ x^{n_1} ⠭⠔⠢⠝⠂⠡

n_1 (n, subscript one) is in the superscript position. "1" is a super-subscript.

⋙ $2^{Y_0} = Y_1$ ⠼⠃⠔⠢⠠⠽⠂⠡ ⠶ ⠠⠽⠂

Y_0 (Y, subscript zero) is in the superscript position. "0" is a super-subscript.

Reminder: The subscript indicator is omitted for a numeric subscript to a letter only for subscripts that are located on the *first level below the baseline of writing.* The super/sub indicator is needed to show a numeric subscript in the superscript position.

Subscripts with Superscripts

Subscript With Superscript	⠡⠔

⋙ x_{n^2} ⠭⠡⠝⠔⠢

n^2 (n, superscript two) is in the subscript position. "2" is a sub-superscript.

⋙ P_{3^n} ⠠⠏⠡⠼⠉⠔⠝

3^n (3, superscript n) is in the subscript position. "n" is a sub-superscript.

Reminder: The subscript indicator is required when a numeric subscript to a letter carries a superscript or subscript of its own.

PRACTICE 8I

Instructions: Descriptions are included in a right-hand column for your benefit—do not braille the descriptions. Analyze the levels as you braille and as you proofread.

Superscripts with Subscripts

$2^{\aleph_0} = \aleph_1$	$\aleph_0$ is in the superscript position. (The aleph is not bold.)
$a = 2^{k_1}$ and $b = 2^{k_2}$	k_1 and k_2 are in the superscript position.
$(ab)^x = 2^{k_1 x} \cdot 2^{k_2 x}$	x, $k_1 x$, and $k_2 x$ are superscripts.
$e^{i\theta_1}$ times $e^{i\theta_2}$ equals $e^{i(\theta_1+\theta_2)}$	$i\theta_1$, $i\theta_2$, and $i(\theta_1 + \theta_2)$ are superscripts.

Subscripts with Superscripts

Z_{5^n}	5^n is in the subscript position.
$7t_{s^4}$	s^4 is in the subscript position.

8.13 Left Subscripts and Superscripts: The appropriate level indicator is brailled before a subscript or superscript printed to the left of its related sign.

⋙ ${}_3\mathrm{X}_1$ ⠸⠬⠸⠩⠸⠆

⋙ ${}_{48}\mathrm{C}_9 \times {}_4\mathrm{C}_4$ ⠸⠬⠸⠲⠸⠆⠸⠦⠸⠲⠸⠆

Reminder: The subscript indicator is not used when a numeral is a right subscript to a letter on the baseline.

⋙ ${}^{14}\mathrm{C}$ ⠸⠼⠸⠂⠸⠆

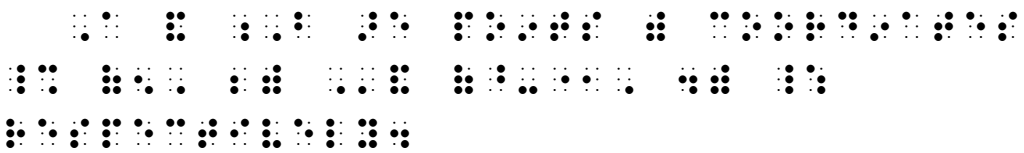
8.13.1 Raised Negative Sign: In some texts, negative numbers are shown with a raised negative sign. The raised position of the negative sign must be shown in braille.

⋙ $^{-}4$ ⠸⠔⠸⠲

Example 8.13-1 Explain why $4 + {}^{-}4 = 0$.

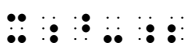
⠸⠭⠸⠐⠸⠣⠸⠲ ⠸⠐⠸⠣ ⠸⠐⠸⠣ ⠸⠐⠸⠣⠸⠲⠸⠭⠸⠲⠸⠐ ⠸⠐⠸⠣ ⠸⠐⠸⠣ ⠸⠐⠸⠣⠸⠲

Example 8.13-2 A and B are points with coordinates (5, 2) and (⁻1, 4) respectively.



Compare the subscripts in the next two examples. The first subscript is −2. A subscript indicator is required because the subscript contains a symbol other than a numeral. In the second subscript, the negative sign is raised—it is a left superscript to the subscript "2".

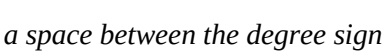
⋙ x_{-2} 

⋙ x_{-2} 

8.13.2 More About the Temperature Abbreviations F (Fahrenheit) and C (Centigrade):
The raised hollow dot representing the degree symbol was introduced in **8.4**. *Review:* When a temperature value is given, the abbreviation "F" or "C" is spaced from the degree symbol in braille, regardless of its print appearance. The next two examples show different spacing in print but the braille transcriptions are identical.

⋙ 98.6° F 

In print, there is a space between the degree sign and the abbreviation "F".

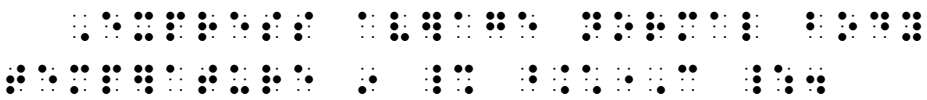
⋙ 98.6°F 

In print, there is no space between the degree sign and the abbreviation "F".

When the temperature abbreviation is shown with the degree symbol and there is no associated value, the letter becomes part of the symbol and is not spaced away from the degree symbol. The "hollow dot" is brailled as a left superscript to a letter. An English letter indicator is not needed because the letter is not standing alone.

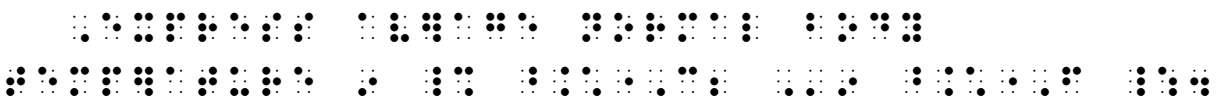
⋙ °C 

Example 8.13-3 Express average normal body temperature in °C.



Punctuation mode is literary.

Example 8.13-4 Express average normal body temperature in °C; in °F.



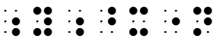
The "C" is followed by a semicolon. No punctuation indicator is used because abbreviations are punctuated in literary mode.

8.14 Further Combinations: The Nemeth codebook illustrates additional combinations of superscripts and subscripts. Due to the obscurity of such complex combinations, only a few examples are shown in this lesson book. Proper interpretation of these characters will require reading the surrounding text in order to apply the correct indicators.

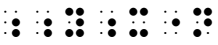
⋙ $_{y}x^{}n$ n is on the baseline of writing and has a left subscript.

Question: Is the subscript y^{x} or $_{y}x$?

This transcription shows left subscript "y" with a sub-superscript "x".



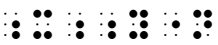
This transcription shows left subscript "x" with a left sub-subscript "y".



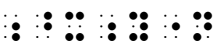
⋙ $_{x_{y}}n$ n is on the baseline of writing and has a left subscript.

Question: Is the subscript x_{y} or ^{x}y ?

This transcription shows left subscript "x" with a sub-subscript "y".



This transcription shows left subscript "y" with a left sub-superscript "x".



8.15 Consecutive Superscripts and Consecutive Subscripts: The appropriate indicator is repeated before a left superscript or a left subscript when two are consecutive—each superscript or subscript has its own indicator. One indicator applies to the expression preceding it and the other to the expression following it.

The space that appears in print helps visually differentiate the two factors, but no space comes between factors in braille unless other spacing rules take precedence.

⋙ $p^{b\,c}q$ ⠏⠘⠆⠉⠘⠰⠟ (There is a space between b and c in print.)

⋙ $P_{x\,y}Q$ ⠠⠏⠰⠭⠀⠽⠴⠠⠟ (There is a space between x and y in print.)

The subscript indicator is or is not brailled according to the rules presented in **8.10**.

⋙ $P_{1\,2}Q$ ⠠⠏⠰⠡⠀⠆⠴⠠⠟ (There is a space between 1 and 2 in print.)

The first numeric subscript does not require a subscript indicator because it is a right subscript to a letter.

8.16 Simultaneous Superscripts and Subscripts: When a superscript and a subscript are printed directly above and below each other, the subscript is brailled first.

⋙ 10^{3}_{8} ⠼⠁⠚⠰⠡⠈⠦

⋙ $^{238}_{92}\mathrm{U}$ ⠰⠼⠃⠓⠰⠡⠊⠦⠠⠥

Example 8.16-1 An example of an I^{-} iodine anion: $^{127}_{53}\mathrm{I}^{-}$.

⠠⠊⠐⠀⠠⠔⠀⠁⠀⠑⠭⠁⠍⠏⠇⠑⠀⠕⠋⠀⠁⠝⠀⠊⠕⠙⠊⠝⠑⠀⠁⠝⠊⠕⠝⠲⠀⠠⠼⠁⠃⠛⠰⠡⠊⠢⠠⠤⠦⠠⠊⠄

The subscript indicator is or is not brailled according to the rules presented in **8.10**.

⋙ x^{2}_{1} ⠭⠰⠡⠂⠈⠆

*The numeric subscript does not require a subscript indicator **because** it is a right subscript to a letter.*

⋙ $^{n}_{m}x$ ⠰⠝⠰⠡⠍⠈⠭

The subscript is brailled first.

⋙ $\mathrm{n}^{x^{y^{z}}}_{a_{b_{c}}}$ ⠝⠰⠡⠰⠭⠘⠽⠘⠵⠈⠦⠰⠡⠁⠘⠃⠘⠉⠈⠦⠈⠦

The entire subscript is brailled before brailling the superscript.

8.17 Nonsimultaneous Superscripts and Subscripts: When a mathematical expression carries both a superscript and a subscript which are <u>not</u> printed directly above and below each other, the superscript and subscript are transcribed *in the same order as in print*, and the baseline indicator is inserted between them.

⋙ $x_1{}^2$ ⠭⠂⠀⠀⠆

⋙ $a^k{}_m$ ⠁⠘⠅⠀⠀⠰⠍

⋙ ${}_y{}^x n$ ⠰⠽⠘⠭⠝

⋙ ${}^b{}_a n$ ⠘⠃⠰⠁⠝

Note: A magnifier and a straightedge can help determine whether superscripts or subscripts are simultaneous or nonsimultaneous. If in doubt whether the expression shows nonsimultaneous super/subscripts or if, instead, the super/subscripts have super/subscripts of their own, compare to the surrounding text for clues.

8.18 Detached Superscripts and Subscripts: When an entire superscript or subscript stands alone, it is brailled without a level indicator. A transcriber's note explains its print position.

Example 8.18-1 In x^2, the ^{2} is the exponent.

⠠⠔ ⠠⠭⠘⠆ ⠠⠊⠝ ⠠⠭⠘⠆⠐⠂ ⠠⠞⠓⠑ ⠆ ⠠⠊⠎ ⠠⠞⠓⠑ ⠠⠑⠭⠏⠕⠝⠑⠝⠞⠲
⠠⠠⠔⠠⠭⠘⠆⠐⠂ ⠠⠞⠓⠑ ⠆ ⠠⠊⠎ ⠠⠞⠓⠑ ⠠⠑⠭⠏⠕⠝⠑⠝⠞⠲

8.19 Literary Symbols and Level Indicators: If a literary symbol is used mathematically and appears in a superscript or a subscript, its level must be indicated.

Example 8.19-1 A dagger may be used as a superscript in quantum mechanics, for example, $A^{\dagger}$.

⠠⠔ ⠠⠁⠘⠔⠠⠡⠐⠂ ⠠⠞⠓⠑ ⠠⠔ ⠠⠊⠎ ⠠⠁ ⠠⠔ ⠠⠙⠁⠛⠛⠑⠗ ⠠⠔
⠠⠠⠔⠠⠁⠘⠔⠠⠡⠐⠂ ⠠⠞⠓⠑ ⠠⠔ ⠠⠊⠎ ⠠⠁ ⠠⠔ ⠠⠙⠁⠛⠛⠑⠗ ⠠⠔
⠠⠠⠔ ⠠⠔ ⠠⠊⠎ ⠠⠁ ⠠⠔

Summary

Here is a summary of the difference in print among four types of superscripts and subscripts as defined in the Nemeth Code.

- **Consecutive**: A right super/subscript belongs to the preceding character and a left super/subscript belongs to the next character.
- **Simultaneous**: A character has both a superscript and a subscript. The superscript is printed directly above the subscript.
- **Nonsimultaneous**: A character has both a superscript and a subscript. The two scripts are not printed directly above and below each other.
- **Detached**: A super/subscript stands alone without being associated with a letter or number. It is printed slightly above or below the baseline and is smaller than the rest of the text.

PRACTICE 8J

1. Here are some expressions with left superscripts: ${}^{3}x,\ {}^{n}x,\ {}^{-}2 + {}^{-}4 = {}^{-}6,\ ({}^{-}3)^{{}^{-}2+{}^{+}2}$.
2. ${}^{12}_{6}\mathrm{C}$ and ${}^{12}\mathrm{C}$ represent the same carbon isotope.
3. $\mathrm{D}_2\,{}^{18}\mathrm{O}$ is the doubly labeled water isotopologue!
4. In CO_2, the subscript $_2$ means "two oxygen atoms".
5. ${}_{n}P_{r} = K({}_{n-1}P_{r-1})$
6. $a_1^2 + b_1^2 + c_1^2$
7. $[t]_0^4$
8. $2 \times 10_6{}^2 + 3 \times 10_6{}^1 + 2$
9. $P_{xy}Q$
10. $\mathrm{NH_4^+ + Cl^- + H_2O}$
11. *Temperature Conversion:* What is 0 K in °C? in °F? (Answers: –273.15° C; –459.67° F.)

GROUPING SIGNS, cont.

8.20 Review of Rules: The following rules have been discussed in earlier lessons. A review of the cited sections is recommended.

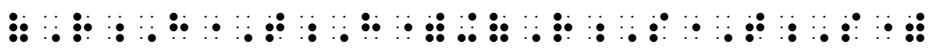
- In mathematical context, enclosure symbols are not considered to be punctuation; they are signs of grouping. Grouping symbols that are part of a mathematical expression must be brailled as Nemeth braille symbols. [3.9]
- The numeric indicator is not used before a numeral which immediately follows an opening sign of grouping unless such numeral is in non-regular type. [3.10, 7.7]
- The English letter indicator is not used when a single English letter or a Roman numeral is entirely enclosed within signs of grouping unless such a letter, or Roman numeral is in non-regular type. This rule does not apply to abbreviations. [4.12.2, 5.3.5]
- Single-letter abbreviations without a related period, or abbreviations whose letters correspond to a shortform without a related period, require an English letter indicator even when enclosed between signs of grouping. [5.19, 5.20]
- When a single English letter, or a Roman numeral is in direct contact with only its opening or only its closing sign of grouping, the English letter indicator is or is not used as though the grouping signs were absent. If the grouping sign carries a prime or other modifying symbol, the English letter indicator is <u>not</u> used. [5.16]

- The appropriate alphabetic indicator must be used with any letter from the German, Greek, Hebrew, or Russian alphabets even when enclosed within, or in contact with, signs of grouping. [5.8.1]
- In mathematical context, non-mathematical symbols of enclosure can be transcribed as Nemeth Code signs of grouping if switching out of Nemeth Code would be cumbersome. [3.13]
- A sequence of mathematical items enclosed in signs of grouping (an "enclosed list") must use the Nemeth braille enclosure symbols. [5.15]
- When square brackets or vertical bars are printed in mathematically-significant boldface, dots 456 are used before the grouping symbol. [7.13.3]

Regarding the main topic in this lesson—superscripts and subscripts—another point can be added.

- When a grouping symbol appears on the baseline level and a level indicator is currently in effect, the baseline indicator is placed <u>before</u> the sign of grouping.

⋙ $(R_H T_H) + (R_S T_S)$



If no subscript indicator is used, a return to the baseline is implied.

⋙ (a_1, a_2, a_3)



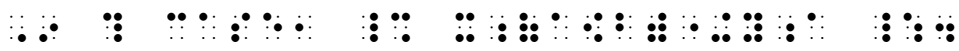
8.21 Grouping Symbols and Level Indicators: Be watchful when determining the level of a grouping sign. Because the bottom margin of grouping signs may extend below the level in effect, it is better to compare the center of the symbol to the surrounding text if its printed level is in question.

Example 8.21-1 Solve $(x^2 + y^2) - (x^2 + y^2)$



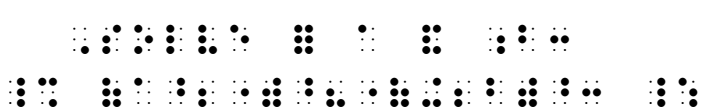
If the grouping symbol starts a different level, the appropriate level indicator is placed before the sign of grouping to properly identify its location.

Example 8.21-2 In this case, $x_{(a,\,b)} + y_a$.



The opening grouping symbol begins the subscript level.

Example 8.21-3 Solve for *a* and *b*: $(a^2)^8(+2b)^3$



All of these grouping symbols are on the baseline of writing.

Review the examples in **<u>8.6.1</u>** with this in mind.

8.22 Grouping Symbols with Super/Subscripts: Lesson 5 taught that when a letter touches only one grouping symbol, the English letter indicator is applied (or is not applied) as though the grouping sign were not present. This rule is illustrated below in the context of a grouping sign that has a subscript and/or superscript.

⋙ $s]_a^b$ ⠰⠎⠻⠁⠆⠃ *Think:* s_a^b ⠎⠁⠆⠃

8.23 More Signs of Grouping: Grouping signs were introduced in **Lesson 3**. Boldface square brackets and vertical bars were shown in **Lesson 7**. The Nemeth Code has devised symbols for eight more grouping signs.

Barred Brackets		
Opening	⟦	⠸⠣
Closing	⟧	⠸⠜
Barred Braces		
Opening	⦃	⠸⠪
Closing	⦄	⠸⠕
Half Brackets		
Upper Left	⎡ or ⌜	⠸⠩
Upper Right	⎤ or ⌝	⠸⠱
Lower Left	⎣ or ⌞	⠸⠫
Lower Right	⎦ or ⌟	⠸⠹

⋙ $A_{n\urcorner i}$ ⠰⠁⠢⠝⠻⠊⠆

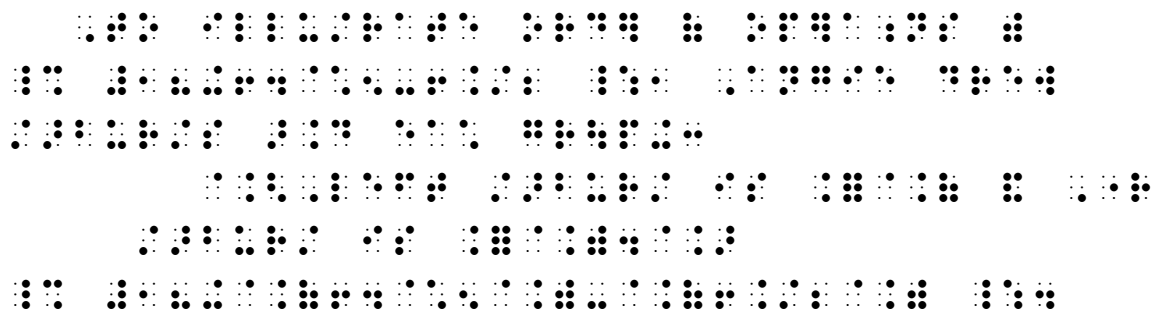
⋙ $\llcorner x \lrcorner$ ⠸⠫⠭⠸⠱

Example 8.23-1 *Integer division* can be defined as $a \backslash b \equiv \lfloor a/b \rfloor$, where "/" denotes normal division and $\lfloor x \rfloor$ is the floor function.

⠠⠔⠞⠑⠛⑇⠗⠀⠙⑇⠧⑇⠎⑇⠕⑇⠝⠀⠉⠁⑇⠝⠀⠃⑇⠀⠙⑇⠋⑇⑇⠝⑇⑇⠙⠀⠁⑇⠎⠀⠁⠸⠳⠃⠀⠨⠅⠀⠸⠣⠁⠸⠌⠃⠸⠜⠂⠀⠺⑇⑇⠗⑇⠀⠸⠌⠀⑇⑇⠝⠕⑇⑇⠞⑇⑇⠎⠀⠙⑇⠧⑇⠎⑇⠕⑇⠝⠂⠀⠁⠝⠙⠀⠸⠣⠭⠸⠜⠀⑇⑇⠎⠀⑇⑇⠞⑇⑇⠀⠋⠇⠕⑇⠕⠀⠋⠥⠝⠉⑇⠞⑇⠕⑇⠝⠲

8.24 Transcriber-Devised Grouping Symbols: If a mathematical sign of grouping is not represented in the Nemeth Code, the transcriber is to devise one. Use two or more braille symbols whose last cell is ⣻ for the opening sign and whose last cell is ⣾ for the closing sign. The transcriber-devised symbol should be identified in a transcriber's note or listed on the Special Symbols page at the beginning of the volume according to the guidelines given in *Braille Formats*.

Example 8.24-1 To illustrate order of operations with $18 + 64 \times 5 - 6 \div 2$, Angie drew starbursts around each grouping: $18 + \text{⣻}64 \times 5\text{⣾} - \text{⣻}6 \div 2\text{⣾}$



Instructions: Treat the vertical bar in sentence (7) as an operation sign.

PRACTICE 8K

(1) $\{f_n\}$

(2) $|a_m - a_n|$

(3) $(x_1 y_1 + x_2 y_2)$

(4) $([\mathrm{CH_3}]_2\mathrm{CH})$

(5) $\mathrm{I}_{\mathrm{ue}}^{2"} = (\mathrm{H}'_{44} x'_{\mathrm{ve}})^{+'}$

(6) $\lfloor x \rfloor = m$ if and only if $m \leq x < m + 1$; $\lceil x \rceil = n$ if and only if $n - 1 < x \leq n$.

(7) The dagger and the asterisk are used as superscripts in quantum mechanics: $\mathrm{A}^{\dagger}$, $(x^{\dagger})^{\dagger} = x$, $\langle \phi | \psi \rangle^{*} = \langle \psi | \phi \rangle$.

INTRODUCTION TO DISPLAYED MATHEMATICAL EXPRESSIONS

FORMAT

8.25 Displayed Mathematical Expressions: Up to this point in the lesson material, mathematical expressions in the examples have appeared in line with the narrative. These are referred to as *embedded expressions*. When mathematical material is set apart from the body of the text in the print copy, it is referred to as a *displayed expression*. Various layouts in the print copy are used to set the material apart, for example, skipped lines, centering or other indentation, or off to the side. In braille, margins for displayed mathematical material depend upon the margins of the surrounding text and are transcribed in one of the following formats.

- *In unitemized explanatory portions of the text*, displayed mathematical material begins in cell 3. Runovers begin in cell 5. In other words: text (3-1); displayed material (**3-5**).
- *In itemized text without subdivisions*, displayed mathematical material begins in cell 5. Runovers begin in cell 7. In other words: text (1-3); displayed material (**5-7**).
- *In itemized text with subdivisions*, displayed mathematical material begins in cell 7. Runovers begin in cell 9. These margins apply to both items and subdivisions, to whatever depth. In other words: main division text (1-5); displayed material (**7-9**). Subdivision text (3-5); displayed material (**7-9**).
- *Within or following instructions*, displayed mathematical material begins in cell 5. Runovers begin in cell 7. In other words: instructions (5-3); displayed material (**5-7**).

Notice that in all four layouts, the first cell of the displayed material is indented two cells to the right of the runover cell of the preceding material. These margins apply regardless of the presence of runover material in the preceding paragraph. A line is not skipped above or below displayed mathematical material unless the preceding or following material requires a blank line. [Refer to **8.26** when the context is not mathematical.]

8.25.1 Placement of Code Switch Indicators: When displayed mathematical material is both preceded and followed by UEB text, the expression and the two switch indicators may be placed all together on one line if they will fit within current margins. If more than one line is required for the expression, the opening Nemeth Code indicator is placed at the end of the text line preceding the displayed material. The Nemeth Code terminator is placed at the completion of the displayed expression. In either case, if the indicator will not fit on the current line, it is placed on the following line in the runover position.

— Math displayed to unitemized text: start in cell 3, runover to cell 5.

Example 8.25-1 (Math is displayed to unitemized text.)

The expression $a(b+c)-d(b+c)$ has the form $ax-dx$ where $x=b+c$. Thus $ax-dx=x(a-d)$ and therefore

$$a(b+c)-d(b+c)=(b+c)(a-d).$$

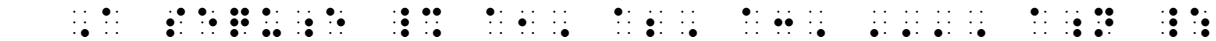
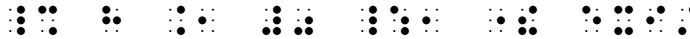
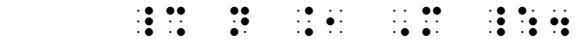
1 ⠀⠀⠀

Example 8.25-3 (Math is displayed to unitemized text.)

A sequence $a_1, a_2, a_3, \ldots, a_n$ is said to *converge* if, for each $h > 0$, there exists

a positive number M such that $|a_n - A| < h$, for all $n > M$.

A sequence that does not converge is said to *diverge*.

1 
2 
3 
4 
5 
6 
7 
8 

Lines 1-3: Narrative paragraph (3-1) with embedded math.

***Lines 4-6: The displayed math expression begins in cell 3, with runovers in cell 5.*

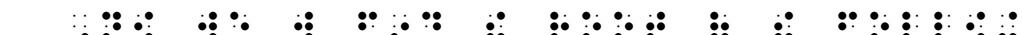
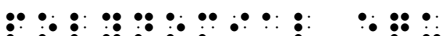
Lines 7-8: Continuation of narrative paragraph.

Example 8.25-4 (Math is displayed to unitemized text.)

Now we will find the root of the following polynomial equation:

$$x^5 + x^4 - 18x^3 + 4x^2 + 88x - 96 = 0$$

How many roots do you predict we will find?

1 
2 
3 
4 
5 

Lines 1-2, 5: Narrative paragraph begins (3-1) and continues on line 5 in runover cell (1).

***Lines 3-4: Displayed math begins in cell 3. Since both switch indicators do not fit on this line, the opening Nemeth Code indicator is on line 2 (in the narrative paragraph), and the Nemeth Code terminator falls in the runover position of the current text – cell 5.*

Line 5: Continuation of 3-1 paragraph (cell 1).

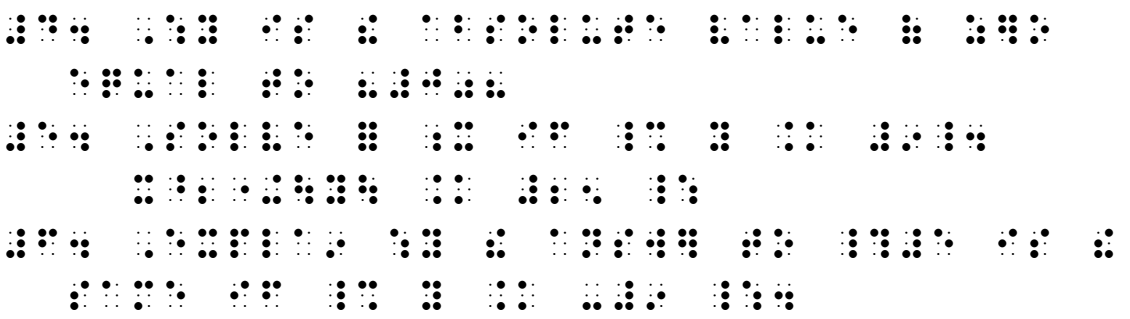
— Math displayed to itemized text: start in cell 5, runover to cell 7.

Example 8.25-5 (Math is displayed to itemized text.)

4. Why is the absolute value of zero equal to "0"?
5. Solve for x if y = 9.

$$x^2 + |y| = 25$$

6. Explain why the answer to #5 is the same if y = –9.



Lines 1-2: Itemized material with no subdivisions (1-3).

Line 3: New item begins in cell 1.

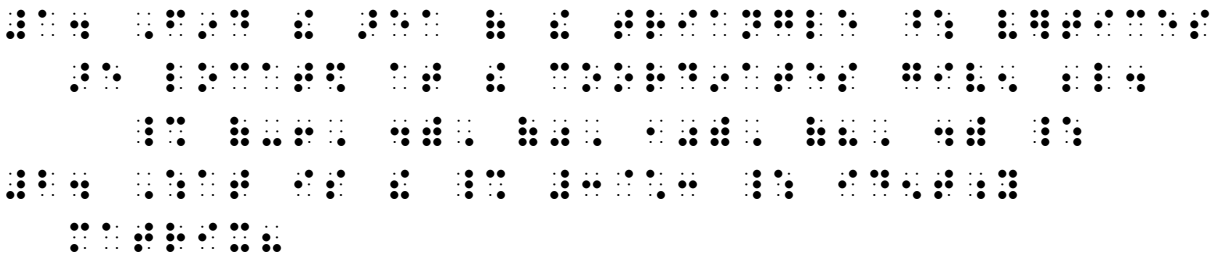
***Line 4: The displayed item begins in cell 5 even though the text to which it applies does not have a runover.*

Lines 5-6: Itemized material continues (1-3).

Example 8.25-6 (Math is displayed to itemized text.)

1. Find the area of the triangle whose vertices are located at the coordinates given below.

   (-6, 4), (0, 10), (8, 4)
2. What is the 3 x 3 identity matrix?



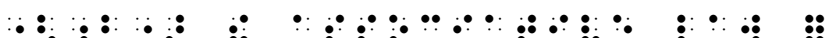
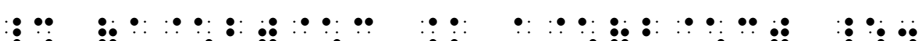
Lines 1-2 and 4-5: Itemized material with no subdivisions (1-3).

***Line 3: Cell 5 is the starting cell for math displayed to itemized text.*

— Math displayed to subitems: start in cell 7, runover to cell 9.

Example 8.25-7 (Math is displayed to subitems.)

5. Give two examples illustrating
   (a) the associative law for addition
   $$(a + b) + c = a + (b + c).$$
   (b) the associative law for multiplication
   $$(a \times b) \times c = a \times (b \times c).$$

1 
2 
3 
4 
5
6 

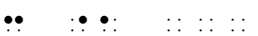
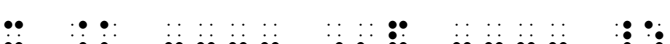
Line 1: Main item begins in cell 1.

Lines 2 and 4-5: Each subitem begins in cell 3. Line 5 shows a runovers in cell 5.

***Lines 3 and 6: Displayed math is in cell 7, which is two cells in from the runover of subitems, whether or not runovers occur.*

Example 8.25-8 (Math is displayed to subitems.)

2. Now solve each of the following equations.
   a. $3(x + 5) = 6x + 6$ b. $x^2 - 25 = 0$
   $x =$ ____ $x =$ ____ and ____

1 
2 
3 
4 
5 
6 

Lines 1-2: Main item begins in cell 1. The opening Nemeth Code indicator is placed at the end of the line of text preceding the math items. Because there is no room for the switch indicator on line 1, it falls in the runover position of the current text (cell 5).

Lines 3 and 5: Each subitem begins in cell 3, not side-by-side as printed.

***Lines 4 and 6: Displayed math begins in cell 7, which is two cells in from the runover of subitems, whether or not runovers occur. (Subitems are 3-5.)*

Line 6: Nemeth Code is terminated at the end of the displayed material.

8.25.2 Look For Context Clues: Each separate expression should start on a new line in the main display cell, even if printed widely-spaced on one line. Notice paragraph patterns in the print copy to determine whether the narrative following displayed material is a continuation of the preceding text or if it is the start of a new paragraph.

Example 8.25-9 (Math is displayed to unitemized text.)

A sentence with an equal sign is called an **equation**. Here are three examples.

$27 = 9(4 - 1)$ $1 + 1 = 3$ $x + 7 = 50$

Which equation is true? Which equation is false? Which equation may be either true or false?

1 ⠀⠀⠀

When punctuation is used to separate displayed items, the items may continue on the same line, running over as necessary in the appropriate runover cell.

Example 8.25-10 (Math is displayed to itemized text.)

21. Use a number line to show the following inequalities:

$$-6 < -5, \quad 0 < +6, \quad -8 < +2, \quad -1 > -5$$

22. ...

1 ⠠⠊⠞⠑⠍⠊⠵⠑⠙ ⠍⠁⠞⠓ ⠊⠎ ⠙⠊⠎⠏⠇⠁⠽⠑⠙ ⠊⠝ ⠞⠓⠑
2 ⠠⠊⠞⠑⠍⠊⠵⠑⠙ ⠍⠁⠞⠓ ⠊⠎
3 ⠠⠍⠁⠞⠓ ⠊⠎ ⠍⠁⠞⠓⠑⠍⠁⠞⠊⠉⠁⠇ ⠁⠝⠙ ⠊⠎ ⠍⠁⠞⠓⠑⠍⠁⠞⠊⠉⠁⠇
4 ⠍⠁⠞⠓⠑⠍⠁⠞⠊⠉⠁⠇ ⠊⠝
5 ⠠⠊⠞⠑⠍⠊⠵⠑⠙ ⠍⠁⠞⠓

Lines 1-2 and 5: Itemized material with no subdivisions (1-3).

***Lines 3-4: The displayed math begins in cell 5 and continues in the runover cell (cell 7). (In print, the inequalities are separated by commas and are all printed on one line.)*

When sentence-structure punctuation is included in the displayed math, each expression may begin in the primary display cell if doing so more closely matches the print layout.

Example 8.25-11 (Two math items are displayed to unitemized text.)

The result of nested floor or ceiling functions is the innermost function:

$$\left\lfloor \lceil x \rceil \right\rfloor = \lceil x \rceil,$$
$$\left\lceil \lfloor x \rfloor \right\rceil = \lfloor x \rfloor.$$

1 ⠠⠞⠓⠑ ⠗⠑⠎⠥⠇⠞ ⠕⠋ ⠝⠑⠎⠞⠑⠙ ⠋⠇⠕⠕⠗ ⠕⠗ ⠉⠑⠊⠇⠊⠝⠛ ⠋⠥⠝⠉⠞⠊⠕⠝⠎
2 ⠊⠎ ⠞⠓⠑ ⠊⠝⠝⠑⠗⠍⠕⠎⠞ ⠋⠥⠝⠉⠞⠊⠕⠝⠒
3 ⠸⠸⠇⠸⠸⠗⠭⠸⠸⠜⠸⠸⠜ ⠶ ⠸⠸⠇⠭⠸⠸⠜⠸⠸⠜⠂
4 ⠸⠸⠗⠸⠸⠇⠭⠸⠸⠜⠸⠸⠜ ⠶ ⠸⠸⠇⠭⠸⠸⠜⠸⠸⠜⠲

Lines 1-2: Narrative paragraph (3-1).

***Line 3: Math displayed to unitemized text begins in cell 3.*

***Line 4: New displayed math item also begins in cell 3.*

8.25.3 Instructions with Displayed Material: If displayed mathematical material appears within or immediately following instructions, the displayed material begins in cell 5 with runovers in cell 7. Instructional text (5-3): displayed material (**5-7**).

— Math displayed to instructions: start in cell 5, runover to cell 7.

Example 8.25-12 (Math is displayed to instructions.)

Use the equation to the right to answer each question. $x + y + z = \$25{,}000$

1. If x = $5,000, express y in terms of z.
2. If y = $10,000, express z in terms of x.
3. If z = $15,000, express x in terms of y.

1
2
3
4
5
6
7
8
9

Lines 1-2: According to UEB and Braille Formats, boldface print used for instructions is disregarded in braille. 5-3 margins characterize instructions in Nemeth Code.

***Line 3: Displayed math begins two cells in from the runover cell of the previous text. The two switch indicators fit on the same line as the displayed math.*

Lines 4-9: Itemized problems (1-3).

8.26 Displayed Narrative Material: Displayed literary text should follow the guidelines in *Braille Formats* with the exception that blocked paragraphs are not used throughout a technical document. If it is difficult to determine whether the material is mathematical or narrative, use your best judgement and apply consistent formatting throughout the transcription.

Example 8.26-1

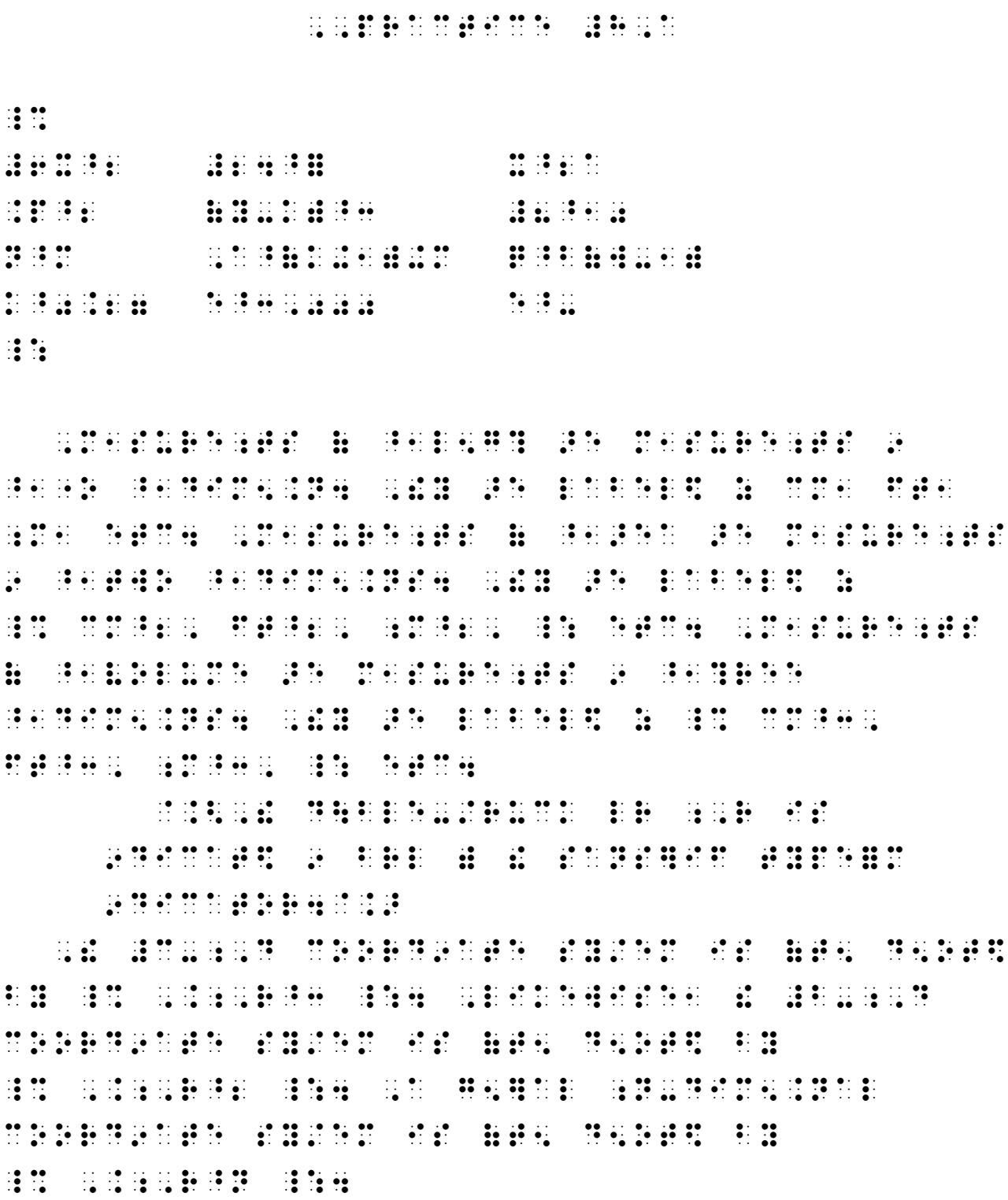
You can guess what your friends are thinking by learning to "operate" your way into their minds! For example, try this math magic trick.

Think of a number. Multiply the number by 8, divide by 2, add 5, and then subtract 4 times the original number.

No matter what number you choose, the answer will always be 5. Try another number and see. You can use what you know about variables to prove it.

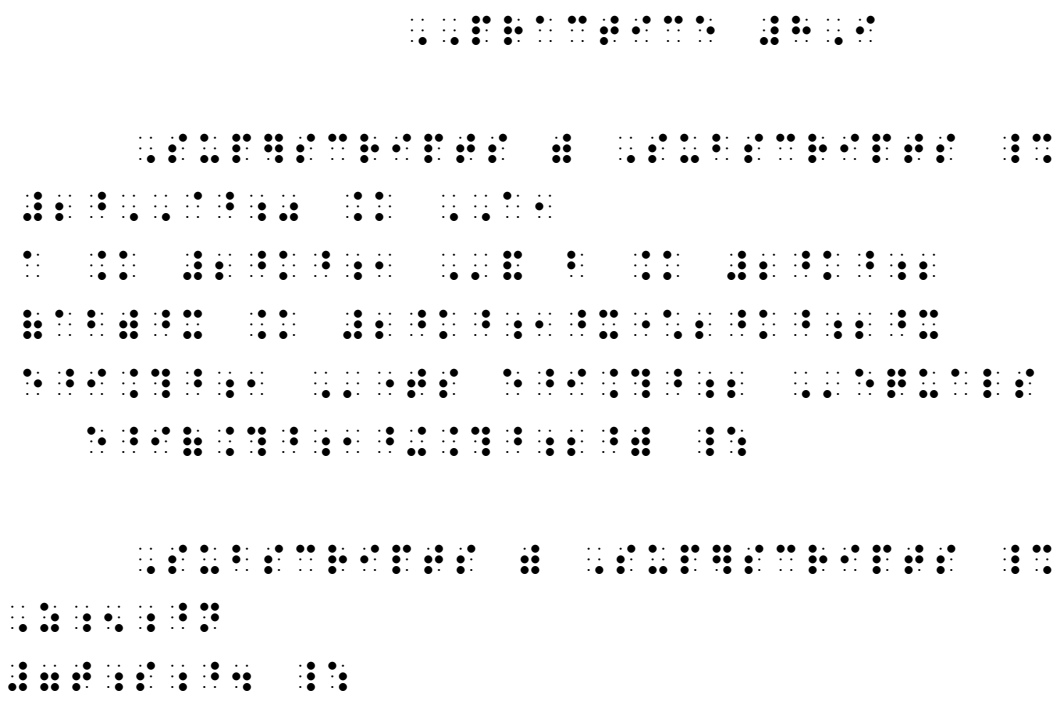
1 ⠀⠀⠀

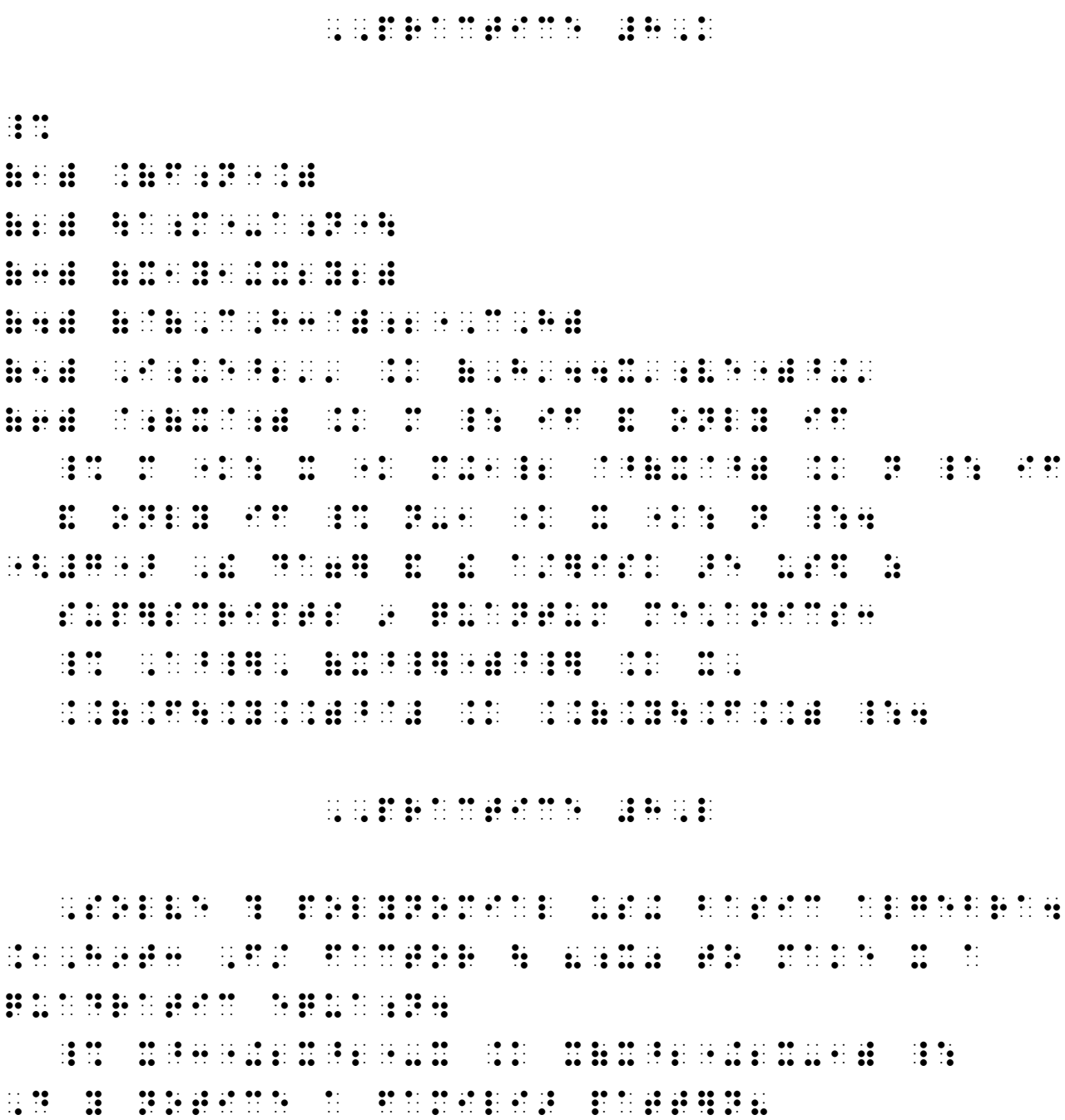
ANSWERS TO PRACTICE MATERIAL



PRACTICE 8H

1. $x_{1,\,2} \neq x^{i,\,j}$
2. $P_{q_{r,\,s}}$
3. $x_{n-1,\,n-1}, x_{n-1,\,n}, x_{n,\,n-1}$
4. $\mathrm{A}^{\mathrm{n+n+n\ all\ n's}}$ are equal.
5. $s]_{t\,=\,a}$
6. $e^{1{,}000}$
7. $a^{m+k} \div a^{m} = a^{k}$
8. $P_{s_1 \ldots\, s_2}$
9. $10_ = 6_8$
10. $a'_1, a'_2, \ldots, a'_n$ are the inverses.





EXERCISE 8

Exercise 8 will be available when this course is finished being written and is no longer "Provisional". Proceed to Lesson 9.

LESSON 9

Read about this PROVISIONAL EDITION in the front matter to this book.
Check the NFB website periodically for updates to this lesson.

- *INTRODUCTION TO FRACTIONS*
  - *Simple Fractions*
  - *Mixed Numbers*
  - *Complex Fractions*
  - *More Fraction Rules*
- *RADICAL EXPRESSIONS*

FORMAT
LINKED EXPRESSIONS
Division of a Long Linked Expression
Special Linked Expression

INTRODUCTION TO FRACTIONS

In a technical transcription, fractions are brailled in Nemeth Code. Two types of fractions are presented in this lesson: simple fractions (including mixed numbers) and complex fractions.

9.1 Recognition and Layout: A fraction is composed of three parts: a numerator, a denominator, and a fraction line.

numerator 3
denominator —
4 ← *fraction line*

Fractions are printed in a variety of ways. The numerator may be printed above the denominator or they may be printed on the same level. The fraction line may be horizontal or diagonal. Here are three examples of the fraction "three fourths" printed in different styles.

$\frac{3}{4}$ $^{3}/_{4}$ 3/4

The numerator and/or denominator may also consist of or contain words or abbreviations. Here are examples.

m/s ("meters per second")

$^{\text{ft.}}/_{\text{sec.}}$ ("feet per second")

$\frac{\text{rise}}{\text{run}}$ ("rise over run ")

3.5%/year ("3.5 percent per year")

Simple Fractions

9.2 Definition of Simple Fraction: A simple fraction is one in which neither the numerator nor the denominator is a fraction.

These are simple fractions: This is <u>not</u> a simple fraction:

$\frac{1}{2}$ $\frac{a^2}{b^2}$ mi./hr. $\frac{1/3}{2/3}$

Note that the fraction line may be printed as a horizontal line or as a diagonal line.

A fraction is also considered to be a simple fraction when its numerator or denominator contains fractions at the superscript or subscript level only.

This is a simple fraction: $\frac{y^{\frac{1}{2}}}{y^{\frac{1}{4}}}$

Regardless of print layout, unless otherwise stated the fraction is *transcribed linearly* so that the numerator, the fraction line, and the denominator are written horizontally across one braille line.

9.3 Simple Fraction Indicators and the Horizontal Simple Fraction Line: Simple fraction indicators are used to enclose a simple fraction whose numerator and denominator are separated by a *horizontal fraction line*.

Simple Fraction Indicators

Opening 

Closing 

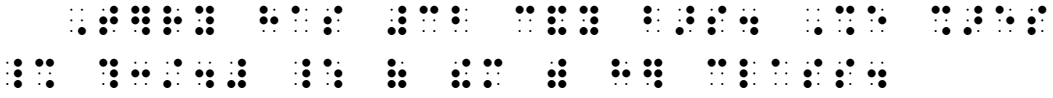
In a simple fraction, the horizontal fraction line is brailled as the symbol shown below.

Horizontal Simple Fraction Line —— 

Note that the horizontal simple fraction line consists of one braille cell.

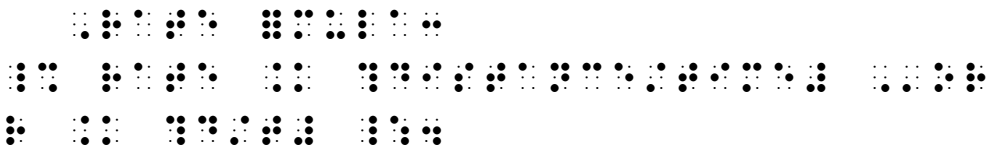
⋙ $\frac{3}{4}$ 

Example 9.3-1 Terry has 32 candy bars. She shares $\frac{3}{4}$ of them with her class.



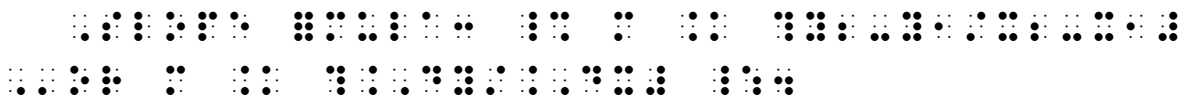
⋙ $\frac{d}{t}$

Example 9.3-2 Rate formula: rate $= \frac{\text{distance}}{\text{time}}$ or $r = \frac{d}{t}$.



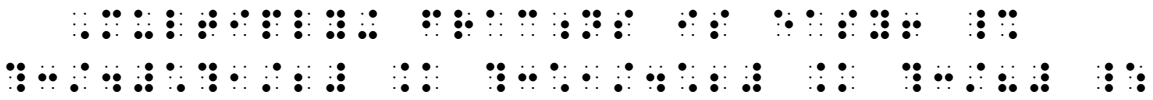
Reminder: Words in Nemeth Code are brailled without contractions.

Example 9.3-3 Slope formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$ or $m = \frac{\Delta y}{\Delta x}$.



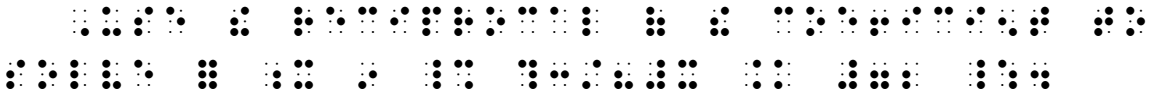
The numerator and denominator are unspaced from the fraction indicators and from the fraction line. Spacing before and after a fraction is subject to the spacing rules for the signs preceding and following the fraction.

Example 9.3-4 Multiplying fractions is easy! $\frac{3}{4} \cdot \frac{1}{2} = \frac{3 \cdot 1}{4 \cdot 2} = \frac{3}{8}$



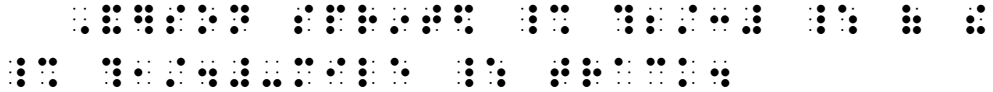
There is no space before or after the operation signs (multiplication dots); there is a space before and after the comparison signs (equals signs).

Example 9.3-5 Use the reciprocal of the coefficient to solve for x in $\frac{3}{8}x = 72$.



The coefficient (fraction) is unspaced from the variable (x).

Example 9.3-6 Anderson sprinted $\frac{2}{3}$ of the $\frac{1}{4}$-mile track.



The fraction in this hyphenated expression is unspaced from the hyphen.

Instructions: Treat the second heading as a connected title heading. (See *Braille Formats,* Section 4, for details.) Transcribe the first series of fractions as a paragraph, starting in cell 3. Begin a new line in the runover cell (cell 1) if the entire fraction or ellipsis will not fit on what remains of the current line. A blank line must precede the itemized portion. When you proofread, check that you closed each fraction, that you returned to the baseline after each superscript, that displayed expressions are placed in the proper cell, and that you terminated Nemeth Code where appropriate. Watch for end-of-sentence punctuation.

PRACTICE 9A

Horizontal Simple Fraction Line

Simple fractions: $\frac{1}{2}$ … $\frac{15}{16}$ … $\frac{x}{y}$ … $\frac{a+b}{c+d}$ … $\frac{\Delta y}{\Delta x}$ … $\frac{(x+y)}{(x-y)}$ …

$\frac{9}{12}$ … $\left(\frac{3}{2}a+\frac{1}{2}b\right)$ … $\frac{3x}{17y}$ … $x-\frac{1}{4}(x-2x)$

1. $V=\frac{1}{3}\pi r^2 h$
2. $\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}$
3. $\left|\frac{a}{b}\right|=\frac{|a|}{|b|}$
4. Write an equation to show that $\frac{3}{4}$ of $\frac{1}{2}$ is $\frac{3}{8}$.
5. $x^2\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{4x^2-x-2}{(x+1)(y+1)}$
6. Solve this differential equation:

$$x\frac{\mathrm{d}y}{\mathrm{d}x}+2y=e^{x^2}$$

7. The number π is the ratio of the circumference of a circle to its diameter. That is,

$$\pi=\frac{\text{circumference}}{\text{diameter}}.$$

8. $\frac{35}{70}=\frac{x}{100}$ 9. $\frac{12}{33}=\frac{m}{11}$ 10. $\frac{x}{15}=\frac{12}{75}$ 11. $\frac{4}{32}=\frac{10.5}{x}$
12. $\frac{1}{4}+\frac{3}{4}-\frac{1}{2}=\frac{1}{2}$

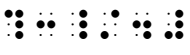
9.4 The Diagonal Simple Fraction Line: The type of fraction line used in the print copy (horizontal or diagonal) is replicated in the braille transcription. In a simple fraction, the diagonal fraction line is brailled as the symbol shown below.

Diagonal Simple Fraction Line	/	

Note that the diagonal simple fraction line consists of two braille cells.

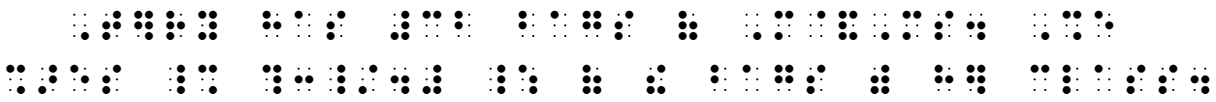
When a diagonal fraction line is printed, it may not be clear where the fraction begins and where it ends. The transcriber must not attempt to analyze the math. Instead, application of the following rules will prevent misinterpretation of the expression.

9.4.1 Use of simple fraction indicators with the diagonal simple fraction line: When the numerator and denominator are *printed at different levels* of writing on either side of the *diagonal line*, the construction is clearly a fraction and so simple fraction indicators are used.

⋙ $^{3}/_{4}$ 

Do not confuse this type style with superscripts and subscripts. In this example, the numeral 3 is the numerator and the numeral 4 is the denominator.

Example 9.4-1 Terry has 32 bags of M&Ms. She shares $^{3}/_{4}$ of the bags with her class.

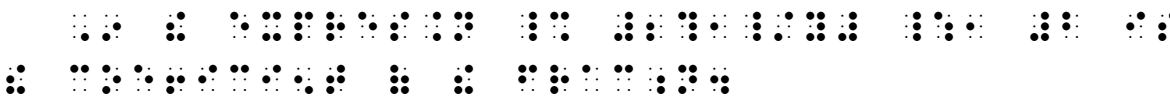


If the numerator and denominator are printed at the same level of writing on either side of the diagonal line the transcriber must notice the <u>type size</u>. If the type is *in a different size* from that normally used for similar expressions throughout the text, identify this as a fraction by using simple fraction indicators. In the example below, note that the fraction is printed on the baseline of writing—it is *not* a subscript.

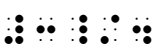
⋙ $2\ ^{1}/_{y}$ 

The numeral 1 is smaller than the numeral 2. The space between the coefficient and the fraction is not shown in braille.

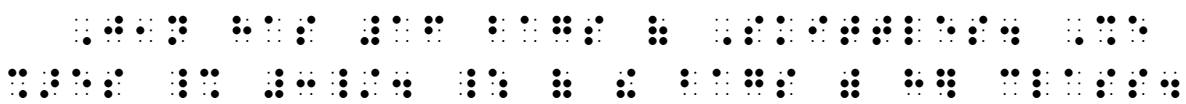
Example 9.4-2 In the expression $2\ ^{1}/_{y}$, 2 is the coefficient of the fraction.



9.4.2 Nonuse of simple fraction indicators with the diagonal simple fraction line: When the numerator and denominator are *printed at the same level of writing on either side of the diagonal line* and the *type size is normal* when compared to similar expressions, fraction indicators are <u>not</u> used. A switch to Nemeth Code is required. (Review **6.4.1** "Slash".)

⋙ 3/4 

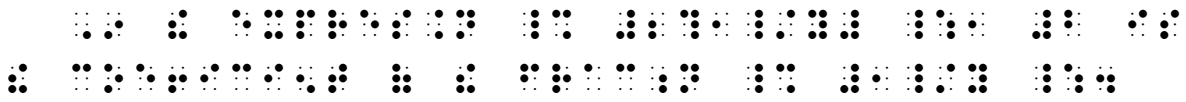
Example 9.4-3 Jean has 16 bags of Skittles. She shares 3/4 of the bags with her class.



Example 9.4-2 is amended below. The first fraction is printed on the baseline but is in smaller type. Fraction indicators are used. However, the second fraction is printed in normal type size. This meets the requirements for nonuse of simple fraction indicators because the numerator and denominator are printed at the same level of writing on either side of the diagonal line.

⋙ 1/y

Example 9.4-4 In the expression $2_{1/y}$, 2 is the coefficient of the fraction 1/y.

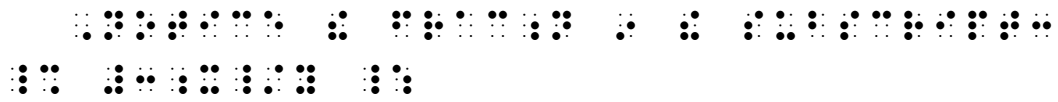


The second occurrence of "1/y" is printed in normal print size, therefore fraction indicators are not used even though the text refers to it as a "fraction".

The next example meets the requirements for nonuse of simple fraction indicators because (a) the numerator and denominator are printed at the same level of writing on either side of the diagonal line; and (b) the type size is normal for similar expressions (subscripts). Simple fraction indicators are not used here, even though the text identifies the expression as a "fraction".

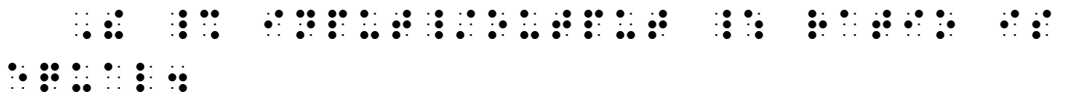
⋙ $3_{x/y}$

Example 9.4-5 Notice the fraction in the subscript: $3_{x/y}$

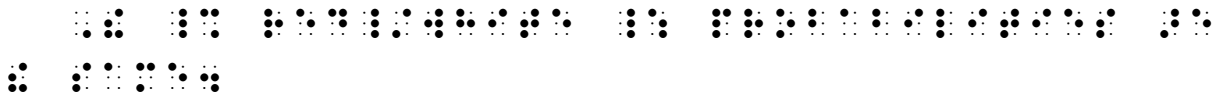


The words next to the diagonal lines in the next two examples are mathematical. Consider the context – the words represent ratios and probabilities. Because the context is mathematical, a switch to Nemeth Code is required for each fraction. Fraction indicators are not used because (a) the numerator and denominator are printed at the same level of writing on either side of the diagonal line; and (b) the type size is normal for similar expressions.

Example 9.4-6 The input/output ratio is equal.



Example 9.4-7 The red/white probabilities are the same.



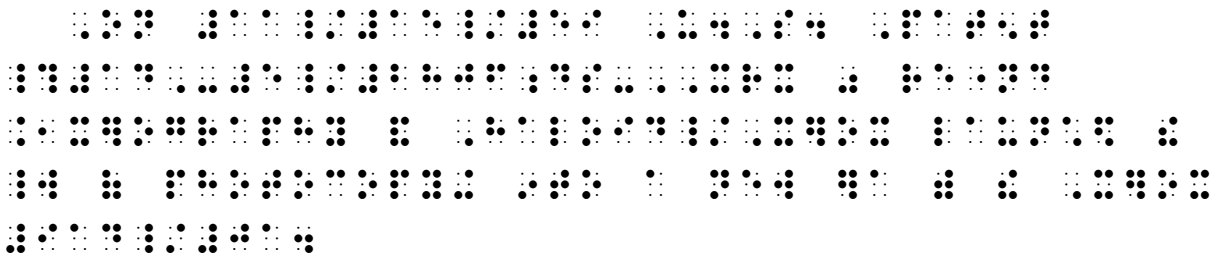
9.4.3 Reminder About the UEB Slash: When the slash is serving as a separator between words or numbers in literary context, it is not mathematical. If the slash does not represent a mathematical operation, the rules of UEB are followed.

and/or, c/o  (UEB)

true/false test  (UEB)

6/6/44 *(date)*  (UEB)

Example 9.4-8 On 11/15/59 U.S. Patent #14–5/2806ds-XRX was renamed *xerography* and Haloid/Xerox launched the world of photocopying into a new era with the Xerox 914/01.



Numerals printed with slashes such as dates, model numbers, etc. are not mathematical and therefore the rules of UEB are followed.

9.5 Additional Considerations

9.5.1 Fractions and Abbreviations: Abbreviations in the numerator or denominator of a fraction are unspaced from the fraction indicators or fraction lines. When the slash means *per, divided by*, or *over* the construction is considered to be a fraction and Nemeth Code is used.

Example 9.5-1 Express m/s in mph.



m/s means "meters <u>per</u> second"

Example 9.5-2 Express $^{ft.}/_{sec.}$ in mph.



$^{ft.}/_{sec.}$ means "feet <u>per</u> second"

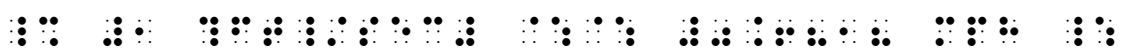
An abbreviation is spaced from the value to which it applies (regardless of the spacing applied in print).

Example 9.5-3 Add 2/3 c. cornstarch—Oobleck!



A space is placed between 2/3 and its associated abbreviation, c.

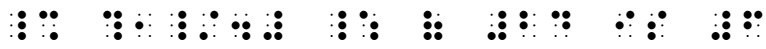
Example 9.5-4 $1\ ^{ft}/_{sec} \approx 0.6818$ mph



A space is between the numeral 1 and its associated abbreviation, ft/sec, as well as between the numeral 0.6816 and its associated abbreviation, mph.

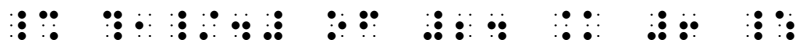
9.5.2 Code Switch Reminders: Revisiting **Lesson 4**, as part of a math problem expressed in symbols and words, the words are included in the switch. Compare the next two examples.

Example 9.5-5 $^1/_4$ of 24 is 6



Only the fraction requires a switch to Nemeth Code.

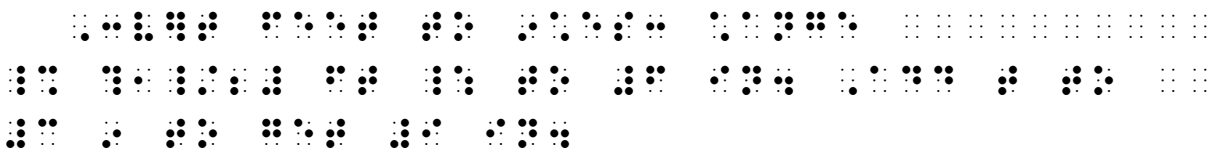
Example 9.5-6 $^1/_4$ of 24 = 6



The entire "math sentence" is a mathematical expression. The word "of" is part of the equation and is uncontracted in Nemeth Code.

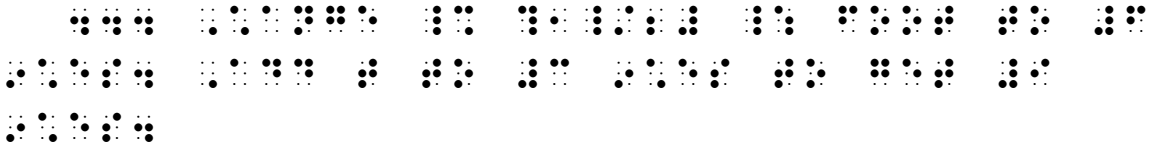
Abbreviations are included inside the switches. Compare the next two examples.

Example 9.5-7 Convert feet to inches: change $^1/_2$ ft to 6 in. Add that to 3 in to get 9 in.



Remember, the abbreviation must fall on the same line as its value.

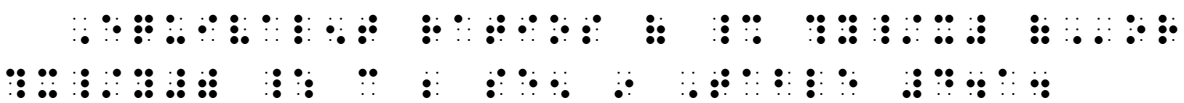
Example 9.5-8 ... Change $^1/_2$ foot to 6 inches. Add that to 3 inches to get 9 inches.



The value need not be forced to fall on the same line as the related word.

Stay in Nemeth Code for a single word in UEB.

Example 9.5-9 Equivalent ratios of $^{y}/_{x}$ (or $^{x}/_{y}$) can be seen in Table 4.1.



Math grouping symbols are used for the parentheses here because we have not switched out of Nemeth Code.

Instructions: First determine whether the slash is used mathematically, that is, does it require a switch to Nemeth Code? If it does, then determine if fraction indicators are required.

PRACTICE 9B

Diagonal Simple Fraction Line

A) How many $^{2}/_{3}$'s are there in $^{5}/_{6}$?

B) Energy is absorbed at the rate of 880 J/s for each square meter of the surface.

C) $y(0) = \pi/4$

D) In y 1/5, y is the coefficient of the fraction $1/5$.

E) True/False: The rise/run ratio is 5 in graph A.

F) $^{a}/_{b} \cdot {}^{c}/_{d} = {}^{ac}/_{bd}$

G) A 5-year CD went from earning interest at the rate of 12.06%/year in 1984 to earning less than 0.87%/year in 2015.

Mixed Numbers

9.6 Definition of Mixed Number: A mixed number is an expression composed of a whole number followed by a simple fraction whose numerator and denominator are both numerals. Numerals in a mixed number may be represented by omission signs. An expression is not a mixed number if it contains any letters, even though the expression appears to be in the form of a mixed number. Here are some examples.

$1\frac{2}{3}$ (Spoken "one and two-thirds")

$99\,{}^{15}\!/_{16}$ (Spoken "ninety-nine and fifteen-sixteenths")

9.6.1 Use of Mixed Number Fraction Indicators: The opening and closing mixed number fraction indicators enclose the fractional part of a mixed number.

Mixed Number Fraction Indicators	
Opening	⠸⠼
Closing	⠼⠸

The fractional part of the mixed number uses simple fraction lines, either horizontal or diagonal, according to the fraction line style used in print.

Horizontal Simple Fraction Line	——	⠌
Diagonal Simple Fraction Line	/	⠸⠌

The examples shown above are brailled as follows.

⋙ $1\frac{2}{3}$ ⠼⠁⠼⠃⠌⠉

⋙ $99\,{}^{15}\!/_{16}$ ⠼⠊⠊⠼⠁⠌⠁⠋

Example 9.6-1 Russ is making a small flowerbed that is $3\frac{1}{2}$ feet by $1\frac{1}{2}$ feet.

⠠⠗⠥⠎⠎ ⠊⠎ ⠍⠁⠅⠊⠝⠛ ⠁ ⠎⠍⠁⠇⠇ ⠋⠇⠕⠺⠑⠗⠃⠑⠙ ⠞⠓⠁⠞ ⠊⠎ ⠼⠉⠼⠃⠌⠛
⠋⠑⠑⠞ ⠃⠽ ⠼⠁⠼⠃⠌⠛ ⠋⠑⠑⠞⠲
⠋⠑⠑⠞ ⠃⠽ ⠼⠁⠼⠃⠌⠛ ⠋⠑⠑⠞⠲

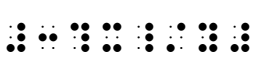
9.6.2 Mixed Numbers and Omissions: If any part of a mixed number contains a sign of omission, the mixed number fraction indicators are used.

⋙ $\frac{7}{4} = 1\frac{?}{4}$ 

⋙ $1\,{}^{15}/_{25} = ?\,{}^{3}/_{5}$ 

9.6.3 Nonuse of Mixed Number Fraction Indicators: If the fractional part of the expression contains a letter, it no longer suits the definition of "mixed number." Appropriate fraction indicators are used (or are not used) according to the rules.

⋙ $\frac{7}{4} = 1\frac{x}{4}$ 

⋙ 3 x/y 

Instructions: Treat "FRACTION REVIEW" as a centered heading.

PRACTICE 9C

Mixed Numbers

1. Find the premium for a 1½–yr. policy at the yearly rate of 24¢ per $100.
2. $2\frac{1}{2}$ ft + 8 in = ? inches
3. $\left(\frac{1}{2} \times 3\frac{1}{2}\right) + \left(3\frac{1}{2} \times 2\right)$
4. $13\,{}^{1}/_{2} + 2\,{}^{2}/_{3} = 16\,{}^{1}/_{6}$
5. $7/4 = 1\,{}^{?}/_{4}$
6. $\frac{9}{4} = 2\frac{x}{4}$

FRACTION REVIEW

Compute each **unit rate** (price/pound).

a. $1.50 for 2/3 pound of potatoes

b. $4.20 for $^{1}/_{2}$ pound of Edam cheese

c. $6.00 for $\frac{3}{4}$ pound of deli smoked turkey

d. $12.50 for $1\frac{1}{2}$ pounds of sliced ham

Complex Fractions

9.7 Definition of Complex Fraction: A complex fraction is one whose numerator and/or denominator are, or contain, one or more simple fractions or mixed numbers.

numerator $4\frac{3}{4}$

$\frac{}{5}$ ← *complex fraction line*

denominator

Here are more examples of complex fractions. In this manual, the complex fraction line is usually longer or thicker than the fraction lines within the numerators and denominators.

$\frac{\frac{4}{3}}{12}$ $\qquad$ $\frac{\frac{a}{b}-\frac{c}{d}}{\frac{a}{b}+\frac{c}{d}}$ $\qquad$ $\frac{1}{3/8}$ $\qquad$ $\frac{1}{2}/\frac{3}{4}$

Reminder: A fraction is <u>not</u> a complex fraction if the only fractions it contains are at the superscript or subscript level. Such a fraction is a simple fraction.

This is a simple fraction, <u>not</u> a complex fraction: $\frac{y^{\frac{1}{2}}}{y^{\frac{1}{4}}}$

9.7.1 Use of Complex Fraction Indicators and Complex Fraction Lines: The opening and closing complex fraction indicators are used to enclose a complex fraction.

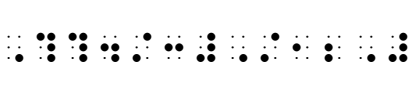
Complex Fraction Indicators	
Opening	⠸⠌
Closing	⠌⠸

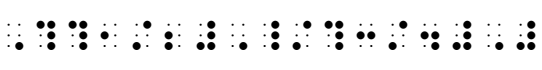
The main complex fraction line is represented by its appropriate braille symbol—either horizontal or diagonal.

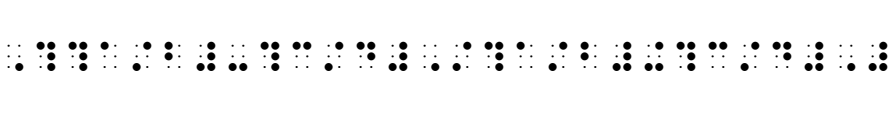
Horizontal Complex Fraction Line	⠐⠂
Diagonal Complex Fraction Line	⠐⠸⠂

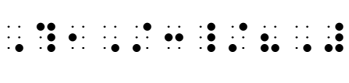
The examples shown above are brailled as follows. Simple fraction indicators enclose each simple fraction when required; complex fraction indicators enclose the entire complex fraction. We suggest that you underline the complex fraction indicators and the complex fraction line in each example in order to analyze each transcription.

⋙ $\frac{4\frac{3}{4}}{5}$ ⠸⠌⠼⠙⠼⠉⠌⠼⠙⠸⠌⠼⠑⠸⠌

⋙ $\frac{\frac{4}{3}}{12}$ 

⋙ $\frac{1}{2}/\frac{3}{4}$ 

⋙ $\frac{\frac{a}{b}-\frac{c}{d}}{\frac{a}{b}+\frac{c}{d}}$ 

⋙ $\frac{1}{3/8}$ 

Although this denominator does not require simple fraction indicators (see ***9.4.2****), it is still a fraction and so the overall construction is a complex fraction.*

Instructions: Begin each complex fraction on a new braille line, not side-by-side as printed. Read left-to-right.

PRACTICE 9D

Complex Fractions

$\frac{\frac{1}{8}+\frac{3}{4}}{7}$ $\frac{\frac{1}{2}+\frac{1}{3}}{\frac{3}{4}-\frac{7}{9}}$ $\frac{1/3+1/4}{4/5-1/2}$ $\frac{\frac{\pi}{8}}{2\pi}$

$\frac{a}{\frac{b}{c}}$ $\frac{33\frac{1}{3}}{100}$ $\frac{3/5}{6}$ $\frac{3}{5}/\frac{7}{9}$

More Fraction Rules

9.8 Fractions and the Baseline Indicator: When a fraction is on the baseline level, assure that the components of the fraction (the fraction indicators and the fraction line) are notated on the baseline of writing. Place the baseline indicator before the fraction line or the fraction indicator when there is a level in effect at the end of a numerator or a denominator. Correct placement of the baseline indicator assures accurate reading.

⋙ $\frac{a^2}{b}$ 

The baseline indicator precedes the fraction line following the superscript in the numerator.

⋙ $\frac{a}{b^2}$ 

The baseline indicator precedes the closing fraction indicator following the superscript in the denominator.

⋙ $\dfrac{\frac{x}{x-1}-1^2}{\frac{x}{x+1}+1^2}$

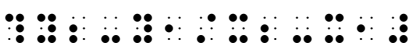


These fractions are all on the baseline.

⋙ $x^{\frac{1}{2}} \times y$ 

This fraction is in the superscript position.

The baseline indicator is not used after a numeric subscript that does not require a subscript indicator.

⋙ $\dfrac{y_2-y_1}{x_2-x_1}$ 

A return to the baseline after each subscript is understood.

9.9 Further Observations Regarding Spacing: As seen in previous examples, spacing before and after a fraction is subject to the spacing rules for the signs preceding and following the fraction.

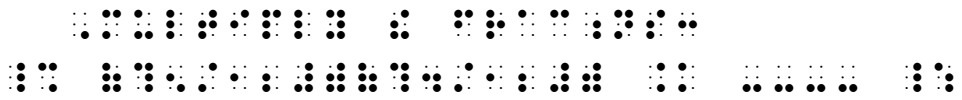
Example 9.9-1 Explain why $\frac{1}{2}+\frac{3}{4}=1\frac{1}{4}$.



The plus sign is unspaced from the fractions before and after it; there is a space before and after the equals sign. There is no space between the components of a mixed number.

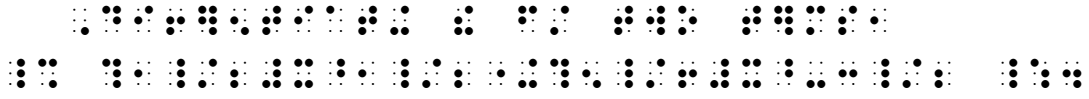
No space is left between a fraction and a letter, a numeral, a sign of grouping, a braille indicator, or another fraction when these items are part of the same expression.

Example 9.9-2 Multiply the fractions: $\left(\frac{5}{12}\right)\left(\frac{4}{12}\right)=$ ___



No space is left between factors even though one may appear in print.

Example 9.9-3 Differentiating the first two terms, $^1/_2\,x^{1/2} + {}^5/_6\,x^{-3/2}$.



No spaces are brailled in this long math expression.

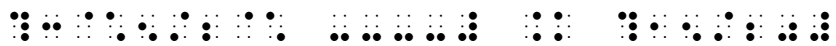
9.10 Fractions and the Ellipsis and Long Dash: No space is left between an opening or closing *fraction indicator* and an ellipsis or long dash in the numerator or denominator of a fraction.

⋙ $\frac{\ldots\times5}{2\times10}=\frac{15}{20}$



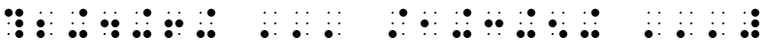
*The space following this ellipsis is required. Review **2.13** in Lesson 2.*

⋙ $\frac{3\times5}{2\times__}=\frac{15}{20}$



*The space preceding this long dash is required. Review **2.13** in Lesson 2.*

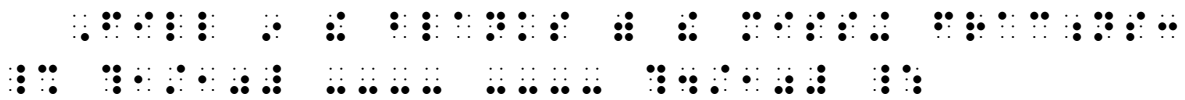
However, a space is left between a *fraction line* and an ellipsis or long dash.

⋙ $\frac{2+4+6+\ldots}{1+3+5+\ldots}$ 

A space is left between a fraction and an ellipsis or long dash preceding or following the fraction.

⋙ $\frac{1}{10}\ \ldots\ \frac{10}{10}$ 

Example 9.10-1 Fill in the blanks with the missing fractions: $\frac{1}{10}$ ___ ___ $\frac{4}{10}$



9.11 Enclosed Lists: Fractions and mixed numbers may be part of an enclosed list. (Review the rules for "enclosed list" in Lesson 5, section **5.15**.)

⋙ $\left\{0,\ \frac{1}{2},\ 1,\ 1\frac{1}{2}\right\}$ 

Instructions: Determine the formatting before beginning your transcription. Where does each paragraph begin? Which expressions are embedded and which are displayed? What is the proper cell placement for the displayed expressions?

PRACTICE 9E

These are simple fractions:

$$\frac{1}{2} \qquad \frac{a^2}{b^2} \qquad \frac{y^{\frac{1}{2}}}{y^{\frac{1}{4}}}$$

This is <u>not</u> a simple fraction: $\frac{1/3}{2/3}$

Review the rules in 8.11.5 regarding an ellipsis on the baseline of writing when it follows a superscript.

$$x^{\frac{1}{2}} \quad \ldots \quad x^{\frac{1}{2}} \cdot y^{-\frac{1}{2}} \quad \ldots \quad \frac{x^{\frac{1}{2}}+1}{y^{\frac{1}{2}}-1}$$

Plot the points $\left(-\frac{1}{2},\ 4\right)$, $\left(3,\ 4\frac{1}{4}\right)$, and $\left(-9,\ \frac{3}{4}\right)$. Then express $\frac{a^{3/4}}{b^{5/4}}$ in radical form.

RADICAL EXPRESSIONS

9.12 Terminology: Here are the parts of a radical expression.

$\sqrt{144}$ $\sqrt{}$ is the *radical sign.*
144 is the *radicand.*
The horizontal bar above the radicand is the *vinculum*. The vinculum shows the extent to which the radical sign applies.

There may be a figure placed to the left and slightly above the radical sign, called the *index* of the radical. For example, this radical sign has an index of three: $\sqrt[3]{}$ When there is no index, the radical sign may be referred to as the "square root" sign.

9.13 The Termination Indicator: When a radical expression has a vinculum, the radical sign is placed before the radicand and the termination indicator is placed after the radicand.

Radical Sign	$\sqrt{}$	⠜
Termination Indicator		⠻

⋙ $\sqrt{x}$ ⠜⠭⠻

⋙ $\sqrt{64}$ ⠜⠼⠋⠙⠻

Reminders: An English letter indicator is not needed for an English letter (in regular type) which occurs in an unspaced sequence of mathematical symbols. A numeric indicator is not used when a numeral immediately follows an indicator.

When a vinculum is not shown in print, or when the radical sign occurs without a radicand, a termination indicator is not used.

⋙ $\sqrt{(x-1)}$ ⠜⠷⠭⠤⠼⠁⠾

Example 9.13-1 The √ is called a "radical sign."

⠠⠜ ⠜⠠ ⠜ ⠠⠜ ⠜⠠ ⠠⠠⠜⠠⠠ ⠠ ⠠⠜⠠⠠⠜⠠⠠⠜⠠⠠⠜⠠
⠜⠠⠜⠠⠠⠜⠠

9.14 Spacing: The spacing before and after a radical expression is subject to the spacing rules for the signs preceding or following the radical expression.

⋙ $\sqrt{9}-\sqrt{4}=1$ ⠜⠼⠊⠤⠜⠼⠙⠶⠼⠁

No space is left between a radical expression and a letter, a numeral, a fraction, a sign of grouping, a braille indicator, or another radical expression when these items are unspaced in print and belong to the same expression.

⋙ $\sqrt{5}y$ ⠜⠼⠑⠽

⋙ $\sqrt{x}^{2}$

⋙ $2a\sqrt{4ab}$

⋙ $\sqrt{4}\sqrt{87} = 2\sqrt{87}$

⋙ $\sqrt{y}\,\mathrm{d}x + (1+x)\,\mathrm{d}y = 0,\ y(0) = 1$

*Reminder: In print, derivative notation dx, dy, etc. is often preceded and followed by a space within an expression, for clarity. In braille, the terms are not spaced unless a space is required with the item preceding or following them. (Review **5.13.1** in Lesson 5.)*

Example 9.14-1 Simplify: $\dfrac{2-\sqrt{\frac{1}{4}}}{3-\sqrt{\frac{1}{2}}}$

9.15 Index of Radical: A small number or letter that may appear next to the radical sign is the *index*. This print example shows an index "3".

$\sqrt[3]{27}$

In braille, the index-of-radical indicator, as well as the index, precede the radical sign.

Index-of-Radical Indicator

⋙ $\sqrt[3]{27} = 9$

⋙ $\sqrt[m]{\dfrac{a}{b}} = \dfrac{\sqrt[m]{a}}{\sqrt[m]{b}}$

Example 9.15-1 Simplify as follows: $\dfrac{\sqrt{9}}{\sqrt[3]{27}} = \dfrac{3}{3} = 1$

PRACTICE 9F

Radical Expressions

1. $\sqrt{\frac{b}{a}+\frac{a}{b}}$ 2. $\sqrt{c/d}$ 3. $\frac{1}{4}\sqrt{\frac{1}{2}}$ 4. $\sqrt{\frac{10}{16}} = \sqrt{10}/4$

5. $(\sqrt{3}+\sqrt{5})(\sqrt{3}-\sqrt{5})$ 6. $2\sqrt{2}+7\sqrt{2}=(2+7)\sqrt{2}=9\sqrt{2}$

7. $\frac{\sqrt{3}}{\sqrt{2}}\times\frac{\sqrt{5}}{\sqrt{2}}=\frac{\sqrt{15}}{2}$ 8. $\frac{\sqrt{2}-\sqrt{\frac{1}{3}}}{\sqrt{3}-\sqrt{\frac{1}{2}}}$ 9. $\sqrt{48x^3y}$

10. $\sqrt{(y-1)}+\sqrt{(2y)}=1$ 11. $\sqrt[n]{d}$ 12. $\sqrt[a+b]{z-y}$

13. $\sqrt[4]{729}+\sqrt[6]{27}=\sqrt[?]{?}$ 14. $7\sqrt[3]{125}\cdot 7\sqrt[5]{2}$ 15. $\sqrt[5]{m}\,\sqrt[5]{n}=\sqrt[5]{mn}$

9.16 Nested Radical Expressions: When radical expressions are nested one within the other, the appropriate number of order-of-radical indicators shows the depth of each inner radical expression.

Order-of-Radical Indicators	
First Inner Radical	⠨
Second Inner Radical	⠨⠨
Third Inner Radical	⠨⠨⠨
Termination Indicator	⠸

The appropriate order-of-radical indicator is placed before its radical sign and before its termination indicator.

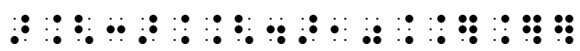
⋙ $\sqrt{x+\sqrt{x+y}+z}$ ⠨⠩⠨⠩⠨⠩⠨⠩⠨⠩⠨⠩⠨⠸

When more than one radical expression is completed at the same point, they are terminated beginning with the innermost expression.

⋙ $\sqrt{x+\sqrt{y+\sqrt{z}}}$ ⠨⠩⠨⠩⠨⠩⠨⠩⠨⠩⠨⠩⠨⠸

9.16.1 Nested Radical Expression with an Index: If an inner radical expression has an index, the appropriate order-of-radical indicator is placed before the index-of-radical indicator as well as before its termination indicator. When more than one radical expression is completed at the same point, they are terminated beginning with the innermost expression.

⋙ $\sqrt{\sqrt[3]{16}} = \sqrt[3]{\sqrt{16}}$ 

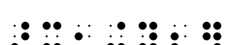
⋙ $\sqrt{\sqrt[3]{\sqrt[4]{10}}}$ 

9.17 Radical Expressions and the Baseline Indicator: When a radical expression is on the baseline level, assure that the components of the construction (the radical sign or indicators) are notated on the baseline of writing. Place the baseline indicator before the component when it follows a superscript or a subscript.

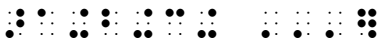
⋙ $\left(r^2\sqrt{r}\right)^2$ 

⋙ $\sqrt{x^2 + y^2}$ 

The baseline indicator is not used after a numeric subscript that does not require a subscript indicator.

⋙ $\sqrt{x_1 + y_2}$ 

9.18 Radical Expressions and the Ellipsis and Long Dash: These two omission symbols are generally preceded and followed by a space, but no space is left between a *braille indicator* and a related ellipsis or long dash. In the context of radical expressions this means no space is left between the ellipsis or long dash and the termination indicator or order-of-radical indicator.

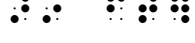
⋙ $\sqrt{a+b+c+\ldots}$ 

⋙ $\sqrt{x+\sqrt{x+\sqrt{\ldots}}}$ 

A space is required between a radical expression and an ellipsis or long dash preceding or following the expression.

⋙ $\sqrt{4}$ ... $\sqrt{64}$ 

9.19 Radical Expressions and Abbreviations: When an abbreviation occurs within a radical, no space is left between the abbreviation and the termination or order-of-radical indicator following it.

⋙ $\sqrt{9\text{ ft}}$ 

However, a space is required between a radical sign and an abbreviation.

⋙ $\sqrt{\text{ft.}}$ 

A space is also required between a radical expression and an abbreviation preceding or following the expression.

⋙ $2\sqrt{12}$ sq. in. ⠼⠃⠩⠼⠁⠃⠀⠎⠟⠲⠀⠊⠝⠲

9.20 Enclosed Lists with Radical Expressions: Radical expressions may be part of an enclosed list.

⋙ $(\sqrt{9},\ 3,\ \sqrt{4},\ 2\sqrt{6})$ ⠷⠩⠼⠊⠆⠀⠼⠉⠆⠀⠩⠼⠙⠆⠀⠼⠃⠩⠼⠋⠆⠾

*See **5.15** in Lesson 5 to review rules regarding enclosed lists.*

PRACTICE 9G

Nested Radical Expressions

(1) $\sqrt{-\frac{1}{2}-i\frac{\sqrt{3}}{2}}$

(2) $\sqrt{\sqrt{13}+\sqrt{15}+\sqrt{117}}$

(3) $\sqrt{1-\sqrt{a-b}}\times\sqrt{1+\sqrt{a-b}}$

(4) $\sqrt{b\sqrt[3]{b\sqrt{b}}}$

(5) $\sqrt[a]{\sqrt[b]{\sqrt[c]{abc}}}$

(6) $\sqrt{a^2}\sqrt{b^4}\sqrt{c}=ab^2\sqrt{c}$

(7) $(s^2\sqrt[3]{s^4})^2$

(8) $\sqrt[3]{x^2\sqrt{64x^6}}$

(9) $\sqrt{\sqrt[3]{\sqrt[4]{\sqrt[5]{b^{48}}}}}$

(10) $\sqrt{x_1+\sqrt{x_2}}$

(11) $q^{\sqrt{r}}+s$

(12) $\sqrt{c+d+e+\cdots}$

LINKED EXPRESSIONS

9.21 Definition of Linked Expression: A linked expression contains *at least one sign of comparison.* The part preceding the first sign of comparison is called the *anchor*. Each remaining part, beginning with a sign of comparison and ending before the next sign of comparison, is called a *link*.

Here are two examples of linked expressions.

$12.5\% > \frac{1}{10}$ *The anchor is* 12.5% *and the link is* $> \frac{1}{10}$

$$6 \times 245 = (6 \times 200) + (6 \times 40) + (6 \times 5) = 1200 + 240 + 30 = 1470$$

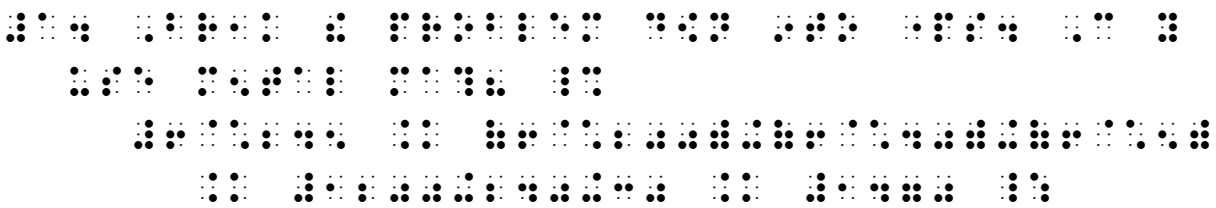
The anchor is 6×245*, followed by three links each beginning with an equals sign.*

9.22 Division of Linked Expressions: Recall that a mathematical expression must not be divided between lines if it will fit on one braille line within the current margins. If a *linked expression* is too long to fit on one line, the expression continues on the next line, beginning with a sign of comparison. If the expression contains more than one link, use as much of the line as possible before dividing the expression. It is not necessary to divide at every comparison sign. Begin the new line in the runover cell of the current format.

Example 9.22-1

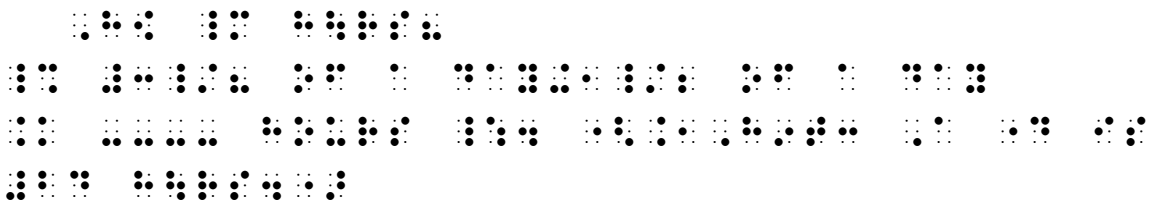
1. Break the problem down into parts. Can you use mental math?

$$6 \times 245 = (6 \times 200) + (6 \times 40) + (6 \times 5) = 1200 + 240 + 30 = 1470$$



The anchor starts in cell 5, displayed to itemized text. The first link fits on the same line. The second link starts on a new line in cell 7, the runover cell in this displayed format.

Example 9.22-2 How many hours? $3/8$ of a day $+ 1/2$ of a day $=$ __ hours.
(*Hint:* A day is 24 hours.)

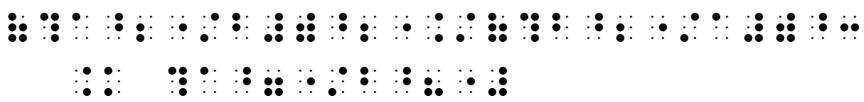


This is a narrative paragraph (3-1) with an embedded equation. The opening switch indicator will fit on the line with the anchor, in the runover position, cell 1. The link begins on the next line with its comparison sign.

9.22.1 Restrictions: In order to divide a long expression before a comparison sign, the comparison sign must be on the baseline of writing. Do not divide before a comparison sign that is part of an item enclosed in grouping symbols, between fraction indicators, or within radical signs. Details will be discussed in **Lesson 14**.

9.22.2 Other Considerations: A transition to a new braille line made before a sign of comparison terminates the effect of any level indicator used on the line above, just as it would if it were not divided between lines.

$$\left(\frac{a^2}{b}\right)^2 \div \left(\frac{b^2}{a}\right)^3 = \frac{a^7}{b^8}$$



A return to the baseline after the superscript "3" is triggered by the presence of the following comparison sign even though it is on the next line.

PRACTICE 9H

Linked Expressions

1. Is the following inequality true? $\frac{3}{5}\left(\frac{2}{3}x - \frac{1}{2}\right) > \frac{2}{5}\left(\frac{1}{4}x + \frac{1}{3}\right)$

2. $33\frac{1}{3}\% + 40\% + 61\frac{2}{3}\% = 134\frac{3}{3}\% = 135\%$

3. In multiplying $5\frac{3}{4} \times 46$, recall that $5\frac{3}{4} = 5 + \frac{3}{4}$. Therefore, $46 \times \left(5 + \frac{3}{4}\right) = (46 \times 5) + \left(46 \times \frac{3}{4}\right) = 264\frac{1}{2}$.

9.23 SPECIAL CASE—Certain Displayed Linked Expressions: Lesson 8 introduced displayed math expressions—how to recognize them in the print copy and what margins to use in the braille transcription. A displayed linked expression is subject to special braille format rules if it appears in print in the following way:

- The expression is displayed (not embedded) in the text.
- Its signs of comparison are vertically aligned.
- Other than the anchor on the first line, no sign of comparison is preceded by any expression on its left. (There is one exception – see the fourth bullet, below)

The following linked expression meets the three "special" requirements.

To factor $ab + c^2 + ac + bc$, the terms can be grouped in pairs with a common factor.

$$\begin{aligned} ab + c^2 + ac + bc &= (ab + ac) + (bc + c^2) \\ &= a(b + c) + c(b + c) \\ &= (a + c)(b + c)\ . \end{aligned}$$

In print it is common for the last line of the expression to contain more than one link. As long as the other conditions are met, this layout is still considered to meet the "special" requirements.

We can reduce $12\frac{1}{2}\%$ to lowest terms in the following way:

$$\begin{aligned} 12\tfrac{1}{2}\% &= 12.5\% \\ &= .125 \\ &= \frac{125}{1000} = \frac{1}{8} \end{aligned}$$

Using the same example only with a different print layout, this <u>does not</u> meet the requirements for a linked expression requiring special margins because there is more than one link on the first line of the displayed expression. (Violation of the third bullet.)

We can reduce $12\frac{1}{2}\%$ to lowest terms in the following way:

$$\begin{aligned} 12\tfrac{1}{2}\% &= 12.5\% = .125 \\ &= \frac{125}{1000} = \frac{1}{8} \end{aligned}$$

Using a similar example, the next layout <u>does not</u> meet the requirements for a linked expression requiring special margins because the expression is not displayed. (Violation of the first bullet.)

$$\begin{aligned} 1.\ 12\tfrac{1}{2}\% &= 12.5\% \\ &= .125 \\ &= \frac{125}{1000} = \frac{1}{8} \end{aligned}$$

If the print layout meets the requirements for a linked expression requiring special margins, one of the following braille formats is applied.

Margin Requirements for a Linked Expression Requiring Special Margins

9.23.1 In Narrative: When a linked expression requiring special margins as defined above occurs in unitemized explanatory portions of the text, the *anchor* begins in cell 3 and *each link* begins in cell 5. Any runovers to the anchor or to a link are indented two more cells, to begin in cell 7. (Runovers are discussed in a later lesson.)

Example 9.23-1

To factor $ab + c^2 + ac + bc$, the terms can be grouped in pairs with a common factor.

$$\begin{aligned} ab + c^2 + ac + bc &= (ab + ac) + (bc + c^2) \\ &= a(b + c) + c(b + c) \\ &= (a + c)(b + c)\,. \end{aligned}$$

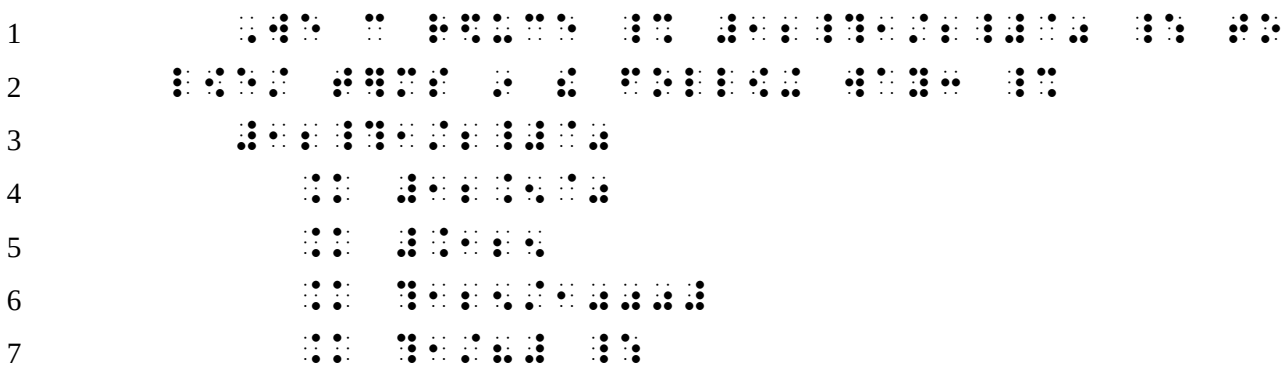
1 ⠀⠀⠀

Even when the print copy shows more than one link on the last line, in braille *each link* begins on a new line.

Example 9.23-2

We can reduce $12\frac{1}{2}\%$ to lowest terms in the following way:

$$12\frac{1}{2}\% = 12.5\%$$
$$= .125$$
$$= \frac{125}{1000} = \frac{1}{8}$$



Lines 1-2: Narrative paragraph (3-1)
***Line 3: Displayed math begins in cell 3.*
***Lines 4-7: Each link of this linked expression requiring special margins begins in cell 5.*

9.23.2 In Itemized Text Without Subdivisions: When a linked expression requiring special margins as defined above occurs in itemized text containing no subdivisions, the *anchor* begins in cell 5 and *each link* begins in cell 7. Any runovers to the anchor or to a link are indented two more cells, to begin in cell 9. (Runovers are discussed in a later lesson.)

Example 9.23-3

1. The example below shows a way of finding 6×245.

$$\begin{aligned} 6 \times 245 &= (6 \times 200) + (6 \times 40) + (6 \times 5) \\ &= 1200 + 240 + 30 \\ &= 1470 \end{aligned}$$

Is there another way to find 6×245?

1 ⠼⠋⠲ ⠼⠃ ⠠⠠⠠

9.23.3 In Itemized Text With Subdivisions: When a linked expression requiring special margins as defined above occurs in itemized text containing subdivisions, the *anchor* begins in cell 7 and *each link* begins in cell 9. Any runovers to the anchor or to a link are indented two more cells, to begin in cell 11. (Runovers are discussed in a later lesson.)

Example 9.23-4

2. Factor $2a(b-c)-3x(c-b)$.

   (a) Factors $b-c$ and $c-b$ are divisible by $(b-c)$ since

   $$c-b=(-1)(b-c).$$

   (b) Thus

   $$\begin{aligned}2a(b-c)-3x(c-b) &= 2a(b-c)+3x(b-c)\\ &= (2a+3x)(b-c).\end{aligned}$$

1
2
3
4
5
6
7
8

Line 1: Itemized text begins in cell 1.

Lines 2-3: Subitem text (3-5).

***Line 4: Displayed math begins in cell 7. This is not a linked expression requiring special margins so the link continues on the same line as the anchor.*

Line 5: Next subitem begins in cell 3.

***Line 6: Displayed math begins in cell 7.*

(Note that the displayed anchor is indented two cells to the right of the runover cell of the preceding material, even if the preceding material does not require a runover line.)

***Lines 7-8: Each link of this linked expression requiring special margins begins in cell 9.*

(Note that each link is indented two cells to the right of the anchor.)

Reminder: A line is not skipped above or below displayed mathematical material unless a blank line is required under other rules or guidelines.

Code Switch Decision: It would not be incorrect to terminate Nemeth Code at the end of line 1, braille the identifier (a) in UEB, and switch back to Nemeth Code after the word "Factors". The transcription shown minimizes code switching which is one of the considerations in the production of a fluent transcription.

Instructions: Treat "MULTIPLYING MIXED NUMBERS" as a centered heading.

PRACTICE 9I

Special Linked Expressions

To test the equation $R_t = \frac{R}{n}$, use four 1000-Ω resistors wired in series to predict a total resistance of 250 Ω.

$$R_t = \frac{R}{n} = \frac{1000\ \Omega}{4}$$

$$\frac{1000\ \Omega}{4} = 250\ \Omega$$

Then, by using the resistance theory equation

$$\frac{1}{R_t} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n},$$

plug the 150-Ω and 1000-Ω resistors into the equation as R_1 and R_2.

$$\begin{aligned} \frac{1}{R_t} &= \frac{1}{150\ \Omega} + \frac{1}{1000\ \Omega} \\ &= 0.007 + 0.001 \\ &= 0.008 \\ R_t &= \frac{1}{0.008} = 125\ \Omega \end{aligned}$$

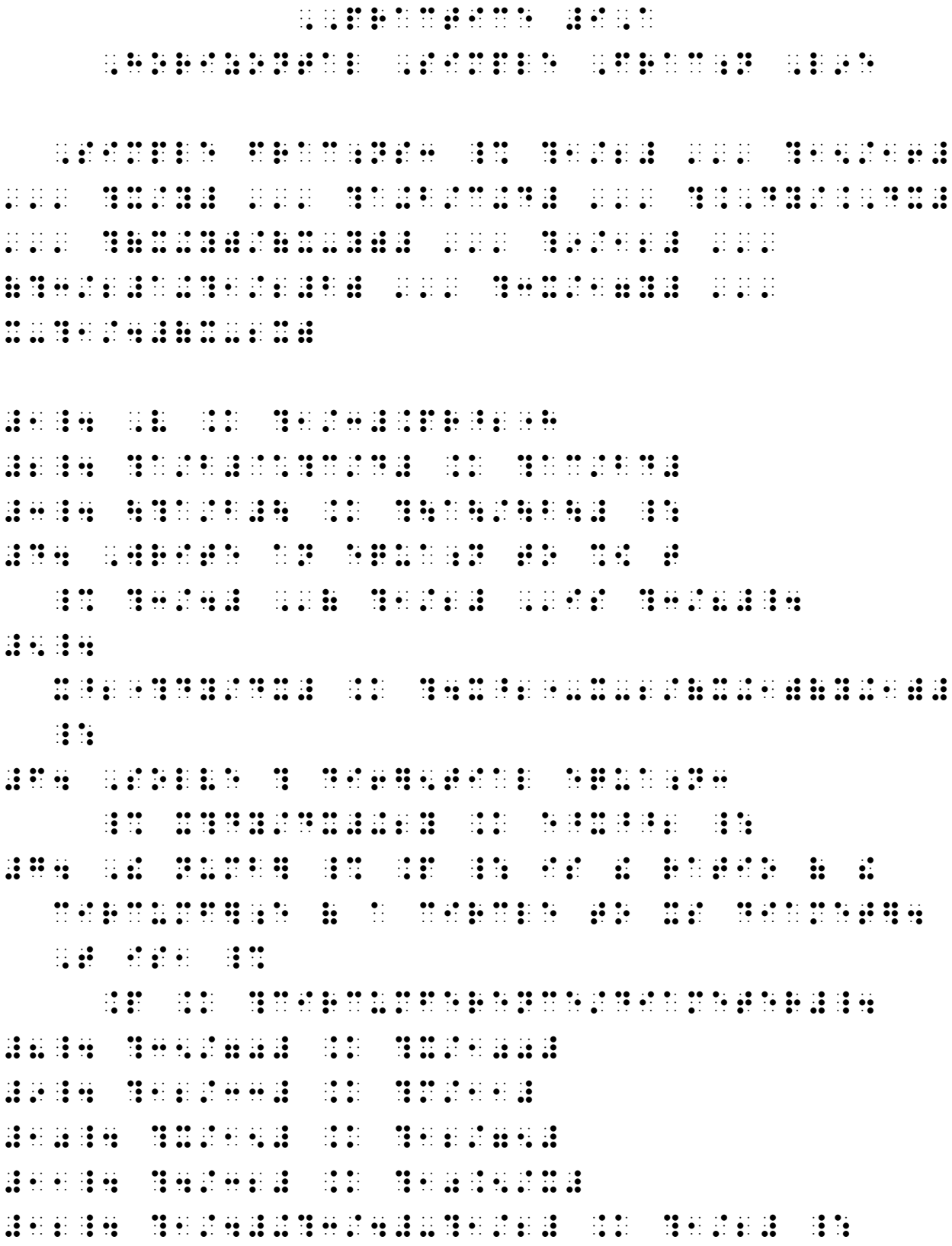
MULTIPLYING MIXED NUMBERS

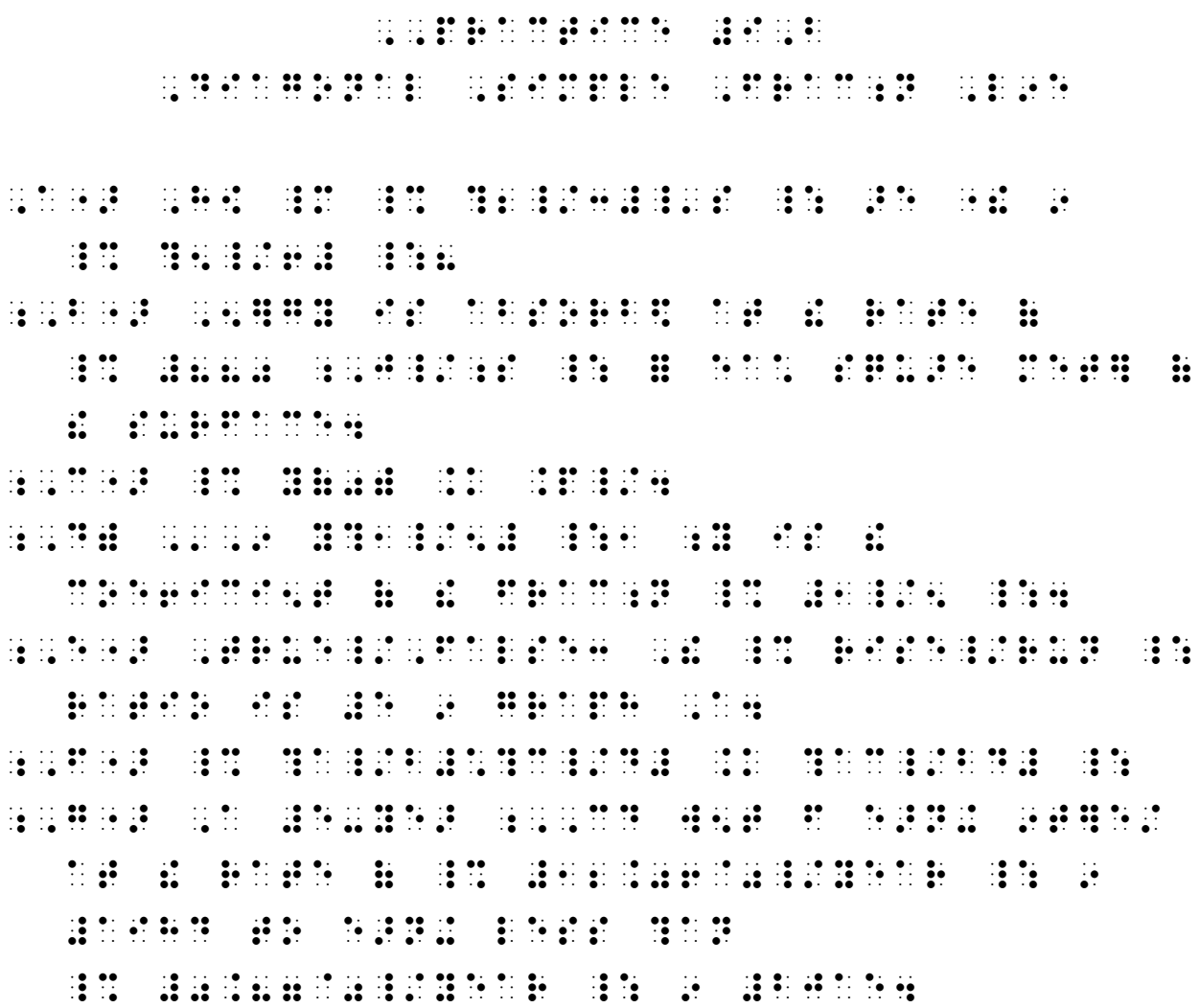
A. Explain each step in the solution to this multiplication problem.

$$\begin{aligned} 2\tfrac{1}{2} \cdot 1\tfrac{1}{4} &= \left(2 + \tfrac{1}{2}\right) \cdot \left(1 + \tfrac{1}{4}\right) \\ &= 2 + \tfrac{2}{4} + \tfrac{1}{2} + \tfrac{1}{8} \\ &= 2 + \tfrac{1}{2} + \tfrac{1}{2} + \tfrac{1}{8} \\ &= 2 + 1 + \tfrac{1}{8} = 3\tfrac{1}{8} \end{aligned}$$

For further practice, see Appendix A—Reading Practice.

ANSWERS TO PRACTICE MATERIAL





NOTES:

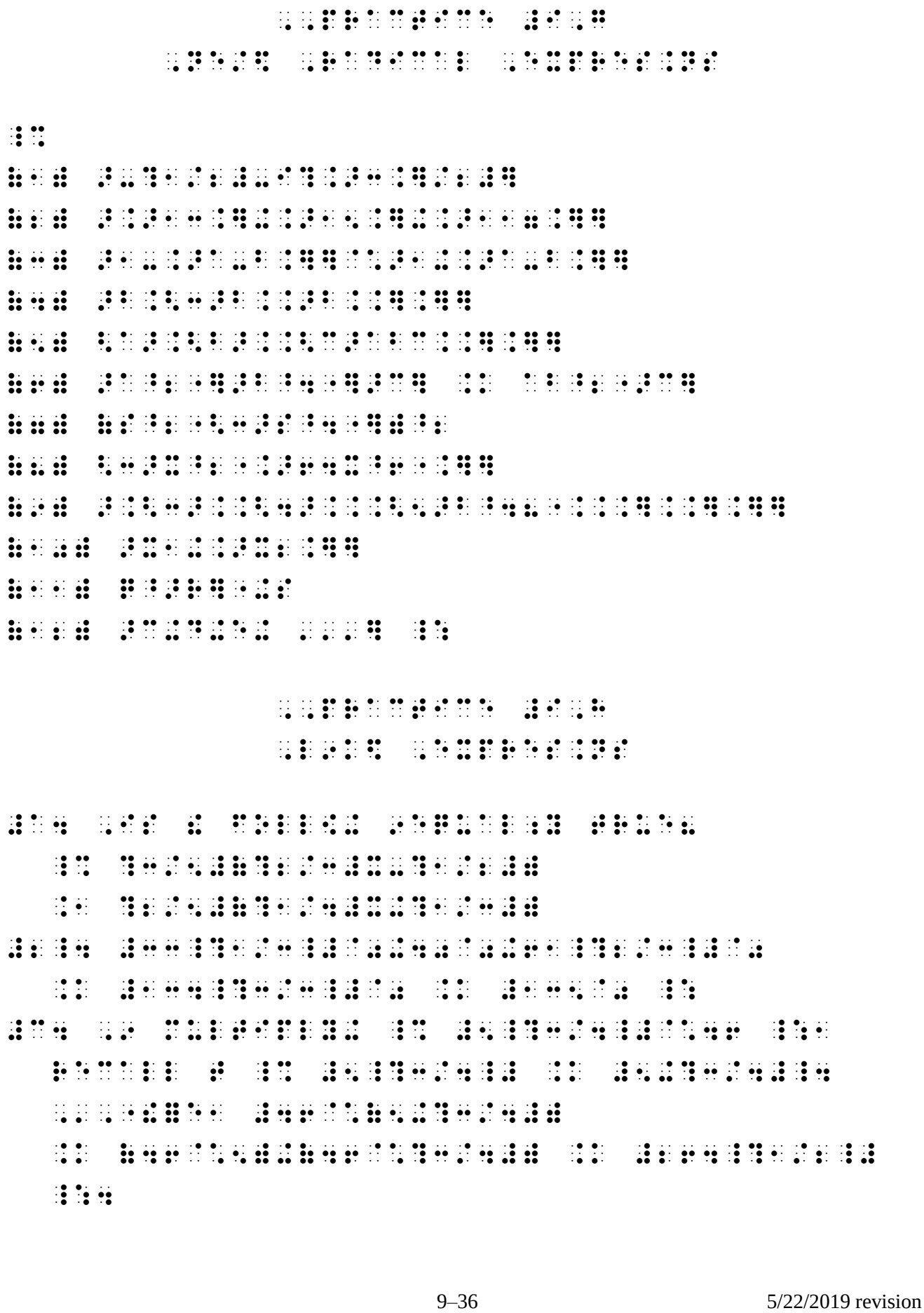
*A) The apostrophe-s is included inside the switch and so a punctuation indicator is needed for the apostrophe. Review **3.7** in Lesson 3.*

B) Because the slash means "per" ("Joules per second") a switch to Nemeth Code is required. The value ("880") is included in the switch. An English letter indicator is required for a single-letter abbreviation with no related period.

C) No fraction indicators are used because "pi" and "4" are of normal size and are printed on the baseline.

E) The first slash is in literary context. A switch to Nemeth Code is required for the second slash because it means "over" ("rise over run") in a ratio.

⠠⠠
⠿⠠⠿ ⠠⠿⠠⠠⠠⠠⠿⠠⠠⠠⠠⠿⠠⠿
⠿⠠⠿ ⠠⠠⠿⠠⠠⠿
⠿⠿⠠⠿ ⠠⠠⠠⠿⠠⠿⠠⠠⠠⠿⠠⠿
⠿⠠⠿⠠ ⠠⠿⠠⠠⠿⠠⠠⠠⠿⠠ ⠠⠠ ⠠⠠⠿⠠⠠⠿⠠
⠿⠠⠠⠿ ⠿⠠⠿⠿⠠⠿⠠⠠⠿⠿⠠⠿⠿⠠⠠⠿⠿
⠿⠿⠿ ⠿⠠⠿⠿⠠⠿⠠⠠⠿⠠ ⠠⠠ ⠿⠠⠿⠠⠿⠠⠠ ⠠⠠ ⠿⠠⠿⠠⠿
⠿⠿⠿ ⠠⠿⠠⠿⠿⠠⠿⠠⠿⠠⠠⠿⠠⠿⠠⠿⠠⠿⠠⠿⠠⠿⠠⠠ ⠠⠠ ⠠⠿⠠⠿⠠⠠⠿⠠⠿⠠
⠿⠿⠿ ⠠⠠⠿⠠⠿⠠⠿⠠⠿⠠⠠⠿⠠⠿⠠⠿⠠⠿⠠⠿⠠⠿⠠⠠⠿⠠⠿⠠⠿⠠⠿⠠⠿⠠⠿⠠⠿⠠
⠿⠿⠿ ⠠⠿⠠⠿⠿⠠⠿⠠⠠⠿⠠⠿⠠⠿⠠⠠⠿⠠⠿⠠⠿⠠⠿⠠⠠⠿⠠⠿⠠⠿⠠⠿⠠⠿⠠⠿⠠
⠿⠠⠿⠿ ⠠⠿⠠⠿⠿⠠⠿⠠⠠⠿⠠⠿⠠⠿⠠⠠⠿⠠⠿⠠⠿⠠⠿⠠⠠⠿⠠⠿⠠⠿⠠⠿⠠⠿⠠
⠿⠠⠿⠠ ⠠⠠⠿⠠⠿⠠⠿⠠⠿⠠⠠⠿⠠⠿⠠⠿⠠⠿⠠⠠⠿⠠⠿⠠⠿⠠⠿⠠⠿⠠⠿⠠
⠿⠠⠿⠠



EXERCISE 9

Exercise 9 will be available when this course is finished being written and is no longer "Provisional".

Proceed to Lesson 10.

LESSON 10

Read about this PROVISIONAL EDITION in the front matter to this book.
Check the NFB website periodically for updates to this lesson.

ARROWS

- *Construction of Braille Arrows*
- *Vertical, Slanted, and Curved Arrow Shafts*
- *Boldface and Compounded Arrows*

INTRODUCTION TO SPATIAL ARRANGEMENTS

- *Spatial Arrangements with Addition and Subtraction*
- *Introduction to Cancellation*
- *Arrangement on the Page*
- *Placement of Code Switch Indicators*

ARROWS

10.1 Arrows Used in Mathematics: Here are some examples of arrows which bear mathematical meaning.

$A \to B$ $\quad \overleftrightarrow{PQ}$ $\quad 2\mathrm{CO} \rightleftharpoons \mathrm{CO_2} + \mathrm{C}$ $\quad N \dashrightarrow M$

$f\colon a \mapsto b$ $\quad P \uparrow Q$ $\quad A \longrightarrow B \leftrightarrow C \longleftarrow D$

Construction of Braille Arrows

10.2 Introduction to the Shape Indicator: The shape indicator is brailled before an arrow symbol.

Shape Indicator	⠳

10.3 Horizontal Arrow Shafts: The length of a shaft is indicated by the number of times the braille arrow shaft symbol is used. Two braille symbols represent the ordinary shaft length for the particular publication. By comparison, one braille symbol represents a short shaft, and three or more symbols indicate a longer shaft.

Arrow Shafts		
Ordinary length, single	——	⠒⠒ ⠒⠒
Short, single	–	⠒⠒
Long, single	———	⠒⠒ ⠒⠒ ⠒⠒

Arrow Shafts, cont.

Ordinary length, double	═	⠶
Short, double	=	⠒
Long, double	═══	⠶⠶
Dashed	– –	⠤ ⠤
Dotted	. . .	⠂ ⠂ ⠂
Wavy	~~~	⠐⠒⠂

10.4 Arrowheads: An arrowhead may be barbed, blunted, straight, or curved, and may occur at the left, at the right, or at both ends of an arrow shaft. An arrowhead may also appear with only its upper or lower barb.

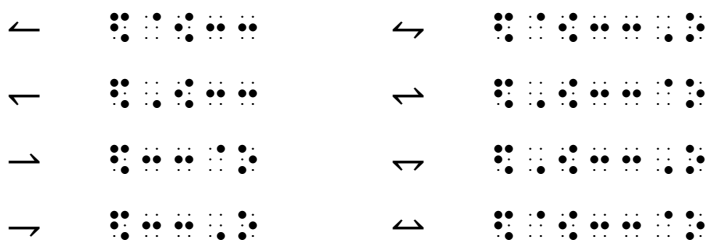
10.4.1 Barbed Arrowheads: This list shows the barbed arrowheads. Rule XXI of the Nemeth Code lists more arrowhead styles and the symbols used to represent them: blunted, curved, and straight arrowheads. (The shortest line in the illustrations below represent the end of a shaft.)

Barbed Arrowheads

Full, left-pointing	<	⠣
Full, right-pointing	>	⠜
Lower only, left-pointing	↖	⠨⠣
Lower only, right-pointing	↗	⠨⠜
Upper only, left-pointing	↙	⠡⠣
Upper only, right-pointing	↘	⠡⠜

10.5 Arrows With Barbed Ends: Consider the shaft in the barbed arrows shown below to be the standard length for the book. A two-cell "ordinary length" shaft is brailled. Notice the similarity between the print and braille symbols. (*Note:* Code switch indicators are omitted in the isolated examples in this section.)

→	⠳⠒⠒⠜	⇒	⠳⠶⠶⠜
←	⠳⠣⠒⠒	⇐	⠳⠣⠶⠶
↔	⠳⠣⠒⠒⠜	⇔	⠳⠣⠶⠶⠜

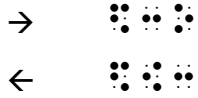


Note: An additional rule may apply to the first arrow shown above (→). See **10.7**, below.

By comparison, the shafts in these barbed arrows are longer. A three-cell shaft is brailled.

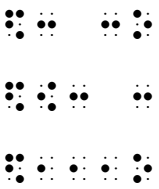


And, by comparison, the shafts in these barbed arrows are shorter. A one-cell shaft is brailled.

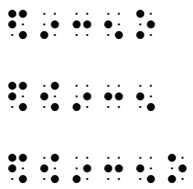


Other shaft styles are illustrated below.

Dashed and dotted: --→ ←-- ⇢



Wavy: ⇝ ⇜ ↭

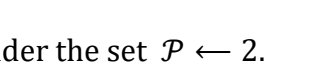


10.6 Spacing and Punctuation with Arrows: When arrows function as a sign of comparison they are spaced according to the rules governing comparison signs.

⋙ $x \rightharpoonup y$



⋙ $X \Rightarrow Y$



Example 10.6-1 Consider the set $\mathcal{P} \longleftarrow 2$.



The script letter P requires an English letter indicator even though the letter precedes a sign of comparison. (A typeform indicator must always be followed by an alphabetic indicator.)

When not functioning as a sign of comparison, an arrow is spaced according to its context. Punctuation associated with an arrow is mathematical.

⋙ (←, ↔, ⇒)



10.7 The Contracted Form of the Right-Pointing Arrow: When a right-pointing arrow in regular type, with a single shaft of ordinary length and a full barb, occurs by itself, it is represented in its contracted form.

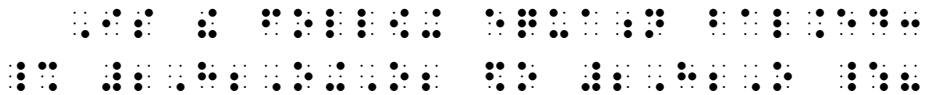
Contracted Arrow	

"Ordinary length" is determined by comparing the arrow shafts printed in the particular book being transcribed.

⋙ $A \longrightarrow B$ 

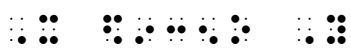
⋙ $p \longrightarrow (q \vee r)$ 

Example 10.7-1 Is the following equation balanced: $2H_2O + O_2 \longrightarrow 2H_2O$?



10.7.1 Nonuse of the Contracted Form of the Right-Pointing Arrow: All other right-pointing arrows require the use of all appropriate shaft symbols.

⋙ X ⇢ Y 

⋙ X ⇝ Y 

⋙ X ⟶ Y 

⋙ X → Y 

Instructions: Leave one space (one blank cell) between the arrows in A. and B. Assume shafts in items A, C, and D to be of regular length, and in item B to be long. Assume all to be in regular type even though they may appear darker. In item D, divide the long expression before the arrow that comes between the double parentheses.

PRACTICE 10A

Arrows with Barbed Ends

A. ⇔ ⇀ ↔ ↼ ⇢ ⇢ ↭ ↼

B. ⟵ ⟻ ⟻ ⟺

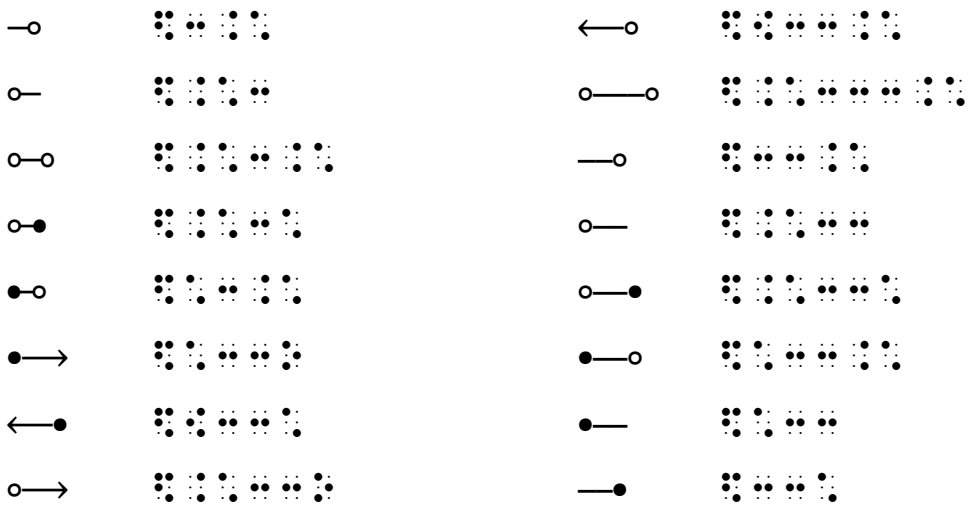
C. The principal operator of the left formula in Fig. 7.2 is ↔, while the principal operator of the right formulas is →.

D. Construct a truth table for $(p \to (q \to r)) \to ((p \to q) \to (p \to r))$.

10.8 Arrows With Dotted Ends: Arrows may also be represented by a shaft preceded or followed by a solid dot or by a hollow dot.

Solid Dot	•	⠁
Hollow Dot	○	⠈

An assortment of possibilities is shown below.



Note: Arrow symbols used with graphic number lines are <u>not</u> represented by the braille symbols shown above. Refer to the most recent edition of "Guidelines and Standards for Tactile Graphics" for techniques.

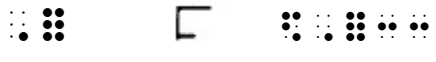
10.9 Arrows With Other Types of Arrowheads: Here are the blunted, straight, and curved arrowheads identified in the Nemeth Code. Examples with a single shaft of ordinary length are shown.

Blunted Arrowheads (the longer line represents the arrow shaft)

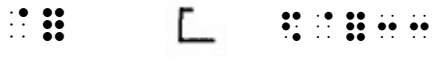
Full (left or right-pointing)



Lower only (left- or right-pointing)



Upper only (left- or right-pointing)

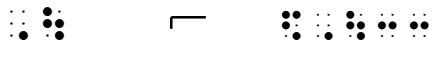


Straight Arrowheads (the longer line represents the arrow shaft)

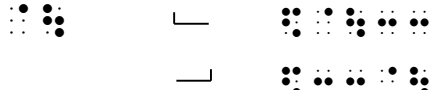
Full (left or right-pointing)



Lower only (left- or right-pointing)



Upper only (left- or right-pointing)

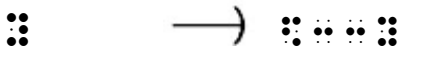


Curved Arrowheads

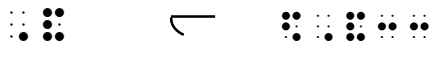
Full, left-pointing



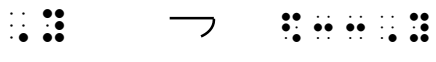
Full, right-pointing



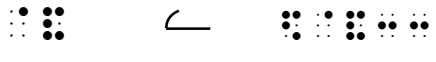
Lower only, left-pointing



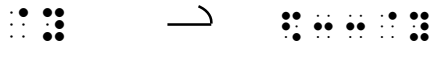
Lower only, right-pointing



Upper only, left-pointing



Upper only, right-pointing



⫸ a' ↦ b'

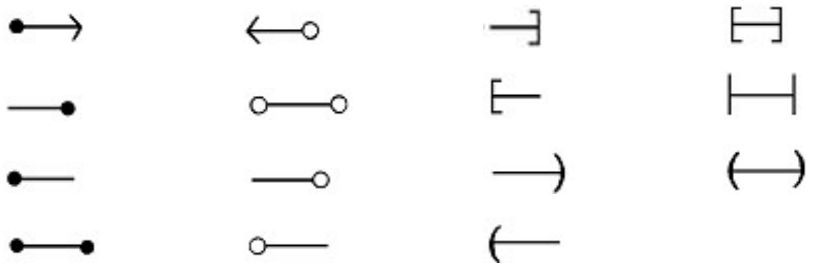
Example 10.9-1 Use $x \mapsto f(x)$ to denote that an element $x \in X$ is mapped to an element $f(x) \in Y$ by the mapping $f: X \to Y$.

In mapping notation, : is an unspaced colon.

Instructions: Transcribe as four columns, as printed. Assume all shafts are of regular length.

PRACTICE 10B

Other Types of Arrowheads



Vertical, Slanted, and Curved Arrow Shafts

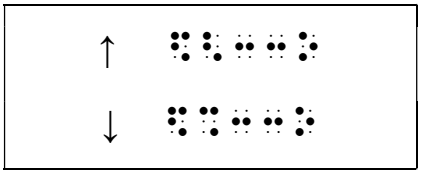
The arrows shown so far point horizontally – to the left, to the right, or both left and right. When an arrow points in a vertical or slanted direction, arrow direction indicators are used.

10.10 Arrow Direction Indicators: An arrow's direction is assumed to be horizontal unless a direction indicator is used. There are two types of direction indicators – one set for vertical arrows and one set for slanted arrows.

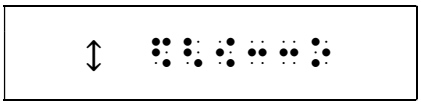
10.10.1 Vertical Arrow Directions: Vertical arrows require one of the following direction indicators. Nemeth Code calls these the "directly over" and "directly under" indicators. The direction indicator is brailled immediately following the shape indicator.

Pointing Up		(applies to the nearest arrowhead)
Pointing Down		(applies to the nearest arrowhead)

When a vertical arrow has *only one arrowhead*, the symbol for the *right-arrowhead* is used (dots 135). The "pointing up" indicator denotes a one-headed vertical arrow pointing up. The "pointing down" indicator denotes a one-headed vertical arrow pointing down.



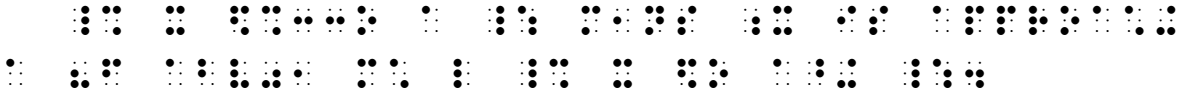
To show a two-headed vertical arrow (pointing up and down) the "pointing up" indicator is brailled before a two-headed arrow.



Functioning as a sign of comparison, the arrow is spaced accordingly.

$x \downarrow a = x \rightarrow a^{+}$ 

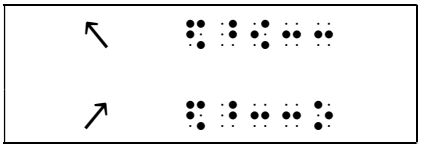
Example 10.10-1 $x \downarrow a$ means *x* is approaching *a* "from above", much like $x \rightarrow a^{+}$.



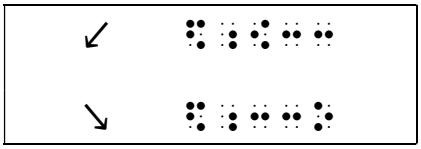
10.10.2 Slanted Arrow Directions: Slanted arrows require one of the following indicators. The indicator is brailled immediately following the shape indicator.

Superscript Indicator	⠘	(Elevates nearer arrowhead by 45 degrees)
Subscript Indicator	⠸	(Depresses nearer arrowhead by 45 degrees)

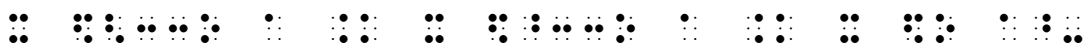
When a slanted arrow has only <u>one arrowhead</u>, the appropriate left- or right-pointing arrowhead is used. The first two examples show a one-headed arrow pointing *upward at a slant*. The superscript indicator is used to show the upward slant of the arrowhead.



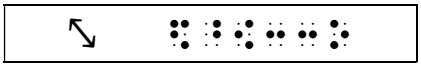
The next two examples show a one-headed arrow pointing *downward at a slant.* The subscript indicator is used to show the downward slant of the arrowhead.



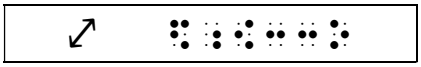
⋙ $x \uparrow a = x \nearrow a = x \rightarrow a^{-}$



To show a two-headed slanted arrow, the left-pointing arrowhead dictates the indicator used. In this example, because the <u>left arrowhead points upward</u>; the superscript indicator is used.



In the next example, because the <u>left arrowhead points downward</u>; the subscript indicator is used.



10.11 Curved Arrows: Direction indicators are not used with curved arrows. The direction of curvature is shown in braille by a left- or right-pointing arrowhead in the following manner. A curved shaft *preceded* by a *left*-pointing arrowhead represents a *clockwise* arrow. A curved shaft *followed* by a *right*-pointing arrowhead represents a *counterclockwise* arrow.

Curved Shaft		
Clockwise	↻ ⤾ ⤿	⠸⠒ ⠒⠇ ⠐⠒ ⠒⠂
Counterclockwise	↺ ⤿ ⤾	⠸⠒ ⠒⠇ ⠐⠒ ⠒⠂

PRACTICE 10C

Other Types of Arrow Shafts

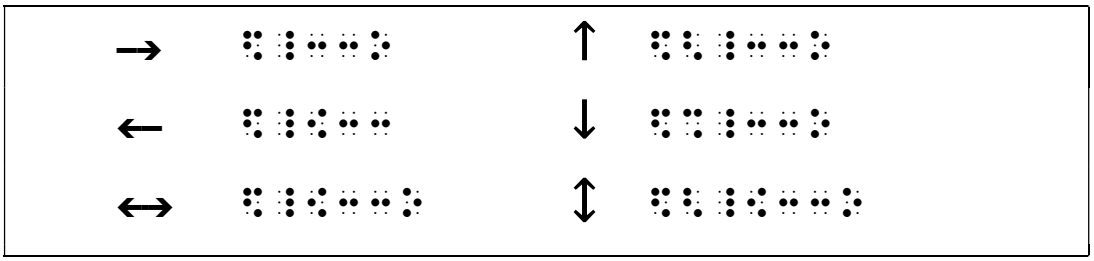
Vertical	Slanted		Curved
↕	↖	↘	↻
↑	↗	⤡	⤾
↓	↙	⤢	

Can you figure out how to construct this spear with a northwest, blunted arrowhead?



Boldface and Compounded Arrows

10.12 Boldface Arrows: When an arrow is printed in boldface type and the typeform is determined to be significant and therefore retained, the boldface typeform indicator (dots 456) is incorporated into the structure of the arrow. The typeform indicator is placed at the beginning of the arrow symbol, before the arrowhead or arrow shaft. When a direction indicator is required, the direction indicator is brailled first, followed by the boldface typeform indicator.

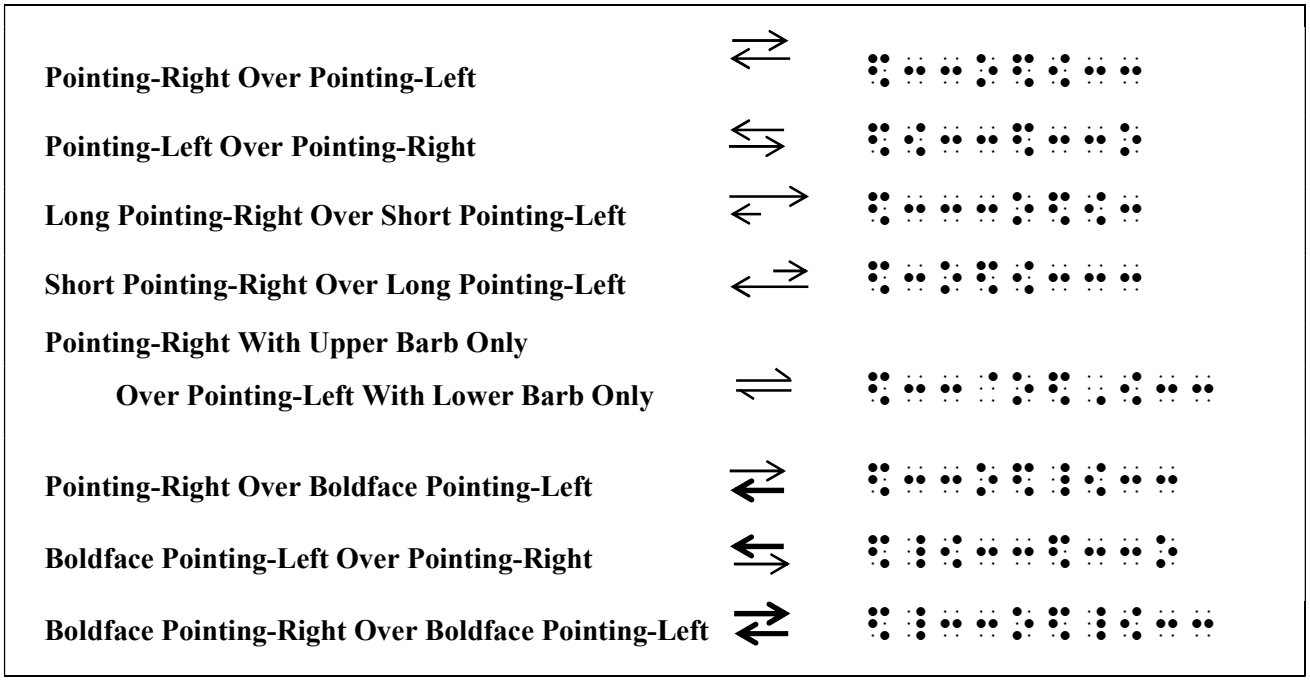


Reminder: A right-pointing arrow in nonregular type requires a shaft. The contracted form is not used.

10.13 Arrows Used as Signs of Comparison Compounded Vertically: Lesson 6 introduced signs of comparison compounded vertically—two or more simple signs of comparison arranged one under the other in print. When arrows are so arranged, the same principle applies. The combination of symbols becomes a single comparison sign "compounded vertically". The uppermost arrow symbol is brailled first, immediately followed by and unspaced from the symbol for the lower arrow. Each arrow requires a shape indicator.

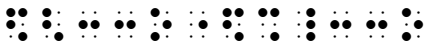
When the arrow is part of a sign of comparison compounded vertically, the uncontracted form of the right-pointing arrow shown below is used. The contracted form of the right-pointing arrow is never used in these constructions.





Arrows compounded vertically which are not shown in the list above should be transcribed in accordance with the principle outlined in this section.

10.14 Arrows Used as Signs of Comparison Compounded Horizontally: Lesson 6 also introduced the topic of signs of comparison compounded horizontally—two or more simple signs of comparison arranged side by side. When arrows are so arranged, the same principle applies. The combination of symbols becomes a single comparison sign "compounded horizontally". A multipurpose indicator (dot 5) is inserted between the unspaced symbols to indicate that the symbols are printed side by side, not one atop another. Arrows compounded horizontally which are not shown in the list below should be transcribed in accordance with this principle.

Pointing-Up Followed by Pointing-Down	↑↓	
Pointing-Down Followed by Pointing-Up	↓↑	
Pointing-Up Followed by Boldface Pointing-Down	↑↓	
Boldface Pointing-Down Followed by Boldface Pointing-Up	↓↑	

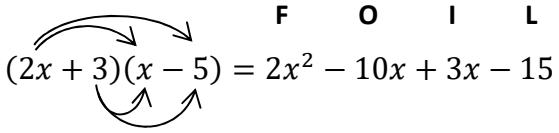
PRACTICE 10D

Boldface and Compounded Arrows

1. $\mathrm{HCl} \rightleftarrows \mathrm{H}^{+} + \mathrm{Cl}^{-}$
2. $+\mathrm{H_2O} \uparrow\downarrow -\mathrm{H_2O}$
3. $\mathrm{NaCl} \rightleftarrows \mathrm{Na}^{+} + \mathrm{Cl}^{-}$
4. $aA + bB \rightleftharpoons cC + dD$
5.  shows that the reaction begins at the left and is completed to the right.

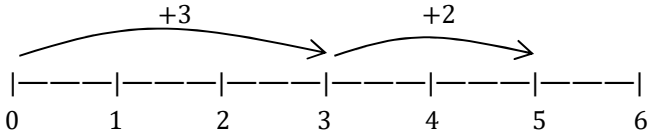
10.15 Other Arrows: It is recommended that Nemeth Code arrows not be used for nonmathematical purposes. Rules and guidelines in BANA sourcebooks examine flowchart arrows, arrows in graphic organizer diagrams, lead lines in tactile graphics, etc. Here are some samples to consider. (Transcriptions are not shown.)

Example 10.15-1



It may be better to present these arrows as a tactile graphic.

Example 10.15-2



Methods for depicting arrows above number lines are given in Guidelines and Standards for Tactile Graphics.

Example 10.15-3 $\frac{3}{4}$ ← *fraction line*

The lead line in this diagram could be omitted without a loss of information.

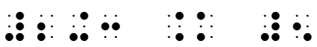
Refer to *Guidelines and Standards for Tactile Graphics* for techniques regarding arrows in specific applications such as box-and-whisker plots, clock faces, graphs, line plots, measurement tools, number lines, and spinners.

INTRODUCTION TO SPATIAL ARRANGEMENTS

10.16 Background: Up to this point we have looked at mathematical material that is read linearly, from left to right, whether it appears embedded within the narrative or set apart from the text as displayed material. When material is arranged on more than one line in print and there is essential vertical alignment of place values or characters, a spatial arrangement is required in braille. This lesson teaches rules for the transcription of spatially-arranged addition and subtraction problems. *Note: The term "alignment" refers to vertical location in the same column (cell).*

Spatial Arrangements with Addition and Subtraction

Lesson 1 introduced linear problems using the symbol + for "plus," – for "minus, and = for "equals".

$2 + 3 = 5$ 

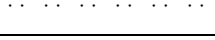
$7 - 6 = 1$ 

When addition and subtraction problems are printed in a vertical arrangement, Nemeth braille rules for spatial arrangements apply. We will use standard terminology to refer to the parts.

2	*addend*	7	*minuend*
+3	*addend*	−6	*subtrahend*
———	*separation line*	———	*separation line*
5	*sum*	1	*difference*

10.17 Numeric Indicator: The numeric indicator is generally not used in arrangements which are aligned for computation. Use of the numeric indicator in certain situations follows rules particular to the specific topic. This distinction will be clarified as each topic is presented.

10.18 Separation Line: The line printed between the addends and the sum or between the subtrahend and the difference (signifying "equals") is called the "separation line" and is brailled as a line of dots 25. The separation line must extend one cell to the left and to the right of the longest entry appearing above or below it.

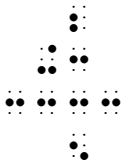
Separation Line (varying in length)	

10.19 Alignment with Addition and Subtraction: In spatial arrangements for addition and subtraction, the corresponding digits, commas, and decimal points are vertically aligned one below the other. In other words, digits under digits, commas under commas, decimal points under decimal points.

Examples Placement of code switch indicators with spatial arrangements is discussed later in this lesson. Assume Nemeth context in all examples in this section. The switch indicators are not shown.

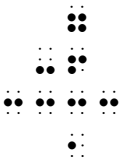
Example 10.19-1

```
  2
+3
---
  5
```



Example 10.19-2

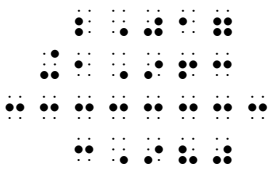
```
  7
-6
---
  1
```



Numeric indicators are omitted. Digits are aligned by place value, as printed. The separation line extends one cell to the left and one cell to the right of the widest row in the arrangement.

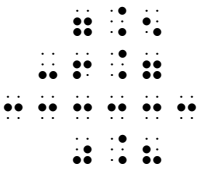
Example 10.19-3

```
  2,017
+1,963
-------
  3,980
```



Example 10.19-4

```
  7.5
-6.7
----
  0.8
```

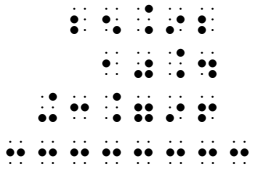


Commas and decimal points are aligned as printed. The separation line extends one cell to the left and one cell to the right of the widest row in the arrangement.

If items have been intentionally misaligned as an exercise for the student, the misalignment is preserved in the transcription. In the next example, the student has been instructed to arrange the digits for proper place value.

Example 10.19-5

```
  25.92
   10.4
+3.796
```

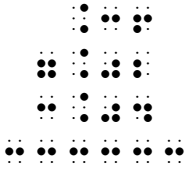


The decimal points are intentionally misaligned. The braille transcription matches the print arrangement.

The plus sign is not always present in addition problems. If there is no operation sign, examine the surrounding text to determine that this is indeed an addition problem. Then apply alignment rules for addition.

Example 10.19-6

```
  .36
 7.02
 3.04
----
```

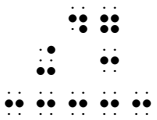


10.20 Placement of Symbols

10.20.1 Operation Symbols: Looking at the portion of the arrangement that lies above the separation line, the plus or minus symbol indicating addition or subtraction is placed at least one column of cells to the left of the widest column of numeric characters, regardless of print layout.

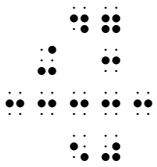
Example 10.20-1

```
47
+3
```



Example 10.20-2

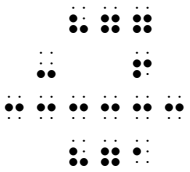
```
47
+3
---
50
```



The plus sign is printed directly to the left of the addend ("3"). In braille, the plus sign is placed one column to the left of the leftmost numeric character that appears above the separation line.

Example 10.20-3

```
877
- 6
---
871
```

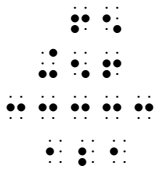


The minus sign is printed under the "8" in the minuend. In braille, the minus sign is placed one column to the left of the leftmost numeric character that appears above the separation line.

Placement of the operation symbol is in relation to the numeric characters above the separation line. If an answer is printed below the separation line, part of it may fall beneath symbols which appear above the separation line.

Example 10.20-4

```
 65
+56
---
121
```

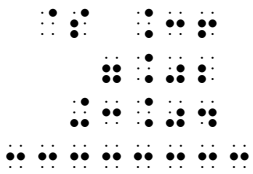


Place values "ones" and "tens" in the sum are aligned with those place values in the addends. The "hundreds" place value in the sum falls in the same column as the plus sign.

10.20.2 Currency Symbols: Currency symbols which appear above the separation line must also be similarly placed—that is, at least one column of cells to the left of the widest column of numeric characters which appears in the part of the arrangement above the separation line.

Example 10.20-5

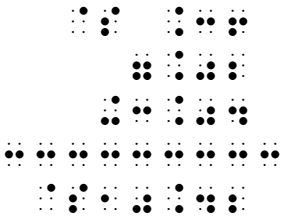
```
$ .36
 7.02
+3.04
```



Placement of the currency symbol applies to the numeric characters above the separation line. If an answer is printed below the separation line, part of it may be shown beneath the symbols which appear above it.

Example 10.20-6

```
 $ .36
  7.02
 +3.04
$10.42
```

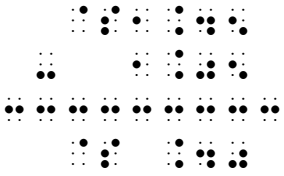


The decimal point and place values in the sum align with the decimal points and place values in the addends. The dollar signs above and below the separation line are not aligned in print and so need not be aligned in braille.

A symbol of operation or a currency symbol may be placed in the same position shown in print as long as the symbols above the separation line are placed at least one column of cells to the left of the widest column of numeric characters appearing there.

Example 10.20-7

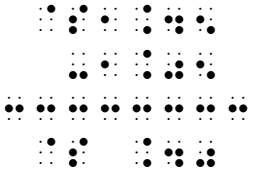
```
 $1.45
- 1.05
 $ .40
```



In print, the minus sign appears to the left of the dollar sign. This alignment is duplicated in braille.

Example 10.20-8

```
$1.45
-1.05
$ .40
```

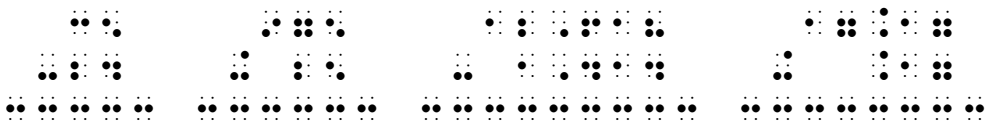


In print, the minus sign appears directly beneath the dollar sign in print and the two dollar signs are vertically aligned. This alignment is duplicated in braille.

10.21 Side-by-Side Layout: Spatial arrangements may be placed side by side across a page. One blank space is left between the end of one separation line and the beginning of the next. This assures that at least three blank cells come between any symbol on any line of one spatial arrangement and any symbol on any line of a neighboring arrangement (separation lines excluded).

Example 10.21-1

```
 35      975      12,618      17.17
-24      +25    - 1,414      +  .17
```

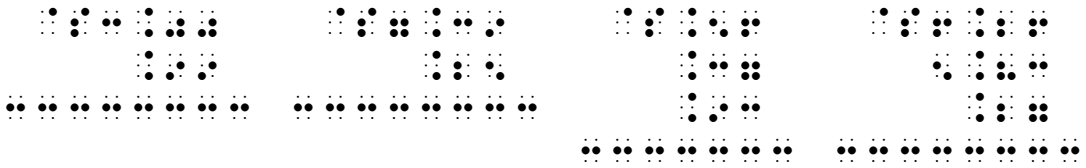


Each separation line extends one cell to the left and one cell to the right of the widest row in its arrangement. One clear column of blank space is between the ends of the separation lines.

Even if the separation lines are not on the same braille line, at least one clear column of blank space (one cell's width) must be between the ends of the separation lines.

Example 10.21-2

```
$3.00    $7.39    $.56     $6.26
  .99      .25     .37      5.83
                    .93      .27
```

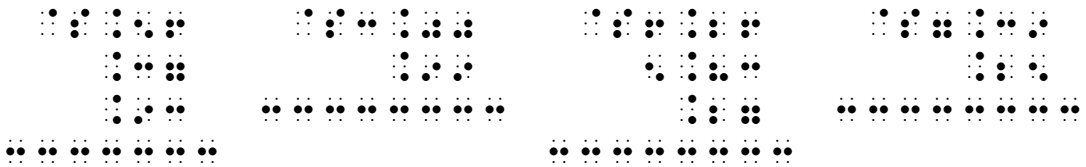


One clear column of blank space is between the ends of the separation lines, even those that do not fall on the same braille line.

The first addend in each problem is brailled on the same line, regardless of the print arrangement.

Example 10.21-3

```
$.56              $6.26
 .37      $3.00     5.83      $7.39
 .93        .99      .27        .25
```



In print, the problems are arranged so that all separation lines are across the same row.

10.21.1 Page Number Restriction: If any portion of a problem falls on lines 1, 2, 24, or 25 of the braille page, at least three blank spaces must be left between the symbol furthest to the right of the overall arrangement including any separation lines and the first symbol of the page number. If this spacing cannot be achieved, the arrangement must end on line 23 or begin on line 3, respectively. An example is given in **10.32**.

10.22 Blank Lines: A blank line is required above and below a spatial arrangement. Practice 10E demonstrates.

Instructions: Braille an opening Nemeth Code indicator on line 1 (cell 1). Line 2 should be blank. Begin the problems on line 3. Use side-by-side layout. Insert one blank line between the two sets of problems. After completion of the practice, leave one line blank and braille a Nemeth Code terminator in cell 1.

PRACTICE 10E

```
 900        4391       25.763      718       5,025
+100        + 81       -4.239     -437      +3,999
1000        4472       21.524      281       9,024

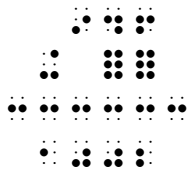
$ .27      $3,854      $14.99
 6.13         602     -  6.37
 2.15       + 918      $ 8.62
```

10.23 Omissions in Work Arranged Spatially for Computation: When a spatial arrangement contains omissions, in order to maintain necessary alignment *only the general omission symbol is used* regardless of how the omission is denoted in print. The number of general omission symbols to be used must be the same as the number of omission signs which occur in print.

General Omission Symbol	⠻

Example 10.23-1

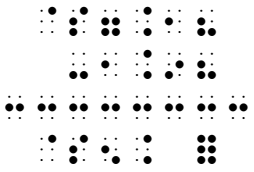
```
 946
 +??
1002
```



In print, the omissions are indicated as two questions marks.

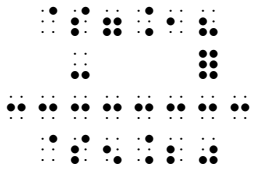
Example 10.23-2

```
$7.18
-1.98
$5.__
```



Example 10.23-3

```
$7.18
-  ...
$5.20
```

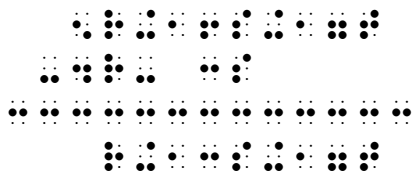


A long dash or an ellipsis does not give a clue regarding the number of characters that are omitted, therefore only one general omission symbol is brailled, aligned to the right.

10.24 Polynomials: In polynomials arranged spatially for addition or subtraction, terms and symbols are aligned.

Example 10.24-1

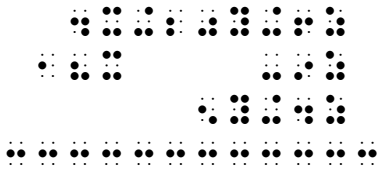
```
 5r+16s+17t
-4r- 3s
  r+13s+17t
```



If terms and symbols are not aligned in print, the braille transcription must show proper alignment unless the terms are intentionally misaligned in an exercise or illustration.

Example 10.24-2

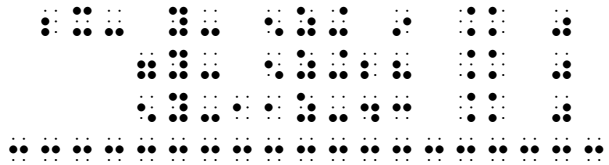
```
 4x+20y+6z
   18x-9z
    5y+4z
```



Terms are not aligned in this print example.

Example 10.24-3

```
 2x-y-5z+9 = 0
  7y-5z+28 = 0
 5y-11z-43 = 0
```

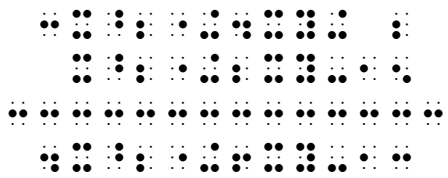


The separation line printed below these three equations characterizes this arrangement as an addition problem. Terms and symbols are aligned in braille even though, in this example, they are not aligned in print.

Superscripts, subscripts, and baseline indicators must align.

Example 10.24-4

```
 3x²+4xy+ 2
  x²+2xy-15
 4x²+6xy-13
```

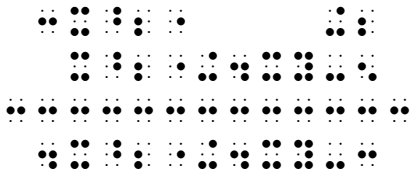


All indicators align, as well as coefficients, and variables.

A space used for alignment does not cancel the need for a baseline indicator. When the baseline indicator is required, it is placed in the first (leftmost) possible position consistent with this alignment. Any spacing required for alignment is placed following the baseline indicator.

Example 10.24-5

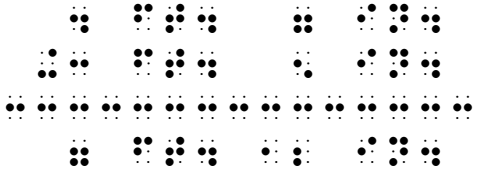
$$\begin{array}{r} 3x^2 \quad +2 \\ x^2+4xy-5 \\ \hline 4x^2+4xy-3 \end{array}$$



10.25 Abbreviations: When abbreviations occur in a spatially arranged addition or subtraction problem, they are vertically aligned and transcribed according to the rules for abbreviations.

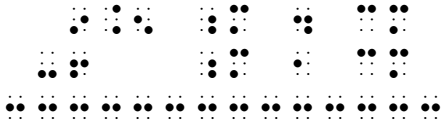
Example 10.25-1

$$\begin{array}{r} 4\ \text{ft.}\ \ 7\ \text{in.} \\ +\ 3\ \text{ft.}\ \ 5\ \text{in.} \\ \hline 7\ \text{ft.}\ 12\ \text{in.} \end{array}$$



Example 10.25-2

$$\begin{array}{r} 9.5\text{m}\ 4\text{cm} \\ -\ 6\ \ \text{m}\ 1\text{cm} \end{array}$$

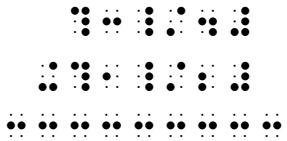


A space is left between an abbreviation and the numeral to which it applies, even if the print copy shows no space.

10.26 Fractions: In a spatial addition or subtraction arrangement containing fractions, the fraction lines and the corresponding parts of fraction indicators are vertically aligned.

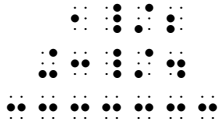
Example 10.26-1

$$\begin{array}{r} \frac{3}{4} \\ +\ \frac{1}{2} \\ \hline \end{array}$$



Example 10.26-2

$$\begin{array}{r} 1/2 \\ +3/4 \\ \hline \end{array}$$

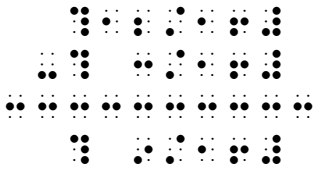


These fractions are printed in regular type and on the baseline of writing, therefore fraction indicators are not used in braille.

Each numerator must be right justified in the column reserved for numerators, and each denominator must be left justified in the column reserved for denominators. Fraction indicators must also be vertically aligned. Blank cells may need to be inserted in order to achieve alignment.

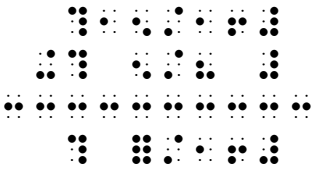
Example 10.26-3

$$\begin{array}{r} \frac{12}{16} \\ -\ \frac{3}{16} \\ \hline \frac{9}{16} \end{array}$$



Example 10.26-4

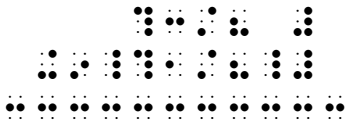
$$\begin{array}{r} \frac{15}{16} \\ +\ \frac{5}{8} \\ \hline \frac{?}{16} \end{array}$$



Fraction indicators must be right-justified in the columns reserved for both opening and closing indicators.

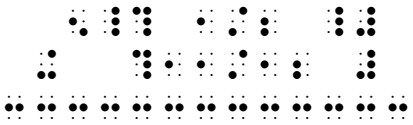
Example 10.26-5

$$\begin{array}{r} \frac{3}{8} \\ +\ 9\frac{1}{8} \\ \hline \end{array}$$



Example 10.26-6

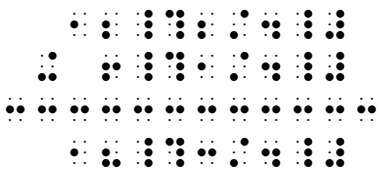
$$\begin{array}{r} 5\frac{1}{2} \\ +\ \frac{11}{12} \\ \hline \end{array}$$



The whole number parts of mixed numbers are vertically aligned by place value.

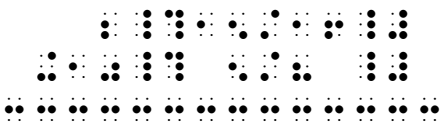
Example 10.26-7

$$\begin{array}{r} 12\frac{2}{4} \\ +\ 6\frac{1}{4} \\ \hline 18\frac{3}{4} \end{array}$$



Example 10.26-8

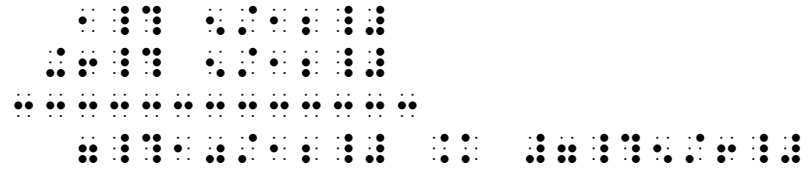
$$\begin{array}{r} 2\frac{15}{16} \\ +10\frac{5}{8} \\ \hline \end{array}$$



The final answer in the next problem is not part of the vertical computation. Rules for linear expressions are followed – a numeric indicator is required.

Example 10.26-9

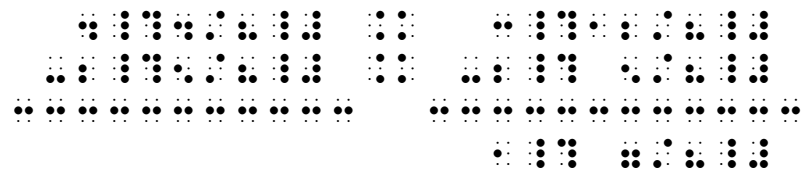
$$\begin{array}{r} 1\frac{5}{12} \\ +\,6\frac{5}{12} \\ \hline 7\frac{10}{12} = 7\frac{5}{6} \end{array}$$



The equals signs in the next problem show a relationship between the two spatial arrangements.

Example 10.26-10

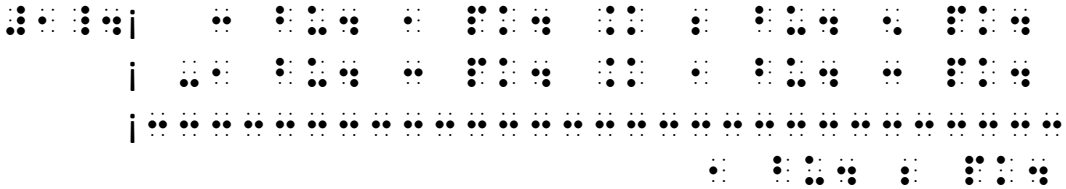
$$\begin{array}{rcl} 4\frac{4}{8} & = & 3\frac{12}{8} \\ -\,2\frac{5}{8} & = & -\,2\frac{5}{8} \\ \hline & & 1\frac{7}{8} \end{array}$$



10.27 Placement of Identifiers: The numeric or alphabetic identifier associated with a spatially arranged addition or subtraction problem is positioned at the top line of the problem—that is, with the first addend or with the subtrahend. One clear column of blank space (one cell's width) must be left between the last symbol in the identifier and the symbol furthest left in the overall arrangement including separation lines.

Example 10.27-1

```
1.  3 bu. 1 pk. = 2 bu. 5 pk.
   -1 bu. 3 pk. = 1 bu. 3 pk.
                  1 bu. 2 pk.
```

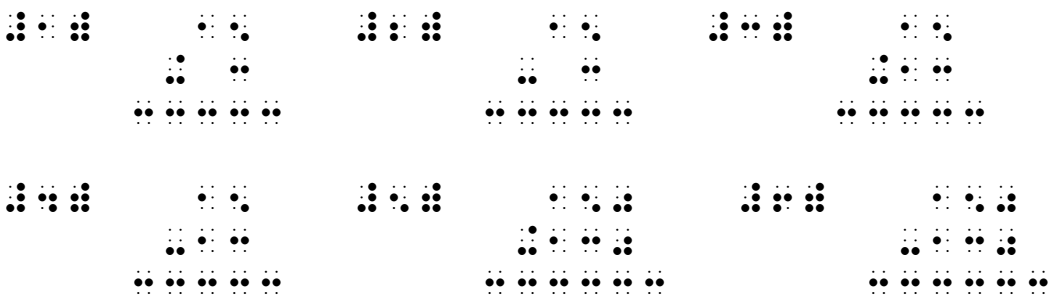


10.27.1 Side-by-Side Arrangement: Spatially arranged itemized material may be arranged side-by-side. The identifiers are located on the same braille line across the width of the page, in sequential order. Problems are spaced so that no less than three blank cells come between any symbol on any line of one spatial arrangement and any symbol on any line of a neighboring arrangement (separation lines excluded) including the neighboring identifier.

Note that this is a special rule pertaining to spatially-arranged itemized material. As taught in **Lesson 3**, each main item in *nonspatial* itemized material must start in cell 1, even if the print copy arranges the items side-by-side across the page.

Example 10.27-2

```
1)  15     2)  15     3)  15     4)  15     5)  150    6)  150
    +3         -3         +13        -13        +130       -130
```



A blank line precedes the second group of spatially-arranged problems (line 4, above). Spacing within and between problems is applied without regard to the arrangements above or below – that is, no attempt is made to align identifiers vertically.

10.27.2 Page Number Restriction: If any portion of a problem falls on lines 1, 2, 24, or 25 of the braille page, at least three blank spaces must be left between the symbol furthest to the right of the overall arrangement including any separation lines and the first symbol of the page number. If this spacing cannot be achieved, the arrangement must end on line 23 or begin on line 3, respectively. An example is given in **10.32**.

Instructions: Use side-by-side layout.

PRACTICE 10F

1) 621
+???
1096

2) 17x- 8y- z
- 2x+17y+6z
15x+ 9y+___

3) $3x^2$ -5x +4
$-5x^2$+12x-12
$-2x^2$ +7x -8

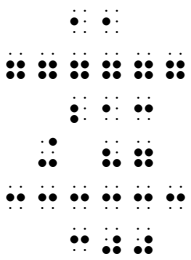
4) $1\frac{2}{3}$ yr
+ $5\frac{5}{12}$ yr
$6\frac{13}{12}$ yr = $7\frac{1}{12}$ yr

10.28 Carried Numbers with Addition: Carried numbers should appear in the same columnar position as in print. A line of carried number indicators the same length as the separation line is inserted between the carried numbers and the first line of the addition.

Carried Number Indicator (varying in length)	⠒⠒⠒⠒⠒⠒

Example 10.28-1

```
  11
  213
+ 87
 300
```

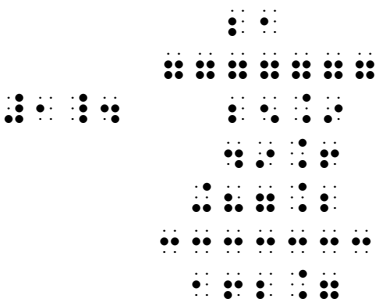


In the print copy, the carried numbers are in a smaller typeface and a carried number line is not present.

10.28.1 Placement of Identifiers with Carried Numbers: An identifier, if present, is placed on the first line of the addition problem (the first addend) regardless of the presence of carried numbers. One blank space must be left between the last symbol in the identifier and the symbol furthest left in the overall arrangement, including separation lines.

Example 10.28-2

```
    21
1.  25.9
    49.6
   87.2
  162.7
```



Introduction to Cancellation

A spatial arrangement is required whenever numbers, letters, abbreviations, or words in a mathematical expression are canceled in print by any type of stroke through them. The opening and closing cancellation indicators enclose the material being canceled.

Opening Cancellation Indicator	⠸
Closing Cancellation Indicator	⠼

⋙ ø ⠸⠼

⋙ ~~b~~ ⠸⠼

⋙ ~~ft~~ ⠸⠼

10.29 Cancellation in Subtraction Problems: When cancellation is shown in subtraction, the material should be aligned for computation. Spaces are inserted where necessary to achieve this.

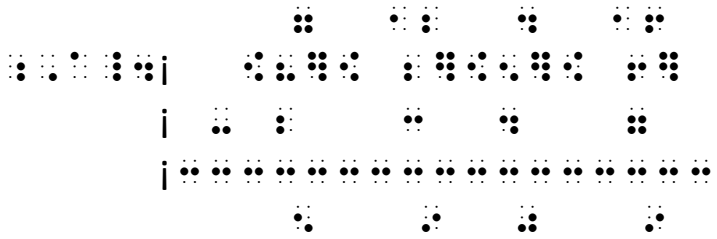
Example 10.29-1

```
  4 10
  1ø0
− 39
  111
```

⠼⠁⠚⠸⠼⠀⠼⠉⠊
⠼⠁⠸⠌⠚⠸⠼⠀⠸⠼⠊⠓
⠸⠼⠉⠊⠀⠼⠊⠓
⠤⠀⠼⠉⠊
⠒⠒⠒

Example 10.29-3 **A.**

```
   7 12  4 16
   8  2  5  6
 − 2  3  4  7
   5  9  0  9
```



PRACTICE 10G

Carried Numbers and Cancellation

1.

```
  1 1
  948
+  75
 1023
```

2.

```
   1 2  2
 $  2.52
   65.89
   91.99
   32.03
 $192.43
```

3.

```
   4 16  1 13
   5  6  2  3
 −    8  0  4
   4  8  1  9
```

Arrangement on the Page

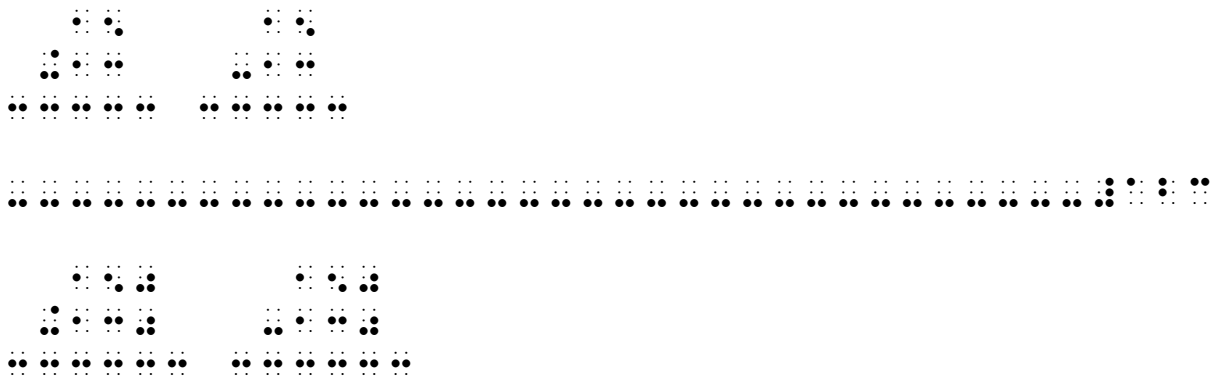
A spatial arrangement should be confined to one braille page. Strategies for larger arrangements are beyond the scope of this course.

10.30 Blank Lines and the Page Change Indicator: A blank line is required above and below a spatial arrangement. The presence of a page change indicator does not affect this requirement. There may be times when a blank line is needed both before and after the page change indicator; for example, when spatial arrangements occur before and after the page change, or when a heading starts the new print page.

Example 10.30-1

```
 15          15
+13         -13
================================ new print page 123
 150         150
+130        -130
```

(blank line)



(blank line)

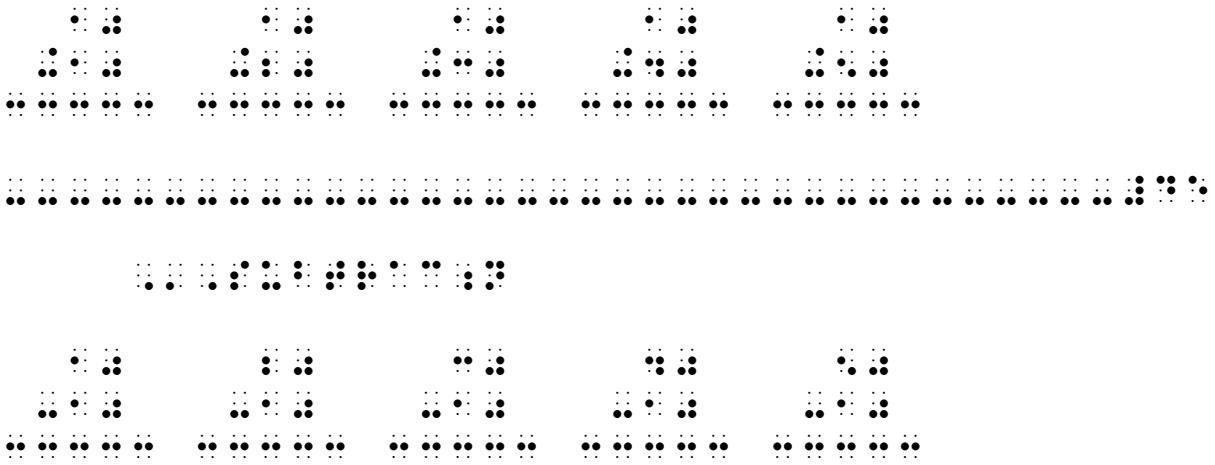
Each row of spatial problems requires a blank line before and after it.

Example 10.30-2

```
  10     10     10     10     10
 +10    +20    +30    +40    +50
====================================== new print page 45
          Subtraction

  10     20     30     40     50
 −10    −10    −10    −10    −10
```

(blank line)



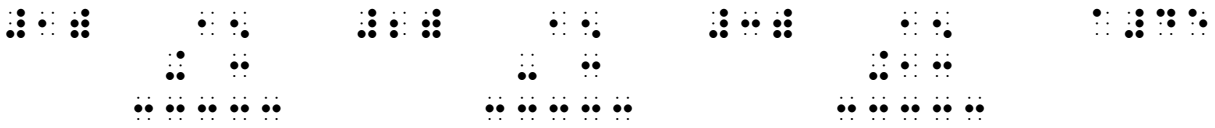
(blank line)

A blank line is required following the spatial problems at the end of print page 44 as well as before the cell-5 heading at the top of print page 45.

10.31 Pagination and Blank Lines: If there is no running head, transition to a new braille page takes the place of the required blank line above or below a spatial arrangement.

10.31.1 Starting a Braille Page with a Spatial Arrangement: When no running head is in use a spatial arrangement may begin on line 1 as long as there is room for at least three blank spaces between the symbol furthest to the right of the overall arrangement *including any separation lines* and the first symbol of the print page number.

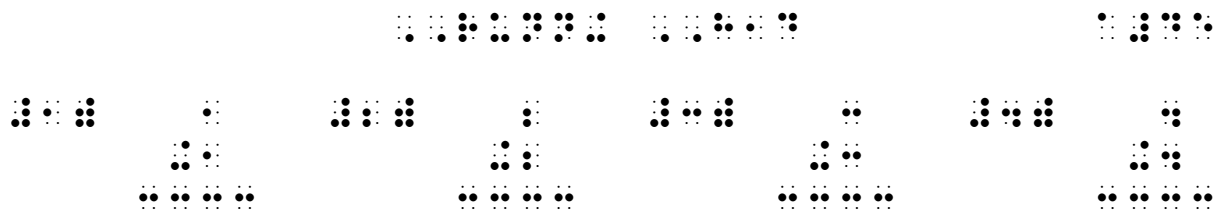
Example 10.31-1



(blank line)

If a running head is in use, line 2 must be blank, beginning the spatial arrangement on line 3.

Example 10.31-2



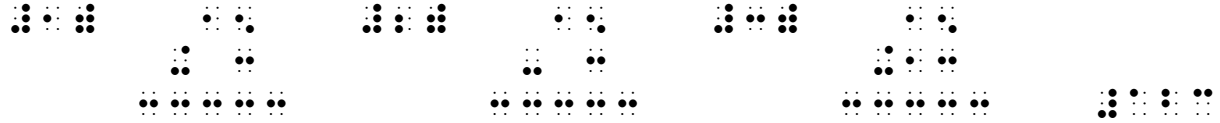
(blank line)

10.31.2 Ending a Braille Page with a Spatial Arrangement: A spatial arrangement may be placed on the last (bottom) line of the braille page as long as there is room for at least three blank spaces between the symbol furthest to the right of the overall arrangement *including any separation lines* and the first symbol of the braille page number.

Example 10.31-3

22 (blank line)

23 24 25



See the restriction regarding wide arrangements, below.

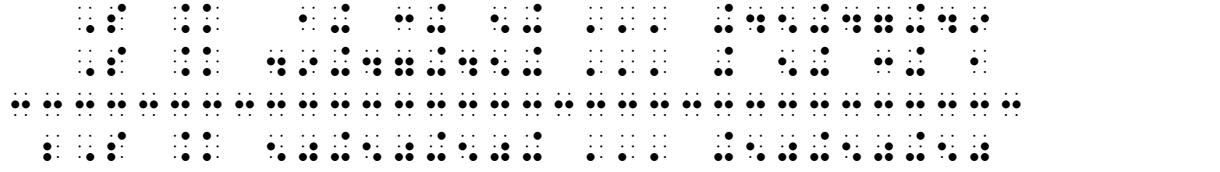
10.32 Wide Arrangements: When a spatial arrangement ends on the 24th or 25th line of a braille page or begins on the 1st or 2nd line of a braille page, at least three blank spaces must be left between the symbol furthest to the right of the overall arrangement *including any separation lines* and the first symbol of the page number. If this spacing cannot be achieved, the arrangement must end on line 23 or begin on line 3, respectively.

Example 10.32-1

```
 S =  1+ 3+ 5+ ... +45+47+49
 S = 49+47+45+ ... + 5+ 3+ 1
-----------------------------
2S = 50+50+50+ ... +50+50+50
```

... at the bottom of a braille page

20 21 22 23



24 (blank line)

25



... at the top of a braille page (with no running head)

1 ⠼⠉

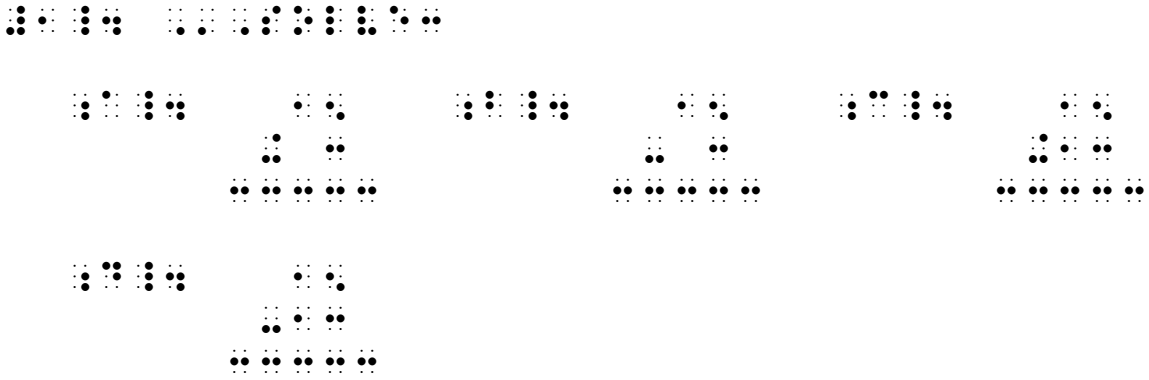
2 (blank line)

3 ⠠⠠ ⠠⠠ ⠠⠠

If there is material associated with the main division number, the subdivisions begin on the next available line (following the required blank line that precedes spatial material) starting in cell 3.

Example 10.33-3

```
1.  Solve: a.  15   b.  15   c.  15   d.  15
               +3       -3      +13      -13
```



Placement of Code Switch Indicators

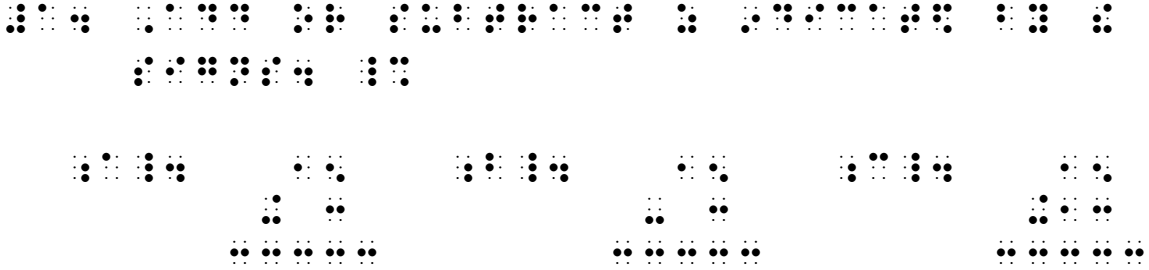
To keep the code switch indicators from interfering with the layout of the problem, they are placed outside of the arrangement. Position the required blank lines *within* the Nemeth Code switches as described below.

10.34 Opening Nemeth Code Indicator: If space permits, an opening Nemeth Code indicator that precedes an itemized spatial problem may be placed on the same line with the end of the text above the problem.

Example 10.34-1

```
1.  Add or subtract as indicated by the signs.

    a.  15   b.  15   c.  15
        +3       -3      +13
```



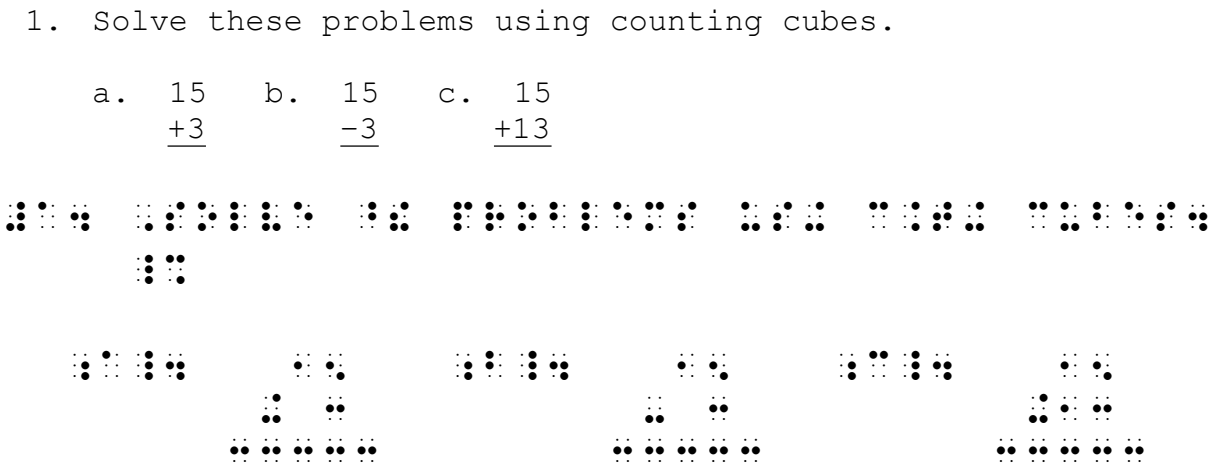
(blank line)

The opening Nemeth Code indicator is placed outside of the arrangement, before the required blank line.

The BANA Nemeth Code Technical Committee is discussing details regarding placement of the opening Nemeth Code indicator preceding itemized spatial arrangements when the indicator does not fit on the same line as the preceding text. The examples below show the opening switch in the runover position of the current paragraph.

If there is not room on the line with the preceding text, the opening Nemeth Code indicator is placed in the runover position. The opening switch indicator will be by itself on that line. The required blank line follows. The switch indicator does not take the place of the required blank line preceding the spatial arrangement.

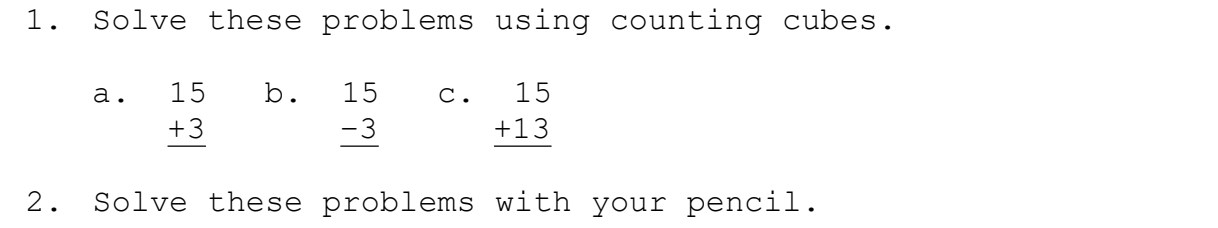
Example 10.34-2

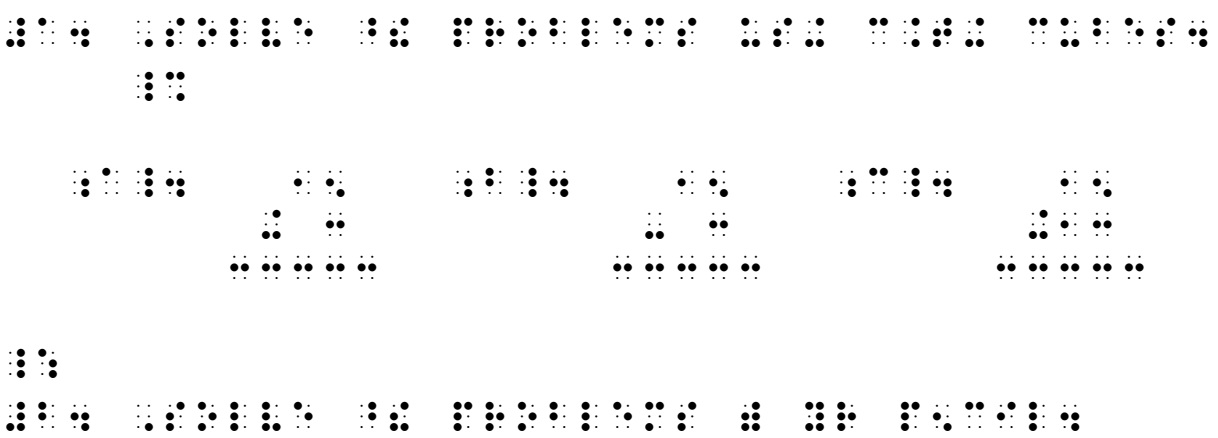


(blank line)

10.35 Nemeth Code Terminator: When Nemeth Code is terminated after a spatial problem, first insert the required blank line that follows the arrangement. Then place the Nemeth Code terminator in cell 1 by itself on the following braille line. The code switch indicator does not take the place of the required blank line following the spatial arrangement. Resume UEB text on the next line.

Example 10.35-1





The Nemeth Code terminator is placed outside of the arrangement, after the required blank line.

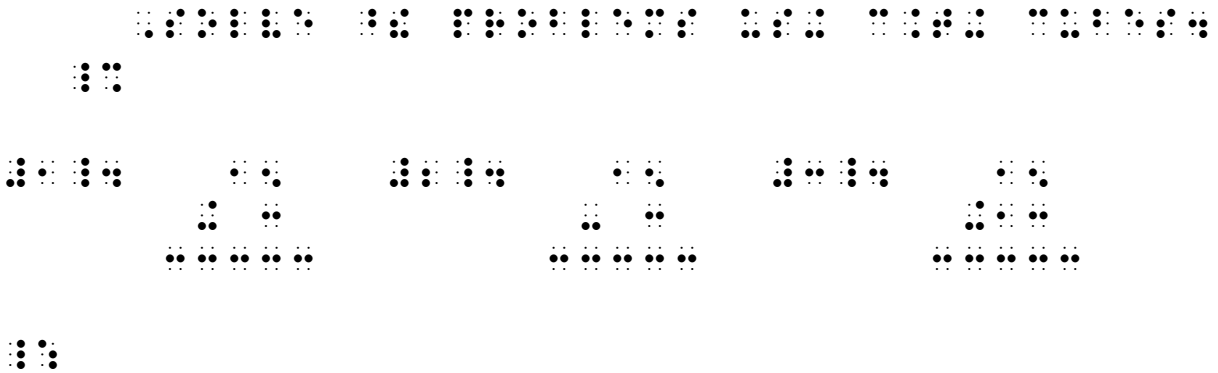
10.36 Braille Page Turn: It is preferable to have the indicators fall on the same braille page as the mathematical material.

10.37 Instructions Preceding Itemized Spatial Arrangements: Recall from **Lesson 6** that instructions begin in cell 5, with any runovers beginning in cell 3, and that one line is left blank above instructions unless they follow a cell-5 or a cell-7 heading. The related itemized material then follows on the next line unless the material itself requires a blank line before it. *One blank line must be left between instructions and a spatial arrangement below them.*

Example 10.37-1

Solve these problems using counting cubes.

```
1.  15   2.  15   3.  15
    +3       -3      +13
```



The opening switch indicator is placed in the runover position for "instructions" (cell 3) because it will not fit at the end of the previous line.

If instructions begin at the bottom of a braille page without room for the spatial arrangement which follows, at least one line of the instructions must be brought to the new page along with the spatial arrangement below it.

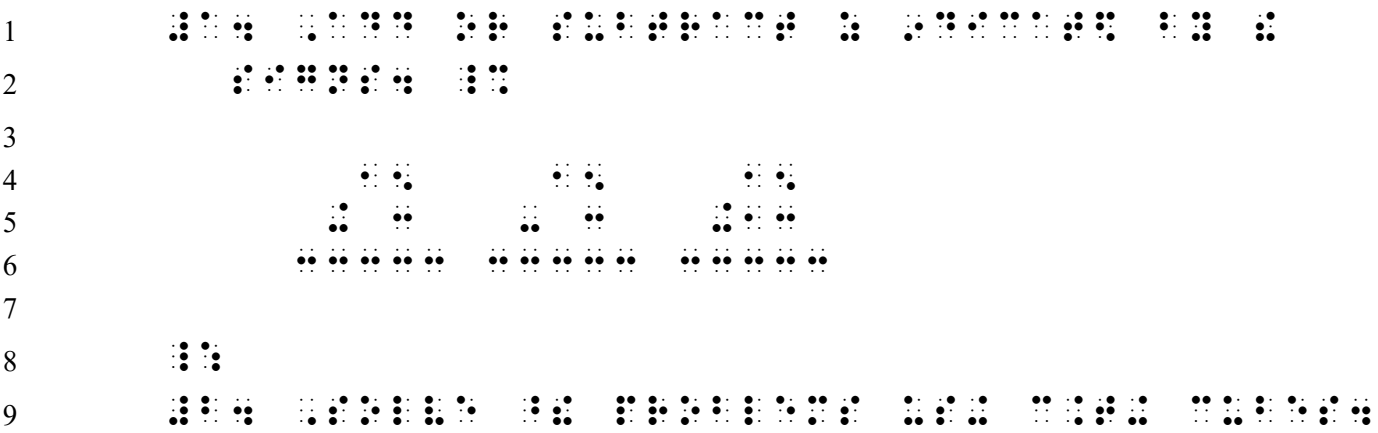
10.38 Displayed Spatial Arrangements: Displayed spatial material is transcribed using the same margin rules for non-spatial material**.** The leftmost cell used by the spatial arrangement should be placed in the appropriate cell for displayed material. *One blank line must be left before and after the spatial arrangement.*

Example 10.38-1

```
1. Add or subtract as indicated by the signs.

       15     15     15
      +3     -3    +13

2. Solve these problems using counting cubes.
```



Line 1: Identifier (main item) in cell 1.

Line 2: Runover starts in cell 3 because there are no subitems. Opening Nemeth Code indicator follows on same line.

Line 3: Blank line precedes spatially-arranged material.

Lines 4-6: The leftmost cell of the displayed material (line 6) is in cell 5, which is the proper margin for "itemized material with no subdivisions". Spatial material may be brailled side-by-side, with one blank cell between separation lines.

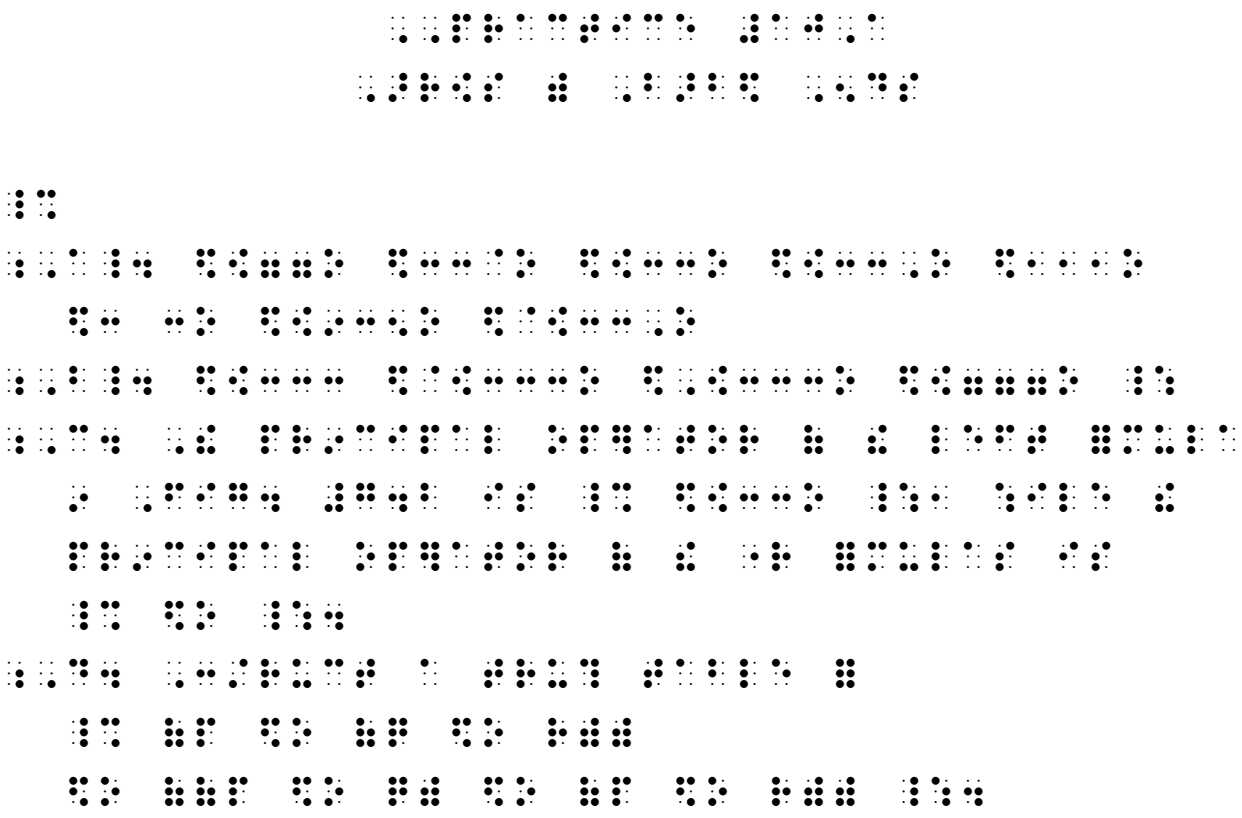
Line 7: Blank line follows spatially-arranged material

Line 8: The Nemeth Code terminator is in cell 1, placed outside of the arrangement (after the required blank line).

Line 9: Item #2 begins in cell 1 on a new line. (Identifiers should each begin in the same cell.)

For further practice, see Appendix A—Reading Practice.

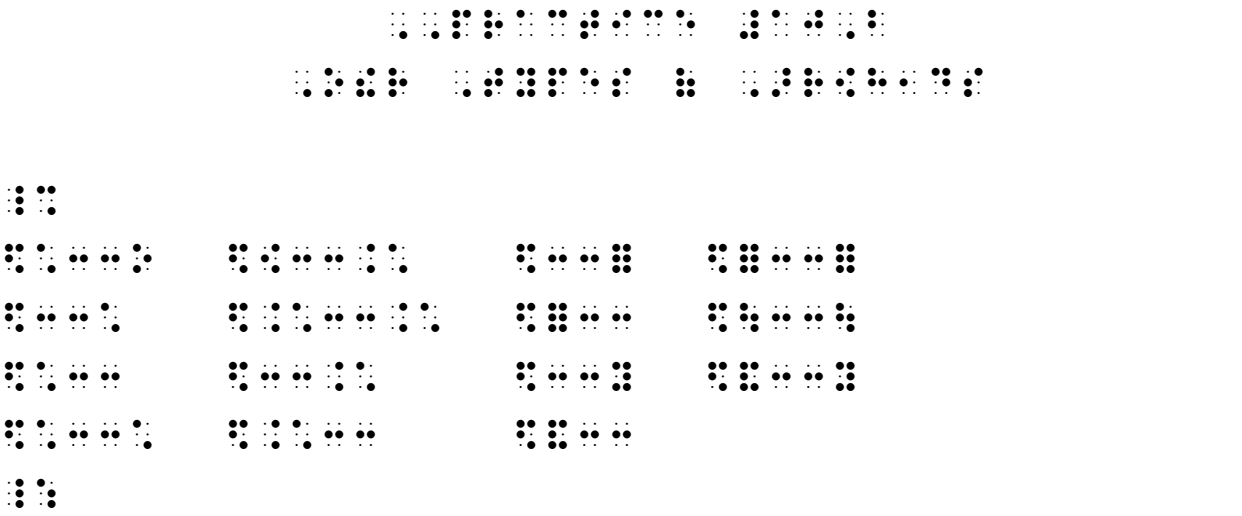
ANSWERS TO PRACTICE MATERIAL

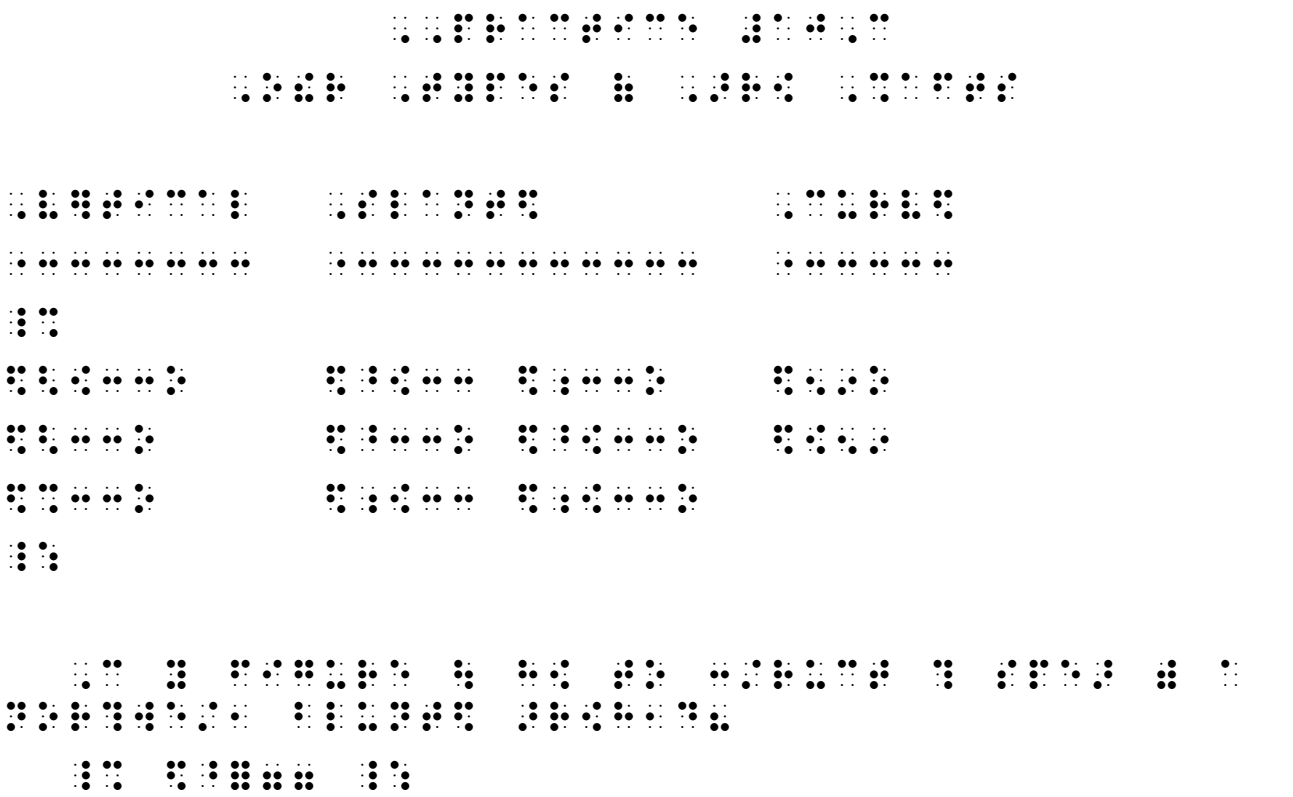


NOTES

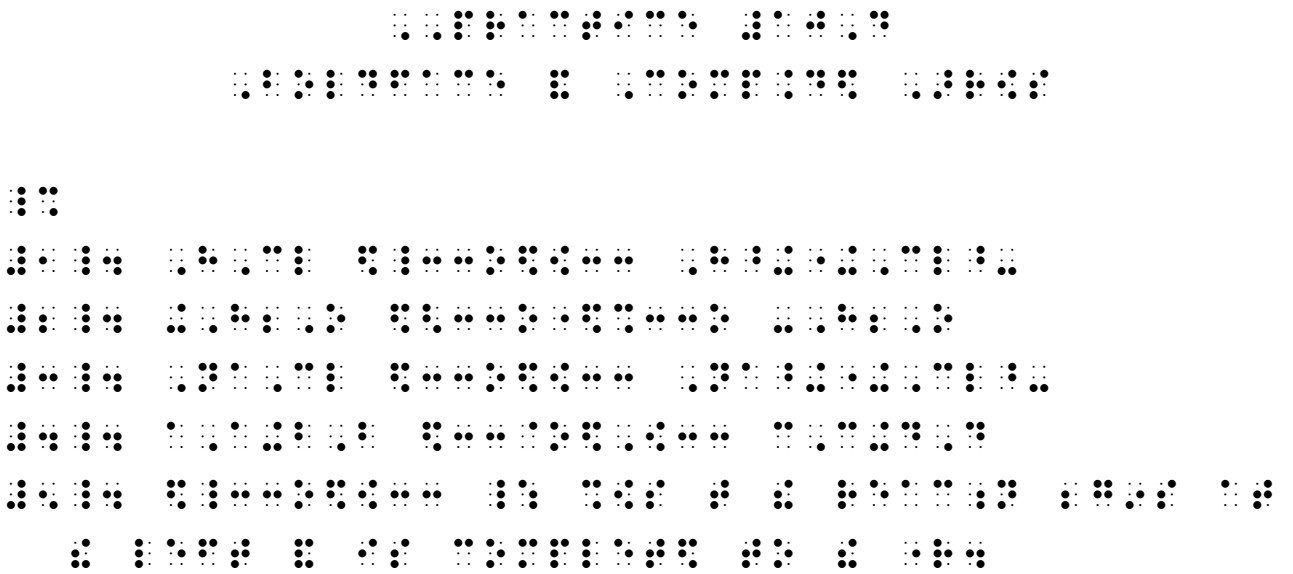
A. The dashed shaft must not be divided between lines.

D. This long expression will not fit on one line. The arrows are comparison signs, making this is a "linked expression". In order to organize this expression into logical mathematical units, divide before the main sign of comparison – the third arrow in this case.



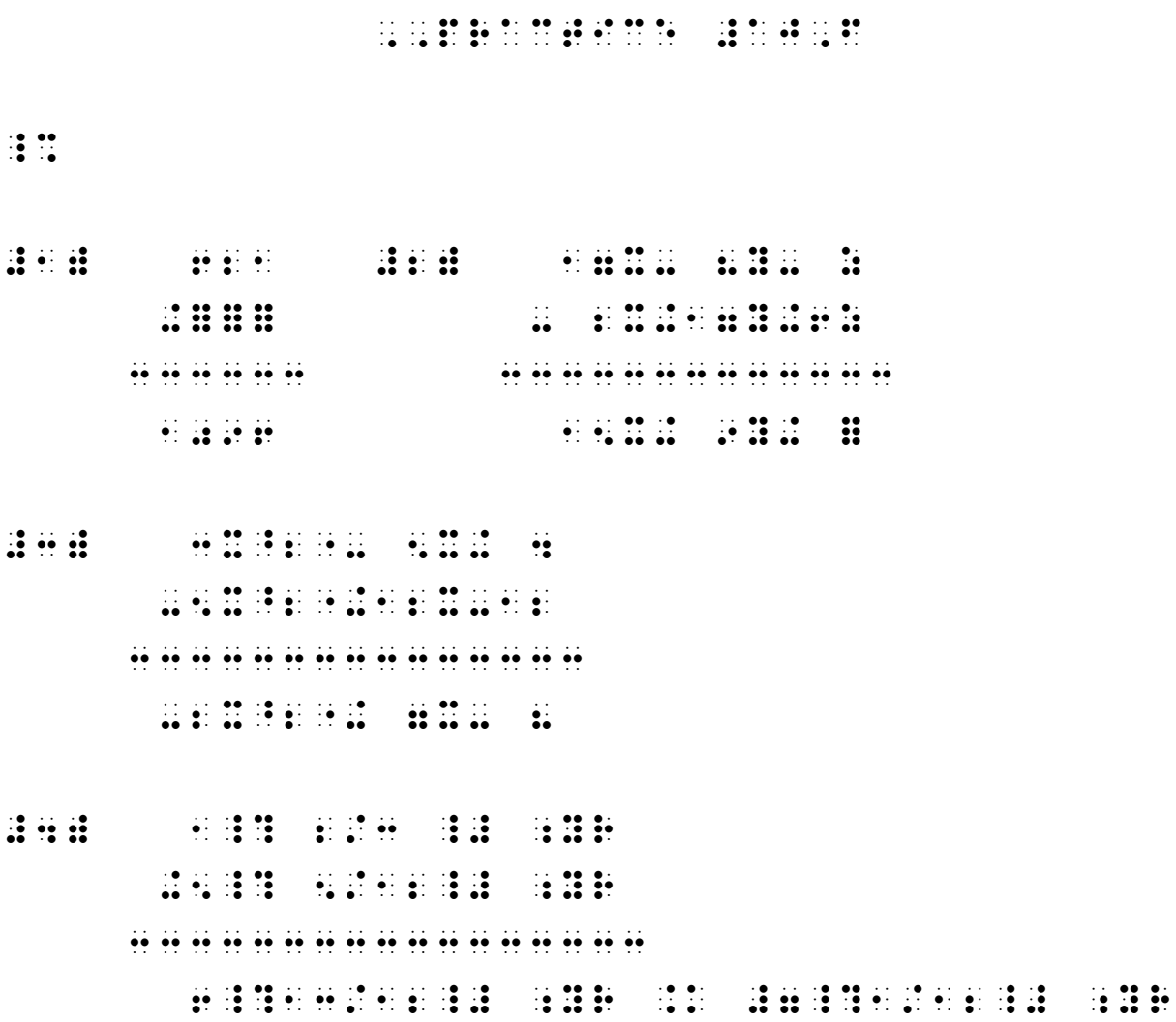


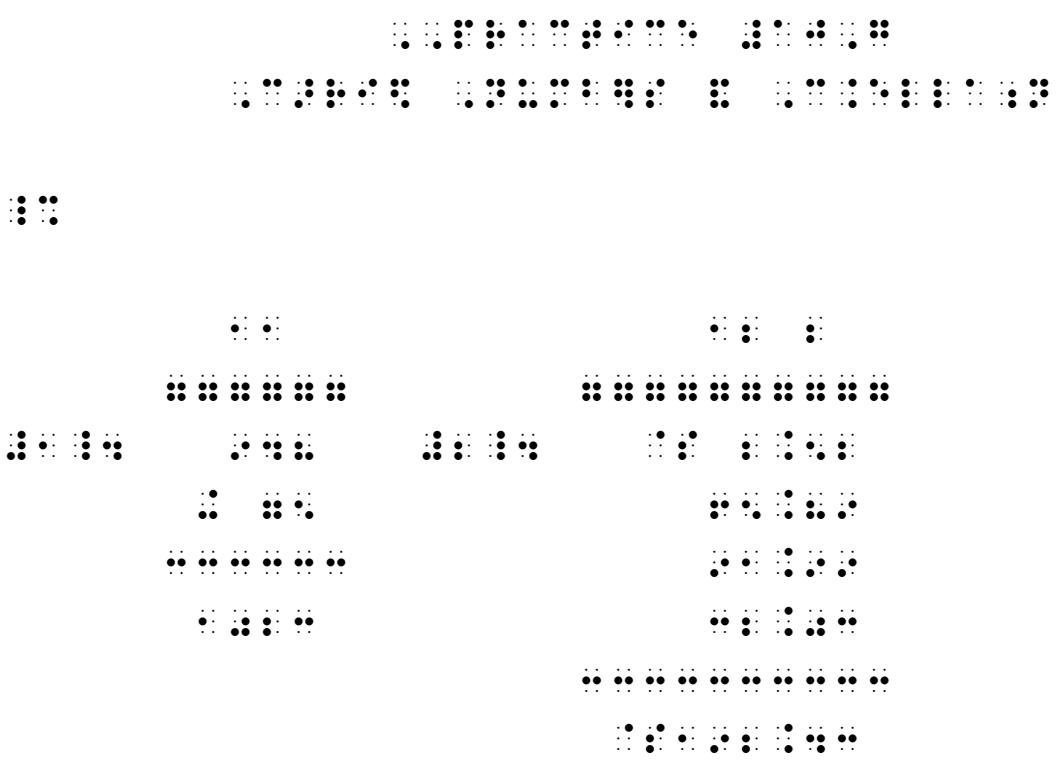
NOTE: It is acceptable to have two blank cells between items in the second column.

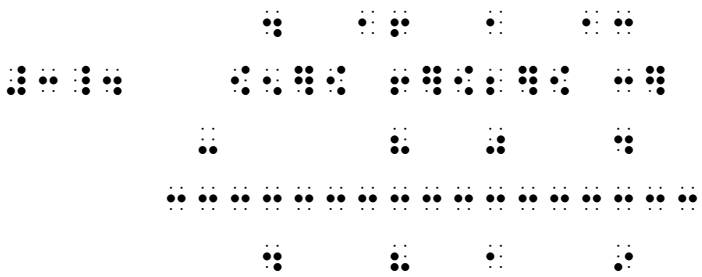


⠠⠠

⠠⠠⠠







EXERCISE 10

Exercise 10 will be available when this course is finished being written and is no longer "Provisional".

Proceed to Lesson 11.

LESSON 11

Read about this PROVISIONAL EDITION in the front matter to this book.
Check the NFB website periodically for updates to this lesson.

SIGNS OF SHAPE

- *Basic Shapes*
- *Shapes with Structural Modification*
- *Shapes with Interior Modification*
- *Other Details*
- *Calculators and Keyboards*
- *Icons*
- *Shapes Used as Signs of Omission*
- *Identified Signs of Shape*

TYPEFORMS

- *Labeled Mathematical Statements*
- *Typeform Indicators for Words/Phrases*

FORMAT

- *Displayed Material with Labels*
- *Labeled Mathematical Statements*

SIGNS OF SHAPE

11.1 Definition: A sign of shape is a miniature picture of a geometric figure or an object.

△ (triangle) ∠ (angle) ○ (circle)

Basic Shapes

A basic shape is represented in braille by the shape indicator followed by a numeral, one or more letters, or a dot combination suggestive of the shape.

Shape Indicator	⠸

11.2 Basic Signs of Shape Represented by Numbers—Regular Polygons: A closed figure that has equal sides and equal angles is called a *regular polygon* and is represented by the shape indicator followed by a numeral specifying the number of sides in the figure.

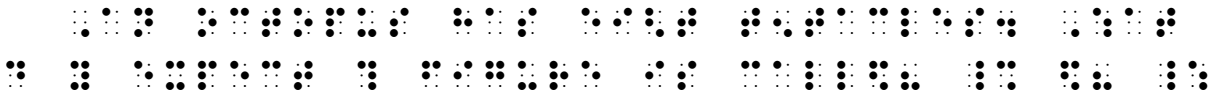
Square (4-sided)	□	⠸⠲
Regular Pentagon (5-sided)	⬠	⠸⠢
Regular Hexagon (6-sided)	⬡	⠸⠖

*Note that the equilateral triangle, which is a regular polygon, is not represented by the number three. See **11.4**.*

11.2.1 Unlisted Regular Polygons: Symbols which represent regular polygons with seven or more sides are not provided for in the Nemeth Code. If the unlisted shape is a *regular polygon*—that is, it is a closed figure with equal sides and equal angles—the transcriber is instructed to devise a symbol in accordance with the principles above, based on the number of sides the shape has. It may be helpful to include a tactile drawing of the shape. Unlisted regular polygon constructions do not require a transcriber's note.

Example 11.2-1 An octopus has eight tentacles. What do you expect this figure is called?

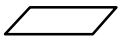






In addition to the transcriber-devised braille symbol, the shape is presented as a tactile graphic at the first mention of this shape. Follow directives in the most recent edition of Guidelines and Standards for Tactile Graphics.

11.3 Basic Signs of Shape Represented by Letters—Irregular Polygons: A closed figure which has at least two unequal sides and/or two unequal angles is called an *irregular polygon* and is represented by the shape indicator followed by a letter or a combination of letters suggestive of the name of the shape. (The derivation of the letter following the shape indicator is underlined in the list below.)

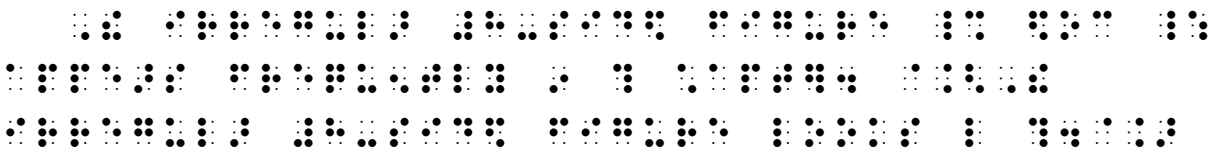
Diamond		
Irregular Hexagon		
Irregular Pentagon		
Parallelogram		
Quadrilateral		
Rectangle		
Rhombus		
Trapezoid		

We apologize for the blurry images in this lesson. We hope to provide better graphics in future editions.

11.3.1 Unlisted Irregular Polygons: You may come across an irregular polygon which is not provided for in the Nemeth Code. If the unlisted shape is an *irregular polygon*—that is, it is a closed figure with at least two unequal sides and/or two unequal angles—the transcriber is instructed to devise a symbol if it appears frequently in the transcription. Construct the symbol in accordance with the principles above. Be careful not to choose a letter or letter combination which already has an assigned meaning in the Nemeth Code. Refer to Appendix B of the Nemeth Code for a list of symbols already in use. Symbols beginning with dots 1246 begin on page 222.

A transcriber's note is required to define the figure unless it is described in the narrative. In your note, give the name or description of the symbol used. Include a drawing of the shape when appropriate.

Example 11.3-1 The irregular 8-sided figure  appears frequently in this chapter.





A raised outline of the shape is included as a tactile graphic at the first mention of this shape. Follow directives in the most recent edition of Guidelines and Standards for Tactile Graphics.

11.4 Other Basic Signs of Shape Represented by Letters: The following shapes are also represented by the shape indicator followed by a letter suggestive of the name. (The derivation of the letter following the shape indicator is underlined.)

Circle	○	
Ellipse	⬭	
Intersecting Lines	×	
Star	☆	
Triangle	△	

Clarification: The triangle shape shown here is an equilateral triangle – sides are equal in length, angles are equal in measurement. The braille symbol  is used to represent a triangle shape when the print copy uses the drawing to replace the word "triangle" or when it is used to specifically represent an equilateral triangle. In the study of triangles, other types of triangles named. These are discussed in **11.7**.

The following shapes were introduced in **Lesson 6** as signs of comparison. They may also be used in print to simply replace the word they represent. Notice that two signs in this category begin with the negation symbol (34) immediately followed by the shape indicator.

Arc, Concave Upward	⌒	⠸⠩
Is Parallel To	∥	⠸⠳
Is Not Parallel To	∦	⠰⠸⠳
Is Perpendicular To	⊥	⠸⠾
Is Not Perpendicular To	⊥̸	⠰⠸⠾

Example 11.4-1 Line AD ∦ Line BC.

⠼⠁ ⠠⠁⠙ ⠸⠩ ⠠⠃⠉ ⠠⠁⠙ ⠸⠳ ⠠⠃⠉

11.4.1 Other Unlisted Basic Shapes: Basic shapes not provided for in the Nemeth Code are formed in accordance with the principles above. One must be careful not to choose a symbol which already has an assigned meaning in the Nemeth Code. Refer to Appendix B of the Nemeth Code for a list of symbols already in use. Symbols beginning with dots 1246 begin on page 222; symbols beginning with dots 34 are on page 231.

A transcriber's note is required to define the figure. Give the name or description of the symbol used. Include a drawing of the shape if it is vital to the mathematical topic at hand.

Example 11.4-2 VALENTINE MATH: Replace the heart with the correct math symbol: $14 \heartsuit 2 = 7$.

⠠⠧⠁⠇⠑⠝⠞⠊⠝⠑ ⠍⠁⠞⠓⠒ ⠗⠑⠏⠇⠁⠉⠑ ⠞⠓⠑ ⠓⠑⠁⠗⠞
⠺⠊⠞⠓ ⠞⠓⠑ ⠉⠕⠗⠗⠑⠉⠞ ⠍⠁⠞⠓ ⠎⠽⠍⠃⠕⠇⠲ ⠼⠁⠙ ⠸⠪ ⠼⠃ ⠨⠅ ⠼⠛
⠼⠁⠙ ⠸⠪ ⠼⠃ ⠨⠅ ⠼⠛

The transcriber represents the heart shape with ⠸⠪ *since* ⠸⠩ *means "rhombus".*

Note: Refer to the most recent edition of *Guidelines and Standards for Tactile Graphics* regarding picture objects (such as counting symbols, pictographs, etc.) used in Kindergarten through third grade materials.

11.5 Basic Signs of Shape Represented by Other Dot Combinations: Three additional shapes are identified in the Nemeth Code.

Angle	∠	
Arc, Concave Downward	◡	
Inverted Triangle	▽	

11.6 Filled-In and Shaded Shapes: A filled-in or shaded closed shape (circle, diamond, square, etc.) is represented as such by the filled-in shape indicator or the shaded shape indicator. The appropriate indicator is placed between the shape indicator and the shape symbol.

Filled-in shape indicator	
Shaded shape indicator	

 (filled-in star)

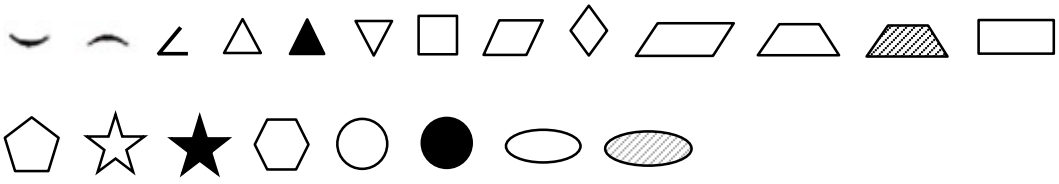
 (shaded circle)

Shapes used as icons in non-mathematical context are discussed later in this lesson. See **11.26**.

Instructions: Leave one space between each shape. Braille as many shapes on one braille line that will fit before beginning a new line. Use "fl" to represent the flower shape and "ch" to represent the chicken. The required transcriber's note may be omitted in this practice exercise.

PRACTICE 11A

Listed Shapes



Unlisted Shapes



Shapes with Structural Modification

11.7 Definition and Construction: A shape with structural modification is one in which the general print form of a basic shape (such as *triangle*) is changed to show a more specific form (such as *right triangle*).

Basic shape: Triangle 

More specific form: Right Triangle 

Composite signs in which two or more signs of shape are combined are also structurally modified shapes, for example, two *angle* shapes in print combine to form the symbol for *adjacent angles*.

Basic shape: Angle ∠

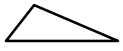
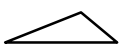
More specific form: Adjacent Angles 

A shape with structural modification is represented by

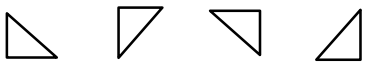
- the basic shape symbol,
- followed by the structural shape-modification indicator,
- followed by a letter or an uncontracted combination of letters suggestive of the change in the shape,
- ending with the termination indicator which signals the end of the modification.

Shape indicator	⠸⠸
Structural shape-modification indicator	⠈⠸
Termination indicator	⠸⠸

11.8 Structurally Modified Triangles: The following five structurally modified triangles are identified in the Nemeth Code. Each symbol starts with the basic shape symbol for "triangle" ⠸⠸. The derivation of the letter following the structural shape-modification indicator ⠈⠸ is underlined in the list of modified triangles below.

<u>A</u>cute Triangle		⠸⠸⠈⠸⠁⠞⠸⠸
<u>I</u>sosceles Triangle		⠸⠸⠈⠸⠊⠎⠸⠸
<u>O</u>btuse Triangle		⠸⠸⠈⠸⠕⠃⠸⠸
<u>R</u>ight Triangle		⠸⠸⠈⠸⠗⠊⠸⠸
<u>S</u>calene Triangle		⠸⠸⠈⠸⠎⠉⠸⠸

Know Your Triangles: Triangles are defined by the measure of angles and sides, not by orientation. For example, each of these is a "right triangle" because each contains a right (90°) angle.



*Definitions can be found in **Appendix B** of this course ("Glossary of Terms").*

11.9 Structurally Modified Angles: The following twelve structurally modified angles are identified in the Nemeth Code. Each symbol starts with the basic shape symbol for "angle" ⠸⠐. The derivation of the letter or letters following the structural shape-modification indicator ⠸ is underlined in the list below.

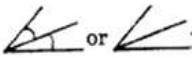
Specific Angles

Obtuse Angle  ⠸⠐⠸⠉

Right Angle  ⠸⠐⠸⠩

Straight Angle ⠸⠐⠸⠍

Combined Angles

Adjacent Angles  ⠸⠐⠸⠬

Alternate Exterior Angles  ⠸⠐⠸⠔⠹⠿

Alternate Interior Angles  ⠸⠐⠸⠔⠼⠿

Complementary Angles  ⠸⠐⠸⠔⠿⠿

Corresponding Angles  ⠸⠐⠸⠔⠩

Exterior Angles  ⠸⠐⠸⠔⠹

Interior Angles  ⠸⠐⠸⠔⠼

Supplementary Angles  ⠸⠐⠸⠔⠿⠿

Vertical Angles  ⠸⠐⠸⠬

11.10 Unlisted Shapes with Structural Modification: Structurally modified shapes which are not provided for in the Nemeth Code are formed in accordance with the principles above. Review the definition of *structural modification* in **11.7** to properly identify the unlisted shape. Be careful not to choose a symbol which already has an assigned meaning in the Nemeth Code. Refer to Appendix B of the Nemeth Code for a list of symbols already in use. Symbols beginning with dots 1246 begin on page 222.

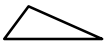
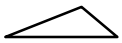
Explain the unlisted shape in a transcriber's note, giving the name or description of the symbol used. Include a drawing of the shape when appropriate.

Instructions: Review simple table format in **Lesson 6**. Do not braille tables side-by-side. After completing the "Angle/Symbol" table, leave one blank line and then begin the "Triangle/Symbol" table. Do not use box lines.

PRACTICE 11B

Structurally Modified Shapes

Angle	Symbol
right	
straight	
obtuse	
complementary	
supplementary	
vertical	

Triangle	Symbol
isosceles	
right	
acute	
obtuse	
scalene	

Shapes with Interior Modification

11.11 Definition and Construction: A shape with interior modification is a basic shape (for example, a *circle*) within which a letter, a numeral, a sign of operation, or other sign appears.

Basic shape: Circle ○

More specific form: Circle with number 8 inside ⑧

More specific form: Circle with asterisk inside ⊛

A shape with interior modification is represented by

- the basic shape symbol,
- followed by the interior shape-modification indicator,
- followed by the symbol corresponding to the interior sign,
- ending with the termination indicator which signals the end of the modification.

Shape indicator	⠸
Interior shape-modification indicator	⠐⠶
Termination indicator	⠸

Note: Symbols, numbers, words, etc. that represent keys on a calculator or a keyboard follow rules for keystrokes. See ***11.23****.*

11.12 Circles with Interior Modification: Eleven circles with interior modification are identified in the Nemeth Code. Each symbol starts with the basic shape symbol for "circle" ⠸⠴ followed by the interior shape-modification indicator ⠐⠶ . Notice that an interior numeral includes a numeric indicator and that the contracted form of the right-pointing arrow is not used.

Circle with Interior Arrow Pointing Right	⊕→	⠸⠴⠐⠶⠳⠕
Circle with Interior Arrow Pointing Left	⊕←	⠸⠴⠐⠶⠪⠕
Circle with Interior Arrow Pointing Up	⊕↑	⠸⠴⠐⠶⠬⠕
Circle with Interior Arrow Pointing Down	⊕↓	⠸⠴⠐⠶⠩⠕
Circle with Interior Capitalized Letter	⊕W	⠸⠴⠐⠶⠠⠺
Circle with Interior Numeral	⊕5	⠸⠴⠐⠶⠼⠑
Circle with Interior Cross	⊕×	⠸⠴⠐⠶⠭

Circle with Interior Dot		
Circle with Interior Minus Sign		
Circle with Interior Plus Sign		
Circle with Interior Vertical Bar		

11.13 Angles with Interior Modification: Three angles with interior modification are identified in the Nemeth Code. Each symbol starts with the basic shape symbol for "angle" ⠐⠣ followed by the interior shape-modification indicator ⠸⠩

Angle with Interior Arc		
Angle with Interior Clockwise Arrow		
Angle with Interior Counterclockwise Arrow		

Note: When the print copy uses the "angle with interior arc" symbol throughout the text to simply mean "angle", the simple braille shape symbol for "angle" ⠐⠣ may be used. A transcriber's note is required to inform the reader of the substitution.

11.14 Rectangles and Squares with Interior Modification: One rectangle and seven squares with interior modification are identified in the Nemeth Code. Each symbol starts with the basic shape symbol for "rectangle" ⠐⠫ or for "square" ⠐⠪ followed by the interior shape-modification indicator ⠸⠩

Rectangle with Interior Horizontal Bar		
Square with Interior Bar		
Interior Horizontal Bar		
Interior Vertical Bar		
Square with Interior Diagonal		
from Lower-Left to Upper-Right		
from Upper-Left to Lower-Right		
Square with Interior Diagonals		

Square with Interior Dot 

Square with Interior Numeral 

11.15 Words Enclosed in Shapes: Words enclosed in shapes are transcribed according to the methods for shapes with internal modification and must be enclosed within Nemeth switches. *Note: Words that represent keys on a calculator or a keyboard follow rules for keystrokes. See* ***11.23****.*

11.16 Two or More Vertically Arranged Modifiers: When two or more vertically arranged symbols occur within a basic sign of shape, the basic shape symbol and the interior shape-modification indicator are followed first by the symbol for the upper and then by the symbol for the lower interior sign. The termination indicator is used only after the last symbol.

Circle with Interior Arrow Pointing Right
Over Interior Arrow Pointing Left ..... 

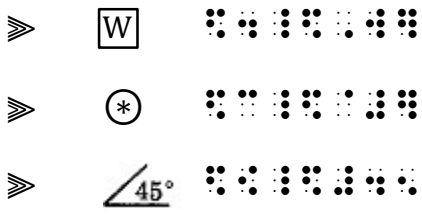
Circle with Interior Arrow Pointing Left
Over Interior Arrow Pointing Right .... 

11.17 Two or More Horizontally Arranged Modifiers: When two or more horizontally arranged symbols occur within a basic sign of shape, a multipurpose indicator (dot 5) is inserted between the interior modifiers to show that they are printed horizontally, not vertically. The termination indicator is used only after the last symbol.

Circle with Interior Arrow Pointing Up
Followed by Interior Arrow
Pointing Down ..... 

Circle with Interior Arrow Pointing Down
Followed by Interior Arrow
Pointing Up ..... 

11.18 Unlisted Shapes with Interior Modification: Shapes with interior modification not provided for in the Nemeth Code are formed in accordance with the principles for the construction of such shapes. Review the definition of *interior modification* in **11.11** to properly identify the unlisted shape.



⋙ ⠷ ⠤⠤⠤

11.20 Punctuation with Signs of Shape: Signs of shape are punctuated mathematically when the punctuation falls within the code switches.

⋙ (◊, ▽, □)

11.21 Plurals/Possessives: The uncapitalized letter "s" or the apostrophe-s combination occurring *inside or after* a sign of shape to show its plural or possessive are placed <u>after</u> the shape symbol in braille. Apply the general rules for the English letter indicator to the plural or possessive ending.

Example 11.21-1 ∠s and △s.

Each "s" is printed inside the shape.

Example 11.21-2 ∠ s and △s.

Each "s" follows the printed shape.

Example 11.21-3 ∠'s and △'s.

A punctuation indicator precedes each apostrophe.

Example 11.21-4 (∠'s, △'s, and ◯'s.)

Each "apostrophe-s" is punctuated mathematically because each is associated with a mathematical item.

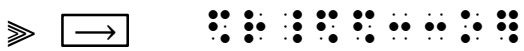
11.22 Further Considerations Regarding Transcriber-Devised Shapes: As previously noted, when encountering a shape not provided for in the Nemeth Code the transcriber may devise a symbol if that shape appears more than occasionally. The print shape should also be drawn as a raised-line diagram the first time the new symbol is introduced. In addition to the guidelines regarding unlisted shapes throughout this lesson, observe the following.

11.22.1 Usage Rules Regarding Interior Numerals and Arrows: Transcriber-devised forms should heed the following principles regarding interior numerals and arrows.

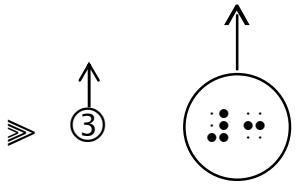
— The numeric indicator is used before a *numeral* or before *a decimal point and a numeral* following the interior shape-modification indicator.

⋙ ⑧

— If a *right-pointing arrow* in regular type with a full barb and single shaft of ordinary length is part of a shape symbol, its contracted form is not used.



11.22.2 Shapes Represented by Drawing: Drawn-in shapes are often more readable than elaborate braille constructions. Since it is not possible to formulate specific rules for the selection of an appropriate form, the decision is left to the experience and judgment of the transcriber. Shapes may also be represented by a combination of drawing and braille symbols. For example, if a modified shape cannot be represented clearly by braille symbols alone, the shape can be drawn and the modification shown in braille.



PRACTICE 11D

1. ▯, ◯, △, ∠, ◎, ∠ .
2. (◯'s, ∠'s, and △'s.)
3. a ⊕ (b ⊕ c)
4. r ⊗ s ⊗ ____ = rst
5. How many Ⓢ△ can you find in the giant ▢ ?

11.23 The Keystroke Indicator: When a print shape with interior modification depicts a labeled calculator or computer key, a contracted form employing a keystroke indicator is used in braille. A keystroke is represented by

— the keystroke indicator,
— followed by the label printed on the calculator key or the computer key,
— ending with the termination indicator which signals the end of the modification.

Keystroke indicator	⠸⠡
Termination indicator	⠸⠱

11.23.1 Shape in Print: The keystroke indicator is used regardless of the shape of the key in the print copy.

⠸⠡ [+] ⠸⠱

⠸⠡ ⊛ ⠸⠱

The actual key shape(s) used in a particular text should be specified on the Transcriber's Notes page. For example, "Calculator keys are depicted in print as square shapes."

11.24 Other Details Concerning Keystrokes

11.24.1 The Label: Regarding the item depicted on the key, note the following.

— Capitalization is duplicated in braille.

⠸⠡ [ENTER] ⠸⠱

⠸⠡ [Enter] ⠸⠱

Reminder: Words are brailled without contractions.

— Follow the usual rules of the Nemeth Code for typeform and use of indicators.

⠸⠡ [x^2] ⠸⠱

Italic typeform for variables is disregarded. A baseline indicator is required before brailling the termination indicator in this example because the keystroke is on the baseline of writing.

Example 11.24-1 Press F1 for help.



*A multipurpose indicator (dot 5) is needed to show that the numeral is not a subscript. Review section **7.16**.*

— The numeric indicator is not required within the keystroke construction.



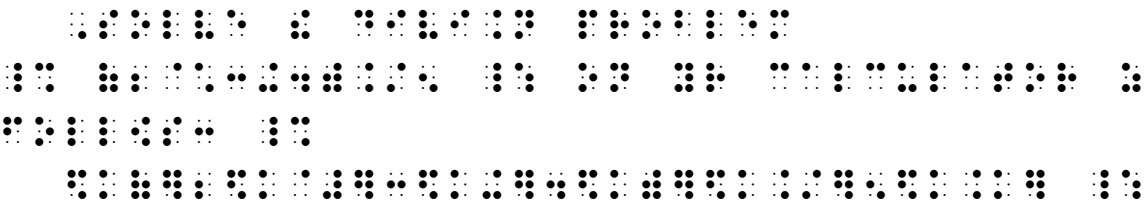
*Compare to a shape with interior modification which <u>does</u> require a numeric indicator. See **11.22.1**.*

11.24.2 Spacing

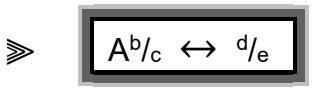
— No space is left between keystroke constructions and other similar constructions or mathematical symbols in a sequence of related calculations.

Example 11.24-2 Solve the division problem $(2 \times 3 + 4) \div 5$ on your calculator as follows:

(2 * 3 + 4) ÷ 5 =



— Arrows contained in the labels on the keys should not be spaced from the material to which they apply.



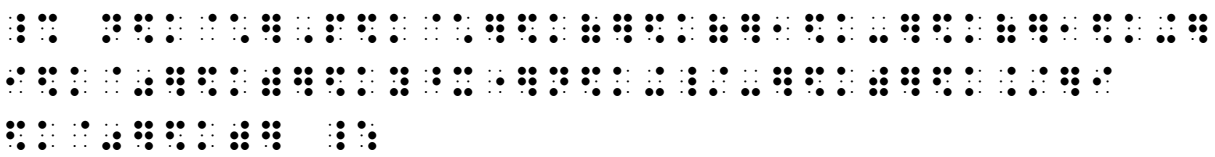






11.25 Long Keystroke Constructions: Use the fullest extent of the current braille line, making sure that a single keystroke construction is not divided between braille lines. (When the print lines are arranged in a logical sequence, duplicate the arrangement of the print lines if it is possible.)

Example 11.25-1 n × P × ((1 − (1 + i %) y^x n +/−) ÷ i %)



Icons

11.26 Consistency in Representation of Icons: When non-word UEB symbols such as icons appear in mathematical context, the symbol may be brailled in Nemeth context using the UEB transcriber-defined shape indicator ⠸⠸ This UEB indicator may be transcribed in either UEB context or Nemeth context without the insertion of switch indicators. List the icon on the Special Symbols page.

Example 11.26-1 (Earlier in the book it is stated that a gold ribbon indicates extra credit problems.)

Evaluate each expression.

13. $\frac{1}{2}(5 + 13) - 4 \cdot 5$

14. $(5 + 11) - (24 - 15) \cdot (3)$



15. $6^2 + 3 \cdot 7 - 9 \div 3$

The BANA Nemeth Code Technical Committee is discussing details regarding alignment of identifiers when some items are marked. The transcription of this example will be revisited after rulings are made.

PROVISIONAL SOLUTION

1 

2 

3 

4 

Line 1: Instructions begin in cell 5. The opening Nemeth Code indicator is placed at the end of this line of text so that all identifiers may start in the same cell.

Line 4: An icon was created to represent the "extra credit" print symbol. There is no need to switch out of Nemeth Code in order to braille the icon. The icon is listed on the Special Symbols page as a UEB symbol as follows.



Shapes Used as Signs of Omission

11.27 Spacing: When a sign of shape is used as a sign of omission or placeholder to represent a numeral, letter, sign of comparison, sign of operation, abbreviation, or any other item, the sign of shape is spaced in accordance with the rules for the omitted material it represents.

⋙ $12 \div 4 = \square$ *(a numeral is omitted—symbol is spaced from comparison sign)*



⋙ 40 dimes = $ ◯ *(a numeral is omitted— symbol is unspaced from monetary symbol)*



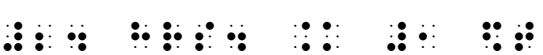
⋙ □ l = 1000 cc *(a numeral is omitted— symbol is spaced from abbreviation "l")*



⋙ 15 ◯ 15 = 30 *(an operation sign is omitted— symbol is unspaced)*

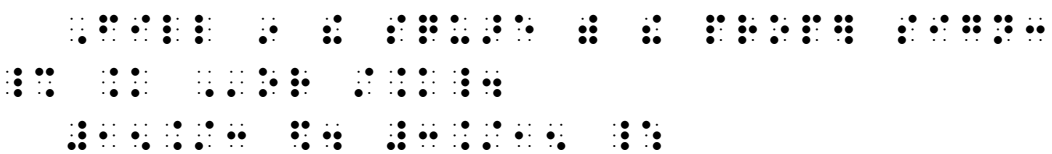


⋙ 24 hrs. = 1 △ *(an abbreviation is omitted— symbol is spaced from preceding numeral)*



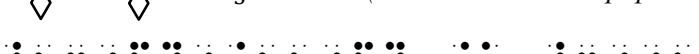
Example 11.27-1 Fill in the square with the proper sign: $=$ or $\neq$.

$15 \div 3 \square 3 \div 15$



A sign of shape is unspaced from any braille indicator which applies to it.

⋙ $24_{\Diamond} + 11_{\Diamond} = 40_5$ *(numerals in subscript position are omitted)*



⋙ $y^{\frac{1}{3}} = \sqrt[3]{\blacksquare}$ *(radicand is omitted)*



11.28 The English Letter Indicator and Comparison Signs: When a sign of shape is used in place of a comparison sign, rules regarding use of the English letter indicator to items immediately preceding or following the shape apply just as they would for any comparison sign. In these three examples, a "square" symbol is used as a sign of omission representing a comparison sign.

⋙ x □ y



Rule: An ELI is not used for a single letter which is immediately preceded or followed by a sign of comparison.

⋙ 1 yr ◯ 200 da



Rule: An ELI is needed for an abbreviation that corresponds to a shortform when the abbreviation has no period.

⋙ ray ab ⬭ ray ac



Rules: "ab" – An ELI is not used for a letter combination corresponding to a shortform when it is immediately followed by a comparison sign. "ac" – An ELI is needed for a lowercase letter combination that corresponds to a shortform that is preceded by a space if it is not immediately next to a sign of comparison.

⋙ iii □ iv



Rule: An ELI is not used for a Roman numeral which is immediately preceded or followed by a sign of comparison.

11.29 Use of the Multipurpose Indicator: Because a regular polygon with four or more sides is represented by a numeral, when an unspaced numeral follows such a shape, a multipurpose indicator is used before the numeral to prevent it from being misread. The shapes in these two examples represent a sign of operation.

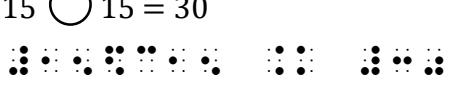
⋙ 15 □ 15 = 30



Without the multipurpose indicator this is read as a 415-sided regular polygon.

The situation stated above occurs only when the shape is represented by a numeral.

⋙ 15 ◯ 15 = 30

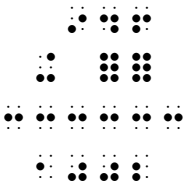


The circle shape is represented in braille with a letter ("c"). The following numeral will not be misread.

11.30 Omissions in Spatially-Arranged Problems: In a spatial arrangement, omissions are indicated with the general omission symbol regardless of the symbol used in print.

Example 11.30-1

```
  9 4 6
+ □□
-----
1 0 0 2
```



In print, the omissions are indicated as two squares.

Instructions: Use the word "pencil" to name the icon shown below. After the completion of this practice drill, show how the icon will be listed on the Special Symbols page.

PRACTICE 11E

(2 * 3 + 9) ÷ 5 =

9 · 35 y^x · 17 +/- =

2 1 2 ° → °C End

1. Fill in the box with the correct exponent.

   a. $x^2 \times x^4 = x^{\square}$

   b. $y^3 \times y^{\square} = y^9$

   c. $z^{\square} \times z^5 = z^{15}$

Show your work with problems marked with ✎ .

A. $436 - \bigcirc = 102$

B. $5_8 + \bigcirc_8 = 22_8$

✎ C. $5\frac{18}{12} = \square\frac{1}{2}$

D. Name two different operation signs that make this a true statement.

$1 \square 1 = 1$

✎ E. $\frac{15}{20} = \frac{3}{\square}$

Identified Signs of Shape

A sign of shape which is followed by a letter, a sequence of letters, or a numeral, is an *identified sign of shape.* The entire unit is mathematical and therefore brailled in Nemeth Code.

11.31 Spacing: There must be a space between the shape symbol and its identification. A space often does not appear in the print copy but it must be present in braille.

⋙ △ABC ("triangle ABC")

⋙ ∠θ ("angle theta")

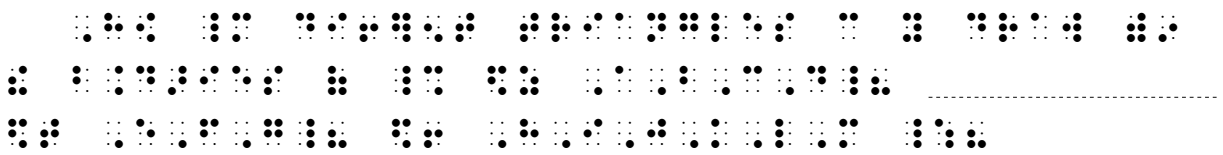
Example 11.31-1 ⏢5 denotes "trapezoid 5."



The first 5 is associated with the mathematical shape and is included inside the switches. A switch to Nemeth Code is not necessary for the second 5 – it is a freestanding numeral in UEB context.

11.31.1 Keep Together: A sign of shape and the letter, sequence of letters, or numeral which follows it is regarded as a single mathematical item and therefore should not be divided between braille lines.

Example 11.31-2 How many different triangles can you draw within the boundaries of ⏢ABCD? △EFG? ⎔HIJKLM?



11.31.2 Surrounding Symbols: The spacing before and after a sign of shape and its identification is subject to the spacing rules for the symbols preceding or following it.

⋙ △PQR ~ △P'Q'R'



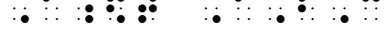
A space precedes and follows the tilde, which, in this case, is a sign of operation meaning "is similar to".

⋙ ∠2 + ∠3 = ∠4

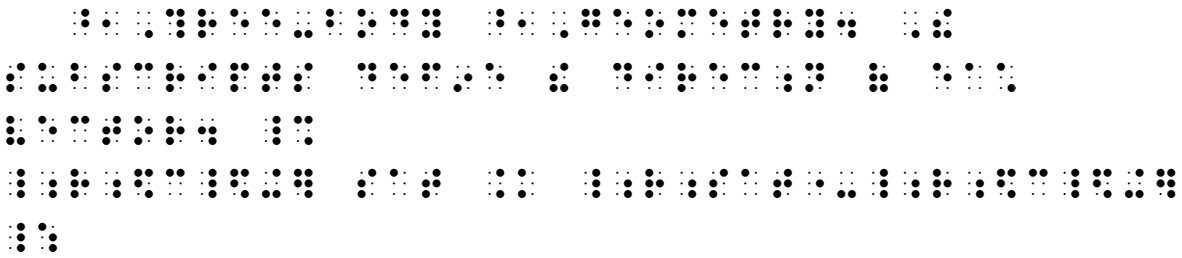


There is no space before or after the operation sign (plus sign).

11.32 A Shape Within a Superscript or a Subscript: An identified sign of shape is read as a unit, therefore when one appears in a superscript or a subscript the effect of the level indicator extends through the space following the sign of shape. That is, the space preserves the superscript or subscript level where the sign of shape appears.

⋙ $A_{\triangle ABC}$ 

Example 11.32-1 **Three-body Geometry.** The subscripts define the direction of each vector. $\mathbf{r}_{\oplus sat} = \mathbf{r}_{sat} - \mathbf{r}_{\oplus}$



"⊕sat" is in the subscript position. Only one subscript indicator is needed because the level continues through the space following the shape. (Reminder: Boldface type used to identify letters as vectors must be preserved.)

11.33 A Shape Which Carries a Superscript or a Subscript: When a sign of shape carries a superscript or subscript, the level indicator is unspaced from the shape.

⋙ $\triangle^2$ 

⋙ $\square_2$ 

The rule for nonuse of the subscript level indicator (see **8.10**) does not apply to a shape represented by a letter. A subscript indicator is required.

⋙ $\triangle_2$ 

With an identified sign of shape, the space required between the sign of shape and its identifier follows the superscript or subscript. When the identified shape is on the baseline of writing, the space following the superscript or subscript terminates the effect of the level indicator and reinstates the baseline level.

⋙ $\triangle^2$ DEF 

Only the numeral "2" is in the superscript position. The shape and letters "DEF" are on the baseline of writing.

11.34 The English Letter Indicator: When an English letter, a Roman numeral, or a shortform letter combination identifies a shape, the English letter indicator is not used.

⋙ $\angle p$  ("angle p")

⋙ □ ii  ("square ii")

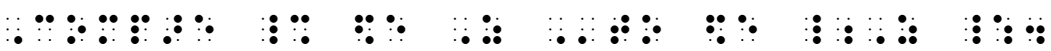
⋙ △ abv ("triangle abv")

Example 11.34-1 ⊙ Q denotes "circle Q."



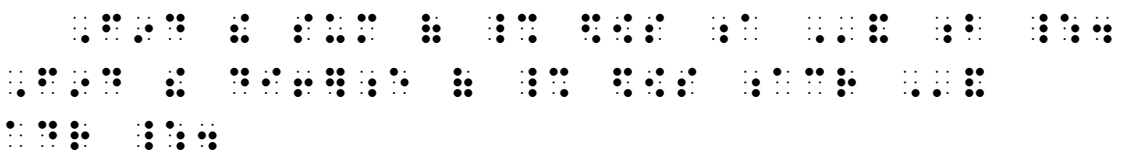
If such letters are in nonregular type, rules regarding typeform are followed and an English letter indicator is required if the variant typeform is retained.

Example 11.34-2 Compare ⬭ Z to ⬭ **Z**.



If the sign of shape has a plural or a possessive ending, an English letter indicator may be required.

Example 11.34-3 Find the sum of ∠s *a* and *b*. Find the difference of ∠s *acr* and *adr*.



Single letters "a" and "b" require an ELI when following Nemeth Code rules.(Review the definition of "single letter" in Lesson 4, section 4.10.) Letter combination "acr" requires an ELI because it is the same as a shortform; letter combination "adr" does not.

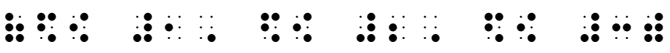
11.34.1 The Letter "m": Notation regarding angle measurement often uses the letter "m" for "measure". The letter "m" is brailled unspaced from the following symbol regardless of spacing used in the print copy.

⋙ m ∠θ ("the measure of angle theta")

⋙ m° ∠α ("the measure in degrees of angle alpha")

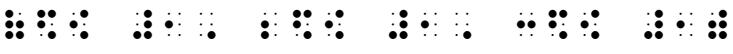
11.35 Use of the Numeric Indicator in an Enclosed List: The "enclosed list" was introduced in **Lesson 5** where it states that a numeric indicator is not used before a numeral in an enclosed list. More specifically, this rule applies to a numeral that occurs at the *beginning* of the item. A sign of shape and an identifying numeral which follows it are a single item even though a space occurs between them. In the context of an enclosed list, the numeric indicator is required for the identifying numeral because the numeral is not at the beginning of the item. Look carefully at the use and nonuse of the numeric indicator in each example below, as described in the comments.

⋙ (∠1, ∠2, ∠3)



In this enclosed list, each numeral needs a numeric indicator because it identifies the angle symbol—the numeral does not begin the item.

⋙ (∠1, 2∠1, 3∠1)



Only the numerals which begin an item ("2"and "3") are brailled without a numeric indicator in this enclosed list.

⋙ $(1\angle a,\ 2\angle b)$ ⠸⠩⠼⠁⠈⠣⠰⠁⠐⠀⠼⠃⠈⠣⠰⠃⠜

The numerals ("1" and "2") begin each item in this enclosed list—a numeric indicator is not brailled. The letters ("a"and "b") are brailled without a letter indicator according to the rules governing identified signs of shape.

PRACTICE 11F

1. □ABCD is a square. ▱EFGH is a parallelogram. ◇ JKLM is a rhombus.
2. Compare triangles: △ADM ≅ △A'D'M'. ◿BEP ≇ ◿CFP.
3. Should △ABV be included in the set {△3, ∠ GHA, ◊2}?
4. $\angle 3 + \angle 4 = 90°$
5. $\text{m}\angle p + \text{m}\angle q = 180°$
6. $\text{m}°\angle\theta = -45$
7. $A_{\Delta DEF} = \frac{1}{2}bh$
8. $\angle ECB = \frac{1}{2}\angle ABC$

LABELED FORMATS, cont.

Displayed Material with Labels

Format for displayed mathematical expressions was introduced in **Lesson 8**. If displayed mathematical material is labeled with a unique identifier, the following details are observed.

11.36 Recognition: Displayed mathematical expressions may be labeled in print with a unique number or letter printed in a location that is visually conspicuous, often in the right margin. That label is then used in place of the actual expression later in an example or in the narrative. This allows for compact presentation of a problem.

11.36.1 Braille Layout: When a number or letter is used to label a displayed mathematical expression it is placed at the left of the expression in braille regardless of the location of the label in the print copy. The label begins in the appropriate cell for displayed material in accordance with the margin rules presented in **Lesson 8**. Nemeth Code symbols are used to label math expressions. Nonstandard typeform is disregarded. The opening Nemeth Code indicator (if needed) is placed at the end of the preceding material in order to allow the label to fall in the first cell of the displayed margin.

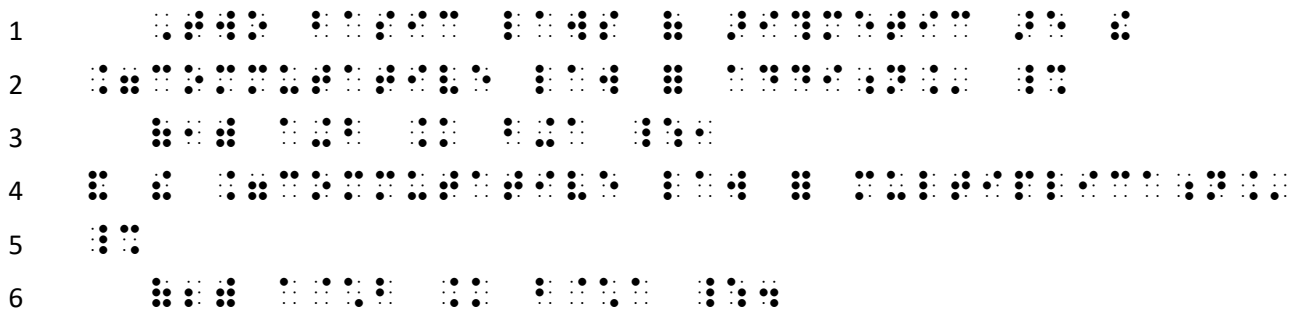
Example 11.36-1

Two basic laws of arithmetic are the
commutative law for addition

$$a + b = b + a, \qquad (1)$$

and the *commutative law for multiplication*

$$a \times b = b \times a. \qquad (2)$$



Lines 1-6: The print style is of a narrative paragraph interrupted twice by displayed math.

Lines 1-2: The paragraph begins in cell 3 and runs over in cell 1.

Line 2: The Opening Nemeth Code indicator is placed at the end of this line to assure that the item number will be the first cell to read on the next line.

Line 3: Displayed math begins in cell 3. The label is brailled first.

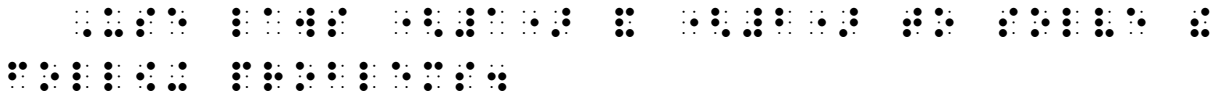
Lines 4-5: The paragraph continues in the runover cell, cell 1.

Line 6: Same comment as line 3. Note that the labels are bold in print but the typeform is disregarded in braille.

When a labeled expression is referred to within narrative, UEB symbols may be used.

Example 10.36-2

Use laws (1) and (2) to solve the following problems.



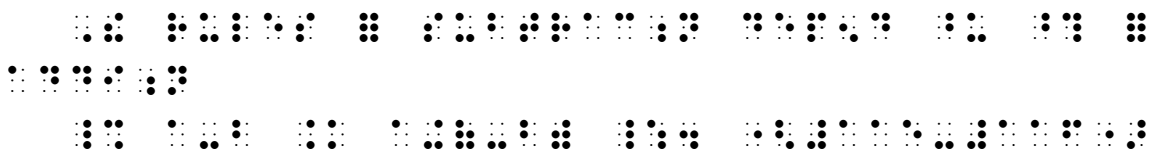
11.36.2 Transcriber's Note Required: A transcriber's note concerning the change in position of the label in the braille copy is required on the Transcriber's Notes page in every volume. *Sample:* "Identifying numbers which are printed to the right of mathematical expressions are brailled to the left." Guidelines for creating a Transcriber's Notes page are given in *Braille Formats*.

11.37 Page Number Citation: The number printed beside a displayed mathematical expression may actually be a page citation, in which case the cross reference immediately <u>follows</u> the expression, as printed. If a range of numbers is shown, you can be fairly confident that they are page numbers, but look for context clues to determine if the label is a page number citation in order to place it in its proper braille location.

Example 11.37-1

The rules for subtraction depend upon those
for addition

$$a - b = a + (-b). \qquad (115\text{-}116)$$

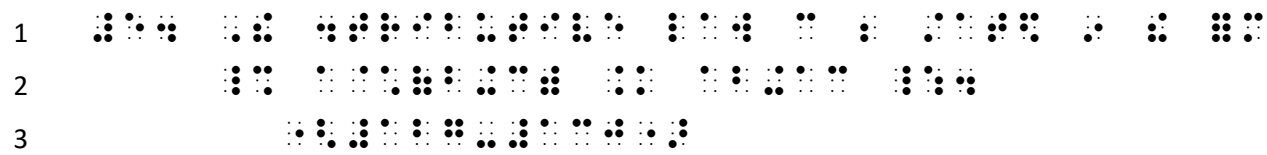


In the next example, there is no room for the citation on the line with the math. It is placed in the runover cell.

Example 11.37-2

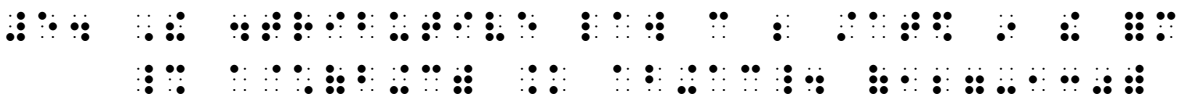
5. The distributive law can be stated in the form

$$a \times (b + c) = ab + ac. \qquad (127\text{-}130)$$



Line 1: Itemized material begins in cell 1.
Lines 2-3: Displayed margins to itemized material, (5-7)

The page numbers may be brailled in either code. If mathematical material continues after the citation, the following transcription is acceptable.



Mathematical material continues ... 

Since the location of the citation is not changed from its location in print, a transcriber's note is not needed.

Instructions: Create a Transcriber's Notes page for this practice. Include the standard statement citing usage of the Nemeth Code: "Mathematical content is transcribed according to *The Nemeth Braille Code for Mathematics and Science Notation*, *1972 Revision*, 2007-2015 Updates including the *Guidance for Transcription Using the Nemeth Code within UEB Contexts*." followed by the note regarding the change in location of identifying numbers, "Identifying numbers which are printed to the right of mathematical expressions are brailled to the left."

PRACTICE 11G

This is the quadratic equation:

$$ax^2 + bx + c = 0 \qquad (1)$$

This is the Pythagorean Theorem:

$$a^2 + b^2 = c^2 \qquad (2)$$

Which equation, (1) or (2), is used to find the length of the sides of a right triangle?

Labeled Mathematical Statements

11.38 Recognition of a Labeled Mathematical Statement: A mathematical or scientific definition, law, theorem, axiom, lemma, etc., is usually printed in a distinctive style to catch the reader's attention. It also may be set off from the main text by different margins or some other means of distinction. In this print example the definition is in boldface and is set off from the text with indented margins.

Definition **A set which can be put into one-to-one correspondence with the natural numbers is called a countable set.**

If the statement is labeled with a heading, as in the example shown above, the Nemeth Code calls it a *labeled mathematical statement* and requires the following format.

11.38.1 The Label: Transcribe the label in full capitals regardless of print style. Nonregular typeface in the label is ignored. Place the label as a paragraph heading regardless of its location in the print copy.

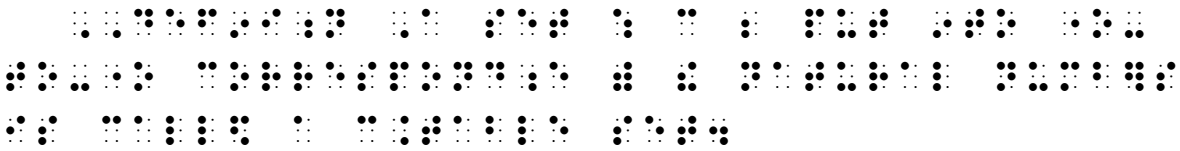
11.38.2 The Statement: If the text of the statement is entirely in the same typeface, the typeface is ignored in the braille transcription. This rule applies even when the statement is entirely in UEB.

11.38.3 Spacing and Margins: A line is left blank before the beginning and after the end of the entire labeled statement. Normal paragraphing (3-1) is applied, with the label beginning the paragraph.

Example 11.38-1

Definition **A set which can be put into one-to-one correspondence with the natural numbers is called a countable set.**

(blank line)



(blank line)

In print, the label "Definition" is a marginal heading. Only the first letter is capitalized and the word is in boldface. In braille, typeface of the label is disregarded, the label is fully capitalized and is placed as a paragraph heading beginning in cell 3. The definition is printed entirely in boldface. Uniform typeface is disregarded in braille. A blank line precedes and follows the labeled statement.

A labeled statement may begin on line 1 of the braille page.

Example 11.38-2 **Theorem 4.** *The diagonals of a rectangle are equal.*

1 ⠠⠠⠮⠑⠕⠗⠑⠍ ⠙⠊⠁⠛⠕⠝⠁⠇⠎ ⠕⠋ ⠁ ⠗⠑⠉⠞⠁⠝⠛⠇⠑ ⠁⠗⠑ ⠑⠟⠥⠁⠇⠦⠠⠠

2 ⠠⠠⠊⠝ ⠏⠗⠊⠝⠞⠂ ⠞⠓⠑ ⠎⠞⠁⠞⠑⠍⠑⠝⠞ ⠊⠎ ⠊⠝ ⠊⠞⠁⠇⠊⠉⠎⠦⠠⠠

(blank line)

In print, "Theorem 4" is bold and the statement is in italics.

A labeled statement may end on line 25 of the braille page.

23 (blank line)

24 ⠠⠠⠮⠑⠕⠗⠑⠍ ⠙⠊⠁⠛⠕⠝⠁⠇⠎ ⠕⠋ ⠁ ⠗⠑⠉⠞⠁⠝⠛⠇⠑ ⠁⠗⠑ ⠑⠟⠥⠁⠇⠦⠠⠠

25 ⠠⠠⠊⠝ ⠏⠗⠊⠝⠞⠂ ⠞⠓⠑ ⠎⠞⠁⠞⠑⠍⠑⠝⠞ ⠊⠎ ⠊⠝ ⠊⠞⠁⠇⠊⠉⠎⠦⠠⠠

If the statement is printed in a box, follow *Braille Formats* guidelines regarding blank lines.

Example 11.38-3

Bauer–Fike Theorem. Let μ be an eigenvalue of $A + \delta A$. Then there exists $\lambda \in \Lambda(A)$ such that

$$|\lambda - \mu| \leq \kappa_p(V)\|\delta A\|_p \, .$$

(blank line)

⠠⠠⠮⠑⠕⠗⠑⠍ ⠃⠁⠥⠑⠗⠤⠋⠊⠅⠑ ⠞⠓⠑⠕⠗⠑⠍⠲ ⠠⠠ ⠇⠑⠞ ⠍⠥ ⠃⠑ ⠁⠝ ⠑⠊⠛⠑⠝⠧⠁⠇⠥⠑ ⠕⠋ ⠁ ⠍⠁⠞⠗⠊⠭ ⠁⠝⠙ ⠇⠑⠞ ⠝⠥⠍⠼⠁ ⠃⠑ ⠁ ⠎⠞⠁⠞⠑⠍⠑⠝⠞ ⠕⠋ ⠏⠑⠗⠞⠥⠗⠃⠁⠞⠊⠕⠝ ⠁⠝⠙ ⠇⠑⠞ ⠙⠑⠝⠕⠞⠑ ⠁ ⠎⠞⠁⠞⠑⠍⠑⠝⠞ ⠕⠋ ⠁⠝⠁⠇⠽⠎⠊⠎⠦⠠⠠

(blank line)

The labeled statement is boxed. Box lines are retained in braille for distinction. The label "Bauer-Fike Theorem." is printed as a paragraph heading. Only the initial letters are capitalized and the label is in boldface. In braille, typeface is disregarded and the label is fully capitalized. The definition itself is printed in normal typeface, with the exception of the letters in the mathematical expressions which are in italics. In braille, italics applied to a variable are disregarded.

Review unfamiliar symbols. This example above includes Greek letters mu, delta, lambda (both lowercase and uppercase), and kappa as well as the comparison sign for

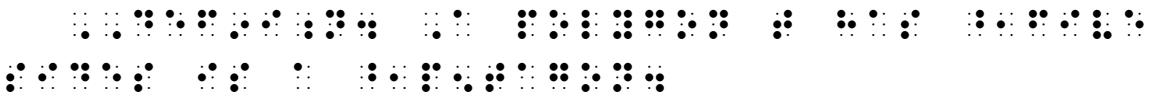
"membership". You can assume the letter A is the English letter, not a Greek Alpha. Several uppercase Greek letters are generally not used as math symbols because they look identical to certain uppercase Latin (English) letters. Review the Greek Alphabet Table in ***Lesson 5****.*

11.39 Significant Typeface: If, in the body of the labeled statement, a word or phrase is singled out for special attention by using a non-regular typeface (for the purpose of definition or other elaboration), the change in typeface is retained in braille. Use the appropriate UEB or Nemeth Code typeface indicators. UEB indicators are used outside of the code switches; Nemeth Code indicators are used inside the code switches.

Analysis: In the next example, the label "Definition." is printed in boldface. Only the first letter is capitalized. In braille, typeface is disregarded and, instead, the label is fully capitalized. The statement is printed entirely in italics but two words are emphasized in bold italics. In braille, the superfluous typeface (italics) is disregarded but the typeface of the emphasized words (boldface) is retained.

Example 11.39-1 **Definition.** *A polygon that has* ***five*** *sides is a* ***pentagon.***

(blank line)



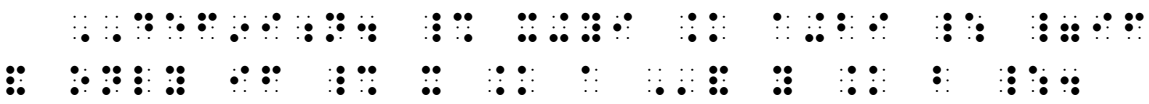
(blank line)

Recall from **Lesson 7** that a switch from UEB to Nemeth Code terminates the effect of a UEB typeform indicator.

Analysis: In the next example, the label "Definition" is printed in italics and only the first letter is capitalized. In braille, the label is fully capitalized. The statement is printed entirely in boldface. This superfluous typeface is ignored in braille. One phrase is emphasized by underlining. The underlining is retained in braille.

Example 11.39-2 *Definition* $\boldsymbol{x+yi=a+bi}$ <u>if and only if</u> $\boldsymbol{x=a}$ **and** $\boldsymbol{y=b.}$

(blank line)



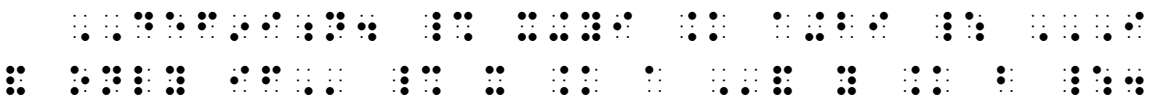
(blank line)

The underlining of the passage is implicitly terminated by the switch to Nemeth Code.

Implicit termination does not apply to capitalization. A fully capitalized passage must be explicitly terminated.

Example 11.39-3 *Definition* $\boldsymbol{x+yi=a+bi}$ **IF AND ONLY IF** $\boldsymbol{x=a}$ **and** $\boldsymbol{y=b.}$

(blank line)



(blank line)

PRACTICE 11H

Labeled Mathematical Statements

Pythagorean Theorem *In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.*

$$c^2 = a^2 + b^2$$

DEFINITION A positive number expressed in the form: $a \times 10^n$, where $1 \leq a < 10$ and n is an integer is said to be written in **scientific notation.**

Typeform Indicators for Mathematical Words and Phrases

11.40 Italic and Boldface Typeform Indicators: When significant typeform is retained inside of the Nemeth Code switches, Nemeth Code typeform indicators are used. **Lesson 7** introduced the Nemeth Code typeform indicators for letters, numerals, symbols, and compound expressions. When typeform is retained for a mathematical *word or phrase*, the following rules apply.

11.40.1 One Word in Italics or Boldface

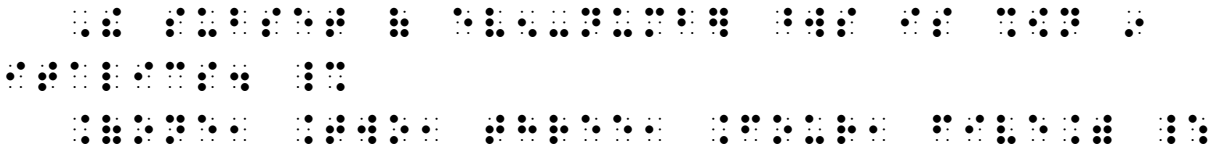
For One Word	
Italic Typeform Indicator	
Opening Boldface Type Indicator	
Closing Boldface Type Indicator	

The one-cell italic typeform indicator is unspaced from the following word.

⋙ *four* 

Example 11.40-1 The subset of even-number words is shown in italics.

{one, *two*, three, *four*, five}

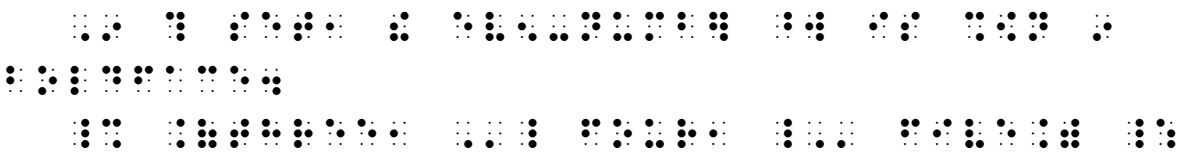


The three-cell boldface type indicators are preceded and followed by a space and must not stand alone on a line.

⋙ **four** 

Example 11.40-2 In this set, the even-number word is shown in boldface.

{three, **four,** five}



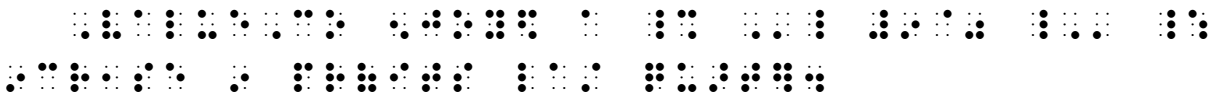
Note: The one-cell boldface typeform indicator of the Nemeth Code  *is applied to a word only in the context of a compound expression. See* ***Lesson 7****.*

When typeform is retained for a fraction, or for a numeral that is associated with a mathematical symbol such as a percent sign or monetary symbol, the three-cell typeform indicator is used.

⋙ $\mathbf{\frac{9}{100}}$

⋙ **9%**

Example 11.40-3 ValueCo enjoyed a **9%** increase in profits last quarter.



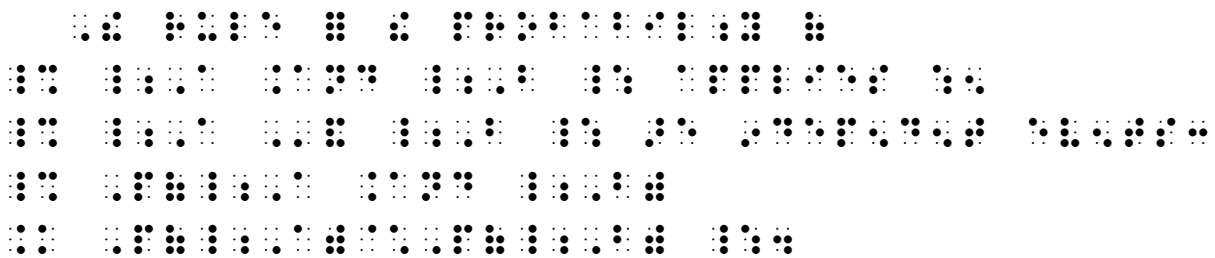
The BANA Nemeth Code Technical Committee is discussing details regarding code switching requirements for freestanding, unmodified mathematical numbers in narrative context when such numbers are printed in a significant typeface. At this time, no such examples are shown.

Significant typeform with a single word is often found in the study of probability. For example,

⋙ $P(A \textit{ and } B) = 0$

⋙ $P(A \textbf{ or } B) = ?$

Example 7.40-4 The rule for the probability of **A** *and* **B** applies when **A** and **B** are independent events: $P(\mathbf{A} \textit{ and } \mathbf{B}) = P(\mathbf{A}) \times P(\mathbf{B})$.



A switch to Nemeth Code is required for the boldface letters because they have mathematical significance. Italic typeform is disregarded for the single letter "P". The italicized word "and" is part of the mathematical expression and is brailled in Nemeth Code. The word "and" printed in normal type is part of the sentence structure and is brailled in UEB. The linked expression is divided before the equals sign because it will not fit entirely on one line (this is a 38-cell line).

11.40.2 A Phrase Italics or Boldface

For a Phrase	
Opening Italic Type Indicator	
Closing Italic Type Indicator	
Opening Boldface Type Indicator	
Closing Boldface Type Indicator	

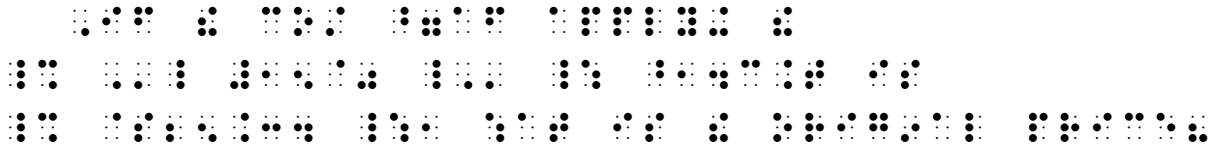
The three-cell typeform indicators are preceded and followed by a space and must not stand alone on a line. When both indicators are required for the same word or phrase, they are unspaced from each other and are closed in the opposite order as opened.

⋙ *4.9 sq. **ft.*** 

Assume the italics is significant in this illustration.

11.41 Code Switching Within an Emphasized Passage: If code switching is necessary within an emphasized technical passage, the beginning typeform indicators are repeated after each switch to show that emphasis continues. Switching from Nemeth Code to UEB does not implicitly terminate a Nemeth Code typeform – the appropriate closing typeform indicator must be brailled before terminating Nemeth Code.

Example 11.41-1 If the cost **after applying the 15% discount** is $25.34, what is the original price?



It is first determined that the boldface should be retained because its purpose here is to focus attention on the phrase.

Line 1: UEB boldface passage indicator applies to three words. The switch from UEB to Nemeth Code terminates the effect of the UEB typeform indicator without the need for a UEB termination indicator.

Line 2: The boldface indicator of the Nemeth Code is used for the boldfaced mathematical item, 15%. The Nemeth Code closing boldface indicator is required before switching out of Nemeth Code.

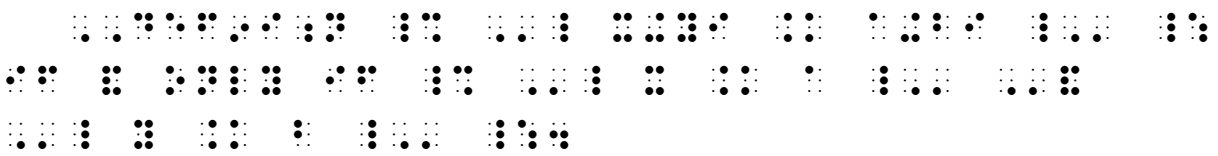
Line 2: The nonregular typeform continues after the termination of the mathematical portion, so a UEB typeform indicator must be re-entered.

11.42 Revisiting Typeform in Labeled Statements: Further examples illustrate the use of UEB and Nemeth Code typeform indicators in the context of labeled statements.

Analysis: In the next example, the label and the statement are printed in a consistent typeface (italics). Each of the three equations is printed in boldface italics. The italic typeform is ignored because italics is universally applied to this statement, but the boldface is retained because it is in a variant typeform in relation to the rest of the statement. The boldface type indicators of the Nemeth Code are used for each equation.

Example 11.42-1 *Definition* $\boldsymbol{x + yi = a + bi}$ *if and only if* $\boldsymbol{x = a}$ *and* $\boldsymbol{y = b}$**.**

(blank line)

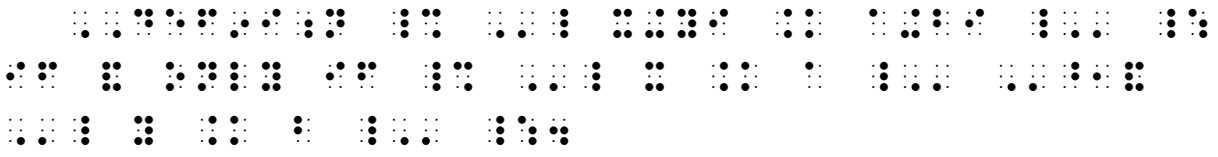


(blank line)

Analysis: In the next similar example, the variant typeform continues in the UEB portion (the word "and" also printed in boldface italics). After terminating boldface in the Nemeth portion of this phrase, the single-word switch indicator initiates UEB and the UEB boldface word indicator is brailled. Boldface continues with an opening Nemeth Code typeform indicator, finally closing before the Nemeth Code terminator.

Example 11.42-2 *Definition* $\boldsymbol{x + yi = a + bi}$ *if and only if* $\boldsymbol{x = a}$ ***and*** $\boldsymbol{y = b}$**.**

(blank line)



(blank line)

PRACTICE 11I

1) In solution b, *only the* $x^2 > 0$ *values* apply.

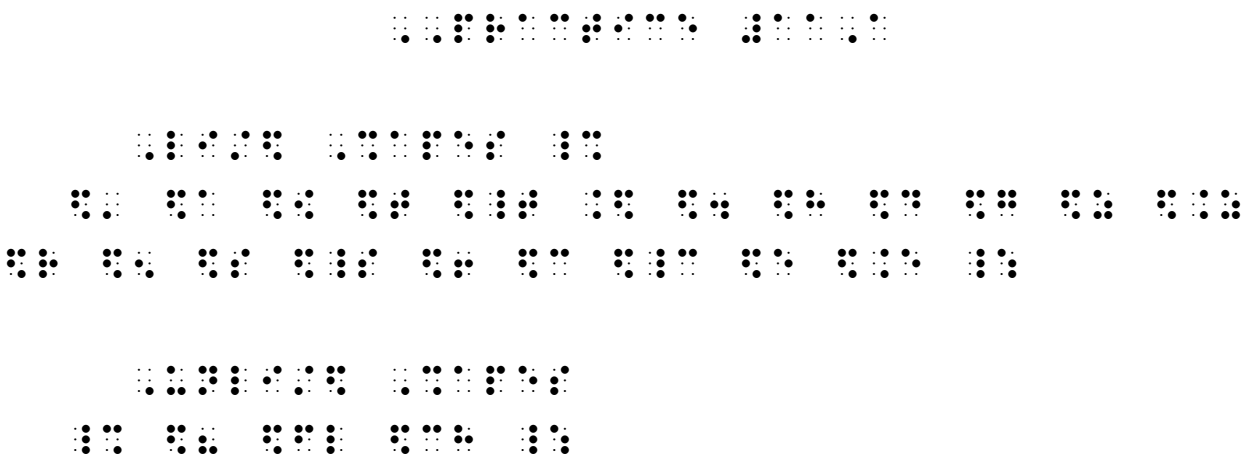
2) The *reciprocal* of a fraction is the fraction turned upside-down. For example, **the reciprocal of** $\frac{2}{3}$ **is** $\frac{3}{2}$**.**

3) The *Specific Addition Rule of Probability* can be expressed as follows**:**

$$P(A \text{ or } B) = P(A) + P(B)$$

For further practice, see Appendix A—Reading Practice.

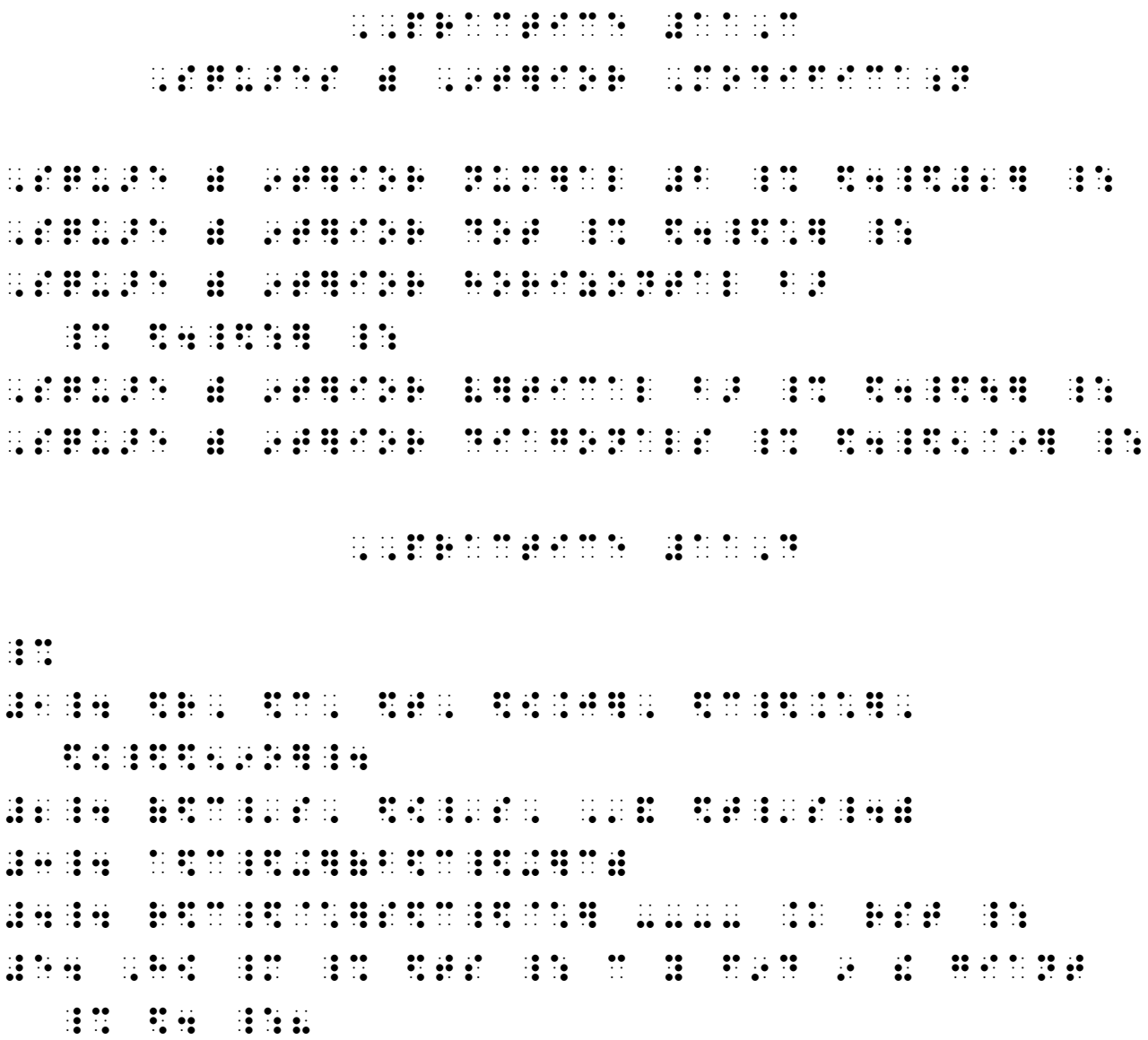
ANSWERS TO PRACTICE MATERIAL



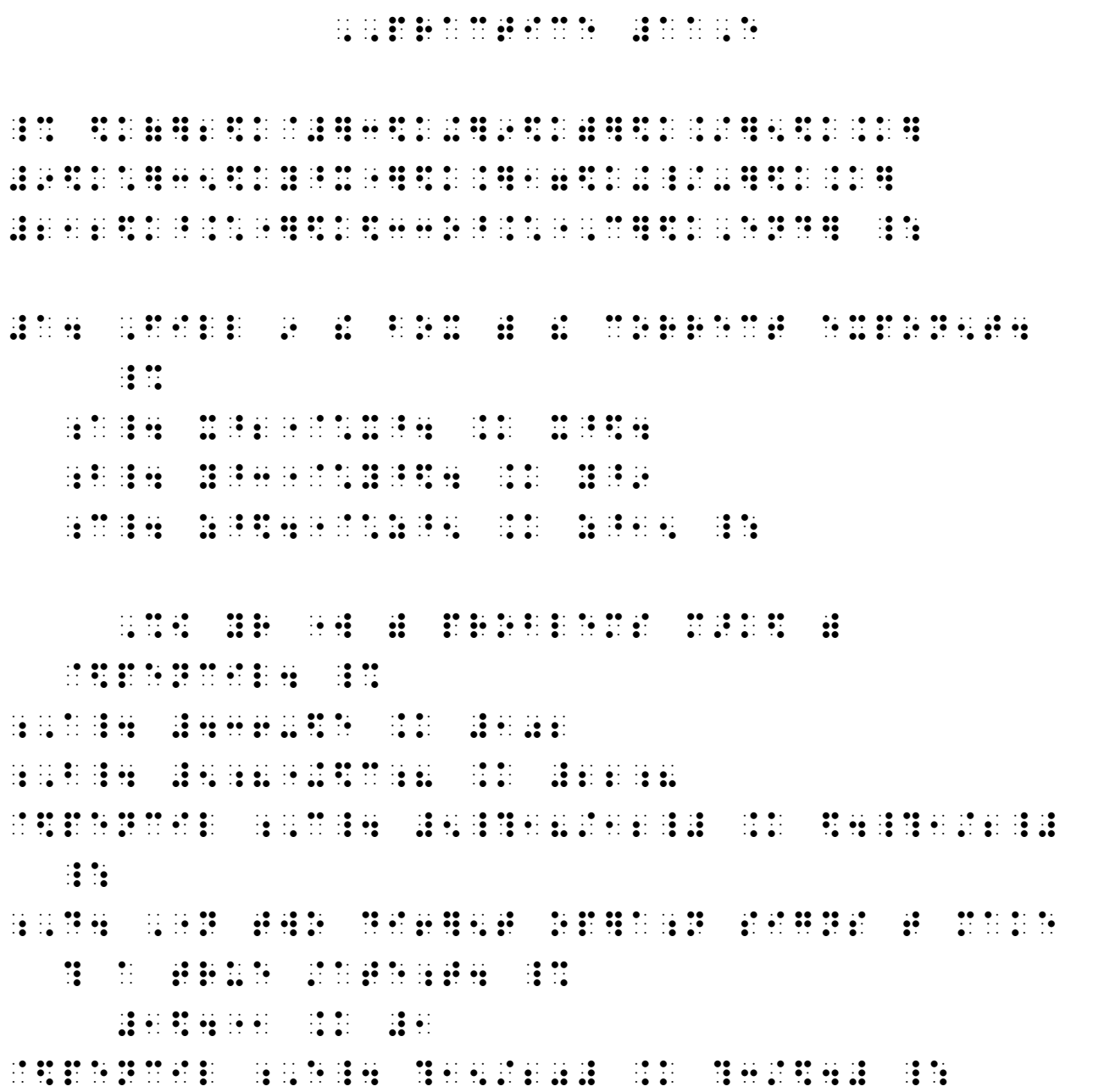
The first opening Nemeth Code indicator may also be placed at the beginning of the series of shapes on line 4.

⠠⠠⠿⠿⠠⠠	⠠⠠⠿⠠⠠⠠
⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	⠠⠠⠠⠠⠠⠠⠠
⠠⠠	
⠠⠠⠿⠿⠠ ⠠⠠⠠⠠⠠⠠⠠	⠠⠠⠠⠠⠠
⠠⠠⠿⠠⠠⠿⠠⠠ ⠠⠠⠠⠠	⠠⠠⠠⠠⠠
⠠⠠⠿⠿⠠⠠⠠ ⠠⠠⠠⠠⠠⠠⠠	⠠⠠⠠⠠⠠
⠠⠠⠿⠠⠠⠿⠠⠠⠠⠠⠿⠠⠠⠠⠠⠿⠠⠠	⠠⠠⠠⠠⠠⠠⠠
⠠⠠⠿⠠⠠⠿⠠⠠⠠⠠⠿⠠⠠⠠⠠⠿⠠⠠	⠠⠠⠠⠠⠠⠠⠠
⠠⠠⠿⠿⠠⠠⠠⠠⠠⠠⠠ ⠠⠠⠠⠠⠠	⠠⠠⠠⠠⠠⠠
⠠⠠⠿	

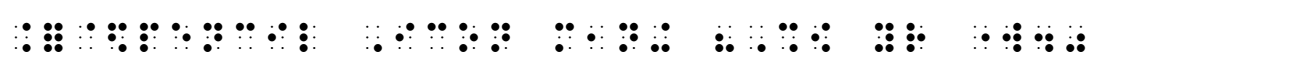
⠠⠠⠿⠿⠠⠠⠠⠠⠠⠠⠠⠠	⠠⠠⠠⠠⠠⠠⠠
⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	⠠⠠⠠⠠⠠⠠⠠⠠
⠠⠠	
⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	⠠⠠⠠⠠⠠⠠
⠠⠠⠿⠿⠠⠠ ⠠⠠⠠⠠	⠠⠠⠠⠠⠠⠠
⠠⠠⠿⠠⠠⠿⠠ ⠠⠠⠠⠠	⠠⠠⠠⠠⠠⠠
⠠⠠⠿⠿⠠⠠⠠ ⠠⠠	⠠⠠⠠⠠⠠⠠
⠠⠠⠿⠠⠠⠿⠠⠠⠠	⠠⠠⠠⠠⠠⠠
⠠⠠⠿	



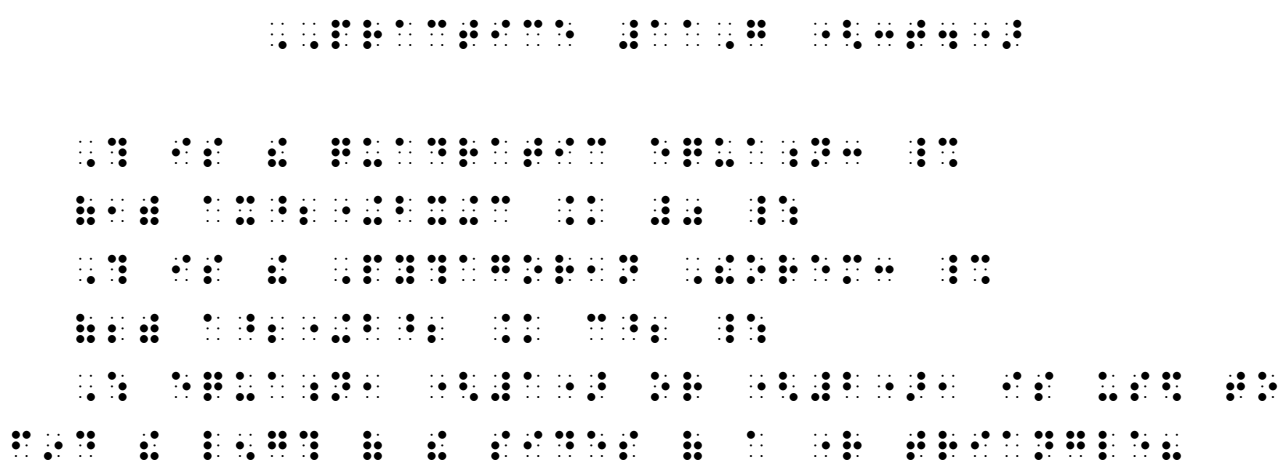
The opening Nemeth Code indicator does not replace the required blank line following the centered heading.



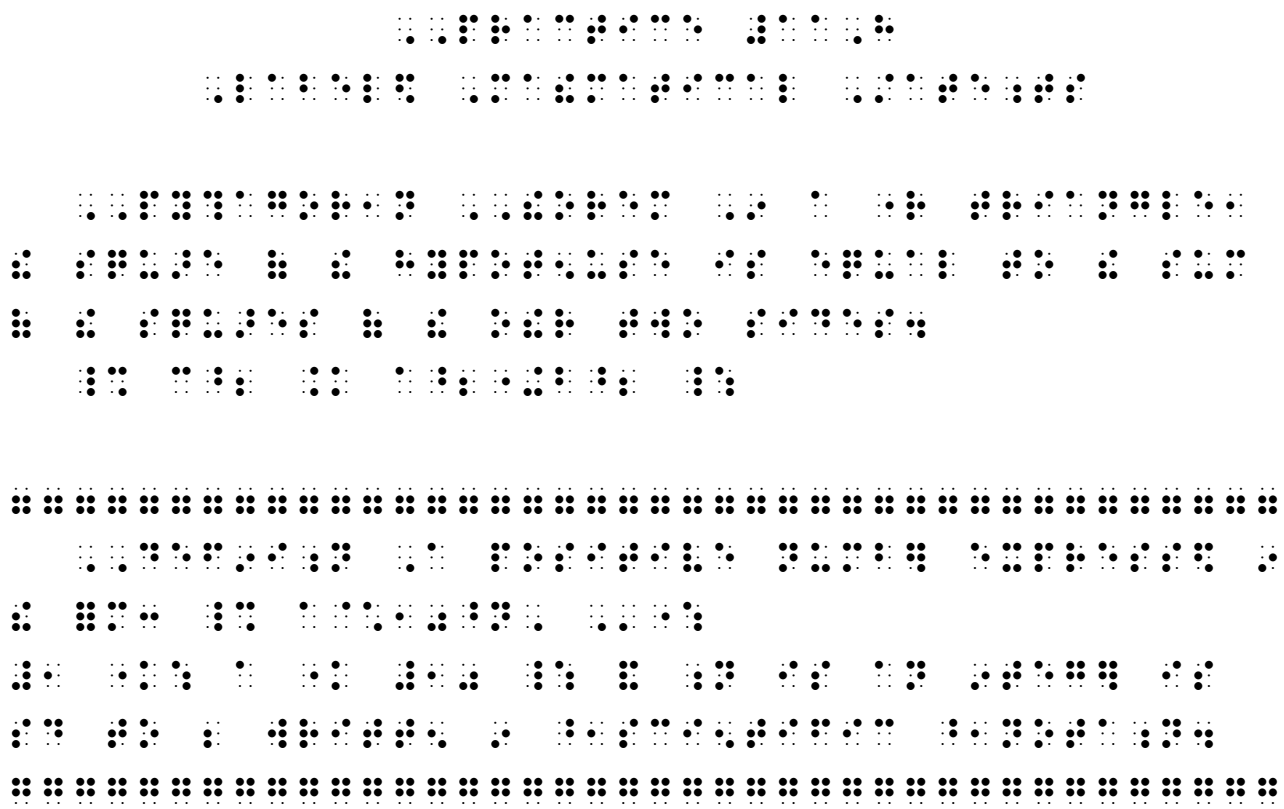
Special Symbols page:

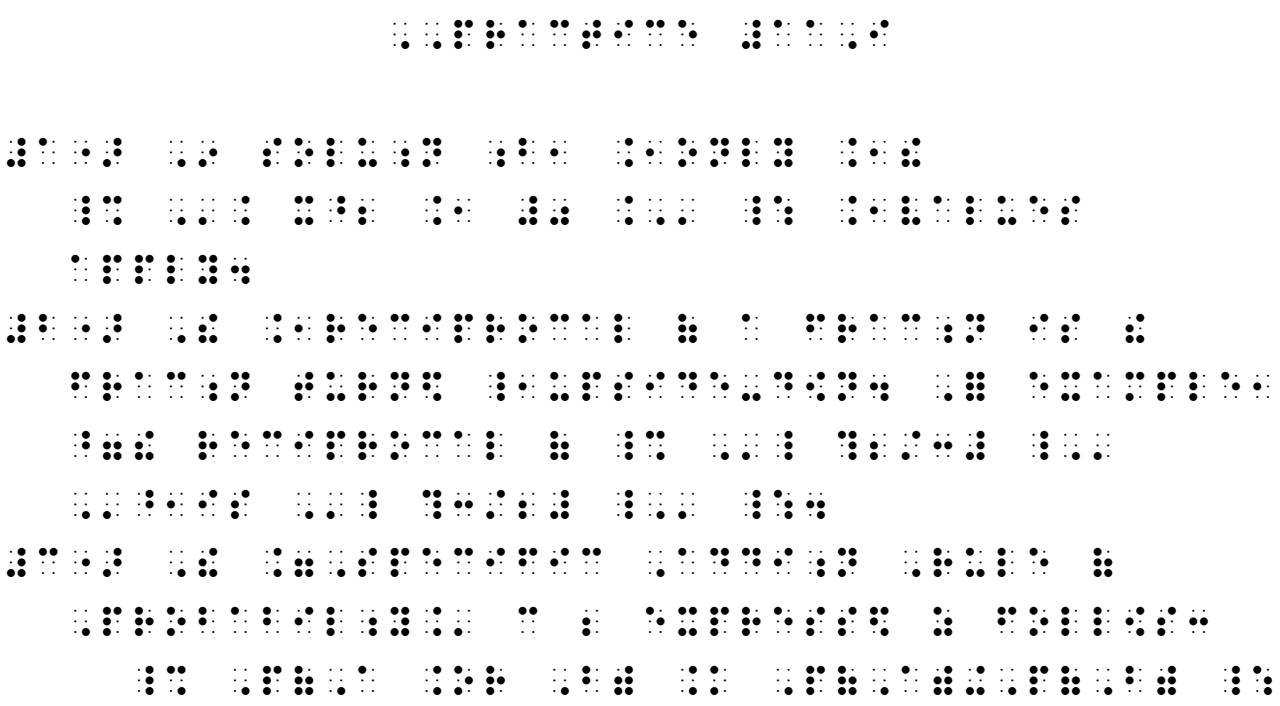


⠠⠠⠠



This is three paragraphs. Math displayed to a 3-1 narrative paragraph begins in cell 3.





EXERCISE 11

Exercise 11 will be available when this course is finished being written and is no longer "Provisional".

Proceed to Lesson 12.

LESSON 12

Read about this PROVISIONAL EDITION in the front matter to this book.
Check the NFB website periodically for updates to this lesson.

- *MODIFIERS AND MODIFIED EXPRESSIONS*
  - *Common Modifiers*
  - *Binomial Coefficient*
  - *Modified Expressions and Superscripts / Subscripts*
  - *Modified Signs of Comparison*
  - *Expressions with More Than One Modifier*

FORMAT
Formal Proof
Spatial Arrangements With Multiplication

Now that you are well along in this introductory course it is a good time to supplement the lesson material by reading about the topic in the Nemeth codebook. Study the examples, follow cross-references, revisit previous subject matter. Become familiar with the arrangement of the codebook and use it as a primary source in the future.

The Nemeth codebook that is currently available applies to a transcription following the rules of *English Braille, American Edition*. The BANA Nemeth Code Technical Committee is updating *The Nemeth Braille Code for Mathematics and Science Notation* to align with the use of Nemeth Code in UEB context. In the interim, supplement the codebook with the paper titled "Guidance for Transcription Using the Nemeth Code within UEB Contexts". The *Guidance* document is available as a free download on the BANA website, www.brailleauthority.org.

MODIFIERS AND MODIFIED EXPRESSIONS

12.1 Definition: A modifier is a symbol or a combination of symbols occurring *directly over* or *directly under* its related symbol or expression. Here are some typical examples.

a question mark over an equals sign	$\overset{?}{=}$	
an underlined digit	$3.1\underline{5}$	
a line over two numerals	$1.\overline{37}$	(signifying a repeating decimal)
an arrow over two letters	$\overleftrightarrow{AB}$	(signifying a line)
a caret over a letter	$\hat{\mathbf{k}}$	(vector notation)

12.2 Construction of Simple Modified Expressions – The Five-Step Rule: An expression modified using the 5-step method is initiated and terminated with special indicators.

Multipurpose Indicator	⠸⠼
Termination Indicator	⠸⠼⠄

The position of the modifier (above or below the expression) is also shown with the use of an indicator.

Directly-Over Indicator	
Directly-Under Indicator	

The process of constructing a modified expression in braille is known as "The Five-Step Rule".

(1) The *multipurpose indicator*  is placed immediately before the expression to be modified.

(2) The *expression* to be modified is written second.

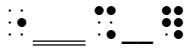
(3) The *directly-over indicator*  or the *directly-under indicator*  is written third to show the position of the modifier.

(4) The *modifying symbol* is written fourth.

(5) The *termination indicator*  is written last to show the end of the modification.

Notice that the multipurpose indicator (dot 5) signals the beginning of the modified expression and the termination indicator signals the completion of the modified expression.

Template for a modifier printed directly above an expression: 

Template for a modifier printed directly below an expression: 

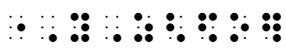
Common Modifiers

The most commonly-used modifiers are presented in this lesson. You have seen many of these symbols in other contexts in previous lessons. Symbols of the code not shown here may also be used as modifiers.

12.3 Arrows as Modifiers: Any of the arrows of the Nemeth Code may be a modifier. Those used in this section are shown below.

Arrow barbed at right, contracted form	⟶	
Arrow barbed at both ends	⟷	
Arrow shaft with hollow dot at both ends	⟜	
Upper barb only, right-pointing	⇀	
Arrow barbed at right with dashed shaft	-->	

A right-pointing arrow in regular type with a full barb and single shaft of ordinary length is brailled in its contracted form when used as a modifier above or below a math expression.

$\overrightarrow{\mathrm{YZ}}$ 

"YZ"  is the expression being modified.
It is modified "directly over"  with a right-pointing arrow

$\underset{\rightarrow}{\mathrm{T}}$ 

"T"  is the expression being modified.
It is modified "directly under" with a right-pointing arrow

The shaft length in print is determined by the width of the expression it modifies. These are not "longer than ordinary" arrow shafts.

$\overrightarrow{M'N'}$ 

The contracted form of the right-pointing arrow is used in braille.
The arrow shaft covers a wide expression.

All other types of arrows require the appropriate symbol.

$\overleftrightarrow{\mathrm{AB}}$ 

"AB" is the expression being modified. It is modified "directly over" with an arrow barbed at both ends.

$\overset{\circ\!\!-\!\!\circ}{\mathrm{AB}}$ 

"AB" is the expression being modified. It is modified "directly over" with an arrow with hollow-dotted ends

$\overrightarrow{\mathrm{OT}}$ 

"**OT**" is the expression being modified. It is modified "directly over" with an arrow with an upper right-pointing barb

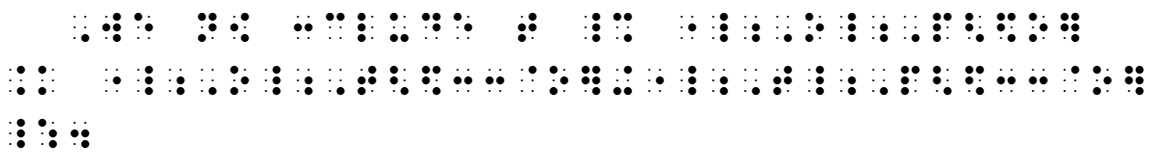
$\overset{\dashrightarrow}{\mathrm{OB}}$ 

"OB" is the expression being modified. It is modified "directly over" with a right-pointing arrow with a dashed shaft.

Example 12.3-1 $\overleftrightarrow{\mathrm{PQ}}$ is a line through points P and Q.



Example 12.3-2 We now conclude that $\overrightarrow{OP} = \overrightarrow{OT} + \overrightarrow{TP}$.



*Reminder from **Lesson 9**: When a linked expression will not fit on one braille line, start a new line with the comparison sign.*

Show exactly what portion of an expression is being modified. In the next example, only the expressions between the grouping symbols are modified. The grouping symbols are brailled outside of the multipurpose and termination indicators.

⋙ $|\overrightarrow{O_1P}| + |\overrightarrow{P_1P_2}|$



"O_1P" and "P_1P_2" are the expressions being modified. Each is modified "directly over" with a right-pointing arrow.

Special Situations Involving Arrows

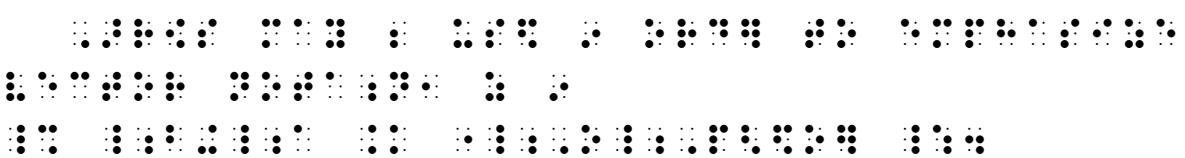
12.3.1 When to Omit Arrows: When *identical arrows* are used above vectors in boldface type *throughout the text*, they are omitted from the braille transcription. (The boldface font attribute of the vectors is considered to be sufficient identification.) If vector arrows are omitted in braille a transcriber's note is required, explaining that the arrows are present in print. For example, on the Transcriber's Notes page the following note should appear: "Arrows printed over vectors are omitted."

⋙ $b + a = \overrightarrow{OP}$

The print copy shows vector OP *with a right-pointing arrow above it. If we assume that <u>all</u> vectors in this book are written this way, the boldface is retained in braille but the arrow is omitted. (Note: Only the letters are printed in boldface. The plus sign and the equals sign are in regular type.)*

However, if the author specifically refers to arrows as a notational device, the arrow is brailled.

Example 12.3-3 Arrows may be used in order to emphasize vector notation, as in $b + a = \overrightarrow{OP}$.



12.3.2 When the Arrow is Being Modified: A right-pointing arrow in regular type with a full barb and single shaft of ordinary length is brailled in its uncontracted form when the arrow itself is being modified.

Arrow barbed at right, uncontracted form	⟶	⠸⠒⠒⠒⠸

$x \xrightarrow{f \circ g} y$ ⠸ ⠐⠸⠒⠒⠸⠒⠒⠸⠒⠒⠸ ⠸

A right-pointing arrow ⠸⠒⠒⠒⠸ is the expression being modified.
It is modified "directly over" ⠐ with the operation "f ∘ g" ⠸⠐⠸

Reminder: In function notation, ∘ is a hollow dot, not the letter "o".

12.3.3 When Other Rules Apply: Some arrangements that may appear to be modified arrows are not brailled using the Five-Step Rule.

Signs of Comparison Compounded Vertically: Horizontal arrows printed one below the other are a sign of comparison compounded vertically. Review this topic in **Lesson 10**.

Chemistry: Words or symbols printed above reaction arrows are not brailled as modifiers. Rules for these constructions are found in the *Braille Code for Chemical Notation.*

$Na_2SO_4 \xrightarrow{H_2O} Na^+ + SO_4^{2-}$

Elementary Mathematics—"ARROW MATH" in Lower Grades: These arrows are not "modified arrows". They are not being used as a sign of comparison, but rather as a "process". Draw these arrow as a tactile graphic. Refer to BANA's *Guidelines and Standards for Tactile Graphics* for techniques.

Example 12.3-4 Use arrow math to add 36 + 23.

$36 \xrightarrow{+20} 56 \xrightarrow{+3} 59 \qquad 36 + 23 = 59$

⠼⠉⠋ ⠼⠑⠉ ⠼⠛⠓ ⠼⠊⠚ ⠼⠑⠓ ⠼⠊⠉ ⠼⠉⠋⠀⠼⠃⠉⠀⠼⠑⠓

⠼⠃⠉ ⠼⠃⠉

⠼⠉⠋ ⟶ ⠼⠑⠓ ⟶ ⠼⠑⠓

⠼⠉⠋⠀⠼⠃⠉⠀⠼⠑⠓ ⠼⠊⠚ ⠼⠑⠓ ⠼⠊⠉

Instructions: Review arrows in **Lesson 10**. *Reminder*: When the print copy arranges non-spatial itemized material side-by-side across the page and there are no subdivisions, the braille format is changed so that all identifiers start in cell 1.

In the sentence at the end of this practice, assume that all vectors in the document are shown using that particular arrow notation. Show the proper way to omit the vector arrows in the transcription. Include the required transcriber's note after the topic heading. (Review "double-struck" letters in **Lesson 7**.)

PRACTICE 12A

Arrows as Modifiers

1. $\overleftarrow{\text{F}}$ 2. $\overleftarrow{\text{AB}}$ 3. $\overleftarrow{\text{CD}}$ 4. $\overrightarrow{\text{OB}} \cup \overrightarrow{\text{OC}}$ 5. $\overrightarrow{\text{EF}}$ 6. $\overrightarrow{\text{GH}}$ 7. $\underleftarrow{\text{T}}$

8. $\overleftrightarrow{\text{XZ}} \parallel \overleftrightarrow{\text{RS}}$ 9. $\overleftrightarrow{\text{AB}} + \overleftrightarrow{\text{CD}}$ 10. $x \overset{g}{\rightarrow} y \overset{f}{\rightarrow} z$

11. $\overrightarrow{\text{OP}} = \mathbf{i}x + \mathbf{j}y$

Vector Addition

$\mathbb{R}$ equals $\overrightarrow{\text{OP}}$ equals $\overrightarrow{\text{OM}}$ plus $\overrightarrow{\text{MC}}$ plus $\overrightarrow{\text{CP}}$.

12.4 Carets as Modifiers

Caret	∧	⠈⠣
Inverted Caret	∨	⠈⠜

The BANA Nemeth Code Technical Committee is discussing the fact that the symbol for the inverted caret is the same construction as the opening Nemeth Code indicator.

⋙ $\hat{k}$ ⠸⠈⠣⠅

Fill in the blanks: "___" is the expression being modified. It is modified "directly _____" with a ____.

⋙ $x_{\vee}$ ⠸⠭⠈⠜

Fill in the blanks: "___" is the expression being modified. It is modified "directly _____" with a ____.

Example 12.4-1 All values of y are located on the regression line $\hat{y} = \alpha + \beta x$.

⠠⠁⠇⠇ ⠠⠧⠁⠇⠥⠑⠎ ⠷ ⠊⠎ ⠍⠁⠙⠑ ⠥⠏ ⠷ ⠎⠊⠝⠛⠇⠑ ⠎⠊⠝⠛⠇⠑ ⠊⠝⠙⠑⠏⠑⠝⠙⠑⠝⠞ ⠧⠁⠗⠊⠁⠃⠇⠑ ⠷
⠠⠽ ⠷ ⠁⠇⠏⠓⠁ ⠠⠃⠑⠞⠁ ⠷ ⠊⠎ ⠇⠕⠉⠁⠞⠑⠙ ⠷ ⠕⠝ ⠷ ⠞⠓⠑ ⠗⠑⠛⠗⠑⠎⠎⠊⠕⠝ ⠇⠊⠝⠑ ⠷ ⠈⠣⠽⠻ ⠿ ⠈⠁ ⠬ ⠃⠭⠲

12.5 Horizontal Bar as a Modifier: When more than one digit or letter or expression is modified by a single horizontal bar, the five-step rule is applied. Only one bar symbol is brailled.

Horizontal Bar	—	⠉

⋙ $\overline{99}$ ⠸⠉⠼⠊⠊

Fill in the blanks: "___" is the expression being modified. It is modified "directly _____" with a ____.

Because the numeral is not preceded by a space, a numeric indicator is not used.

The BANA Nemeth Code Technical Committee is discussing details regarding the treatment of underlined numbers. Transcriptions shown below are according to the current Nemeth Code.

⋙ $\underline{419}$ 

Fill in the blanks: "___" is the expression being modified. It is modified "directly _____" with a ____.

Because the numeral is not preceded by a space, a numeric indicator is not used.

⋙ $\$11.\underline{25}$ 

Fill in the blanks: "___" is the expression being modified. It is modified "directly _____" with a ____.

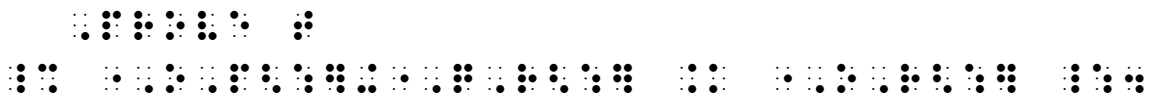
The decimal point is not part of the modified expression. The bar is under the digits only.

⋙ $1.\overline{37}$ 

Fill in the blanks: "___" is the expression being modified. It is modified "directly _____" with a ____.

The decimal point is not part of the modified expression. The bar is over the digits only.

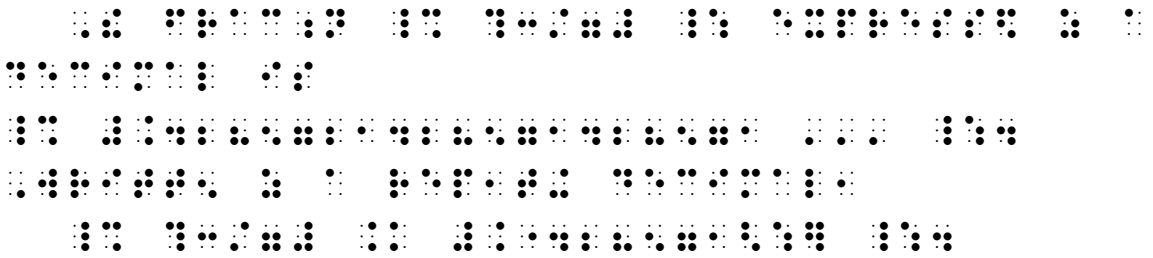
Example 12.5-1 Prove that $\overline{\mathrm{OP}} + \overline{\mathrm{QR}} = \overline{\mathrm{OR}}$.



Example 12.5-2

The fraction $\frac{3}{7}$ expressed as a decimal is .4285714285714285 ... .
Written as a repeating decimal,

$\frac{3}{7} = .\overline{428571}$.



The decimal point is not part of the modified expression. The multipurpose indicator is brailled in the cell immediately preceding the numeral "4".

12.5.1 The Contracted Form of Bar Over/Bar Under: When a single digit or a single letter in any typeform or any alphabet is modified only by a single horizontal bar either directly over or directly under it, the five-step rule is not applied. Instead, the contracted form of modified expressions is brailled.

12.5.1.a Bar Above: When a single digit or a single letter (in any typeform or any alphabet) is modified only by a single horizontal bar directly over it, the symbol for the bar is placed immediately after the digit or letter modified.

⋙ $\overline{x}$

⋙ $\overline{3}$

⋙ $\overline{\mathrm{C}}'$

⋙ $\overline{\lambda}_t$

Example 12.5-3 The $\overline{7}'$s are repeated.

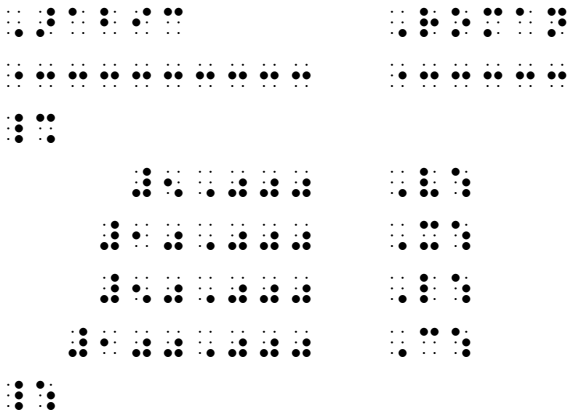


The apostrophe-s relates to the numeral, therefore it is brailled inside the switches. The punctuation indicator prevents the apostrophe from being misread as a prime sign.

Example 12.5-4

Arabic	Roman
5,000	$\overline{\mathrm{V}}$
10,000	$\overline{\mathrm{X}}$
50,000	$\overline{\mathrm{L}}$
100,000	$\overline{\mathrm{C}}$

The Arabic numbers in the first column can be brailled in either UEB or Nemeth Code. The Roman numerals in column two require a code switch because they are modified letters. Smoother reading is accomplished by brailling both columns in Nemeth Code.

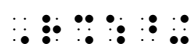


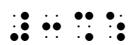
*(Review simple table format in **Lesson 6**.)*

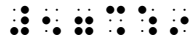
The BANA Nemeth Code Technical Committee is discussing details regarding the treatment of underlined single numbers and single letters in narrative. Transcriptions shown below are according to the current Nemeth Code.

12.5.1.b Bar Below: When a single digit or a single letter (in any typeform or any alphabet) is modified only by a single horizontal bar directly under it, the directly-under indicator followed by the symbol for the bar is placed immediately after the digit or letter modified.

⋙ $\underline{x}$ 

⋙ $\underline{\mathrm{R}}^{+}$ 

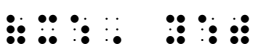
⋙ $\underline{3}$ 

⋙ $5\underline{7}9$ 

More Examples

Study the examples below. Notice when the contracted form of the bar can be used. All simbraille is in Nemeth Code. Switch indicators are omitted.

Examples 12.5-5-11

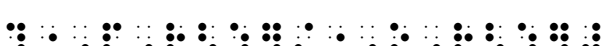
(1) $(\overline{x},\overline{y})$ 

(2) $\overline{x}$'s and $\overline{y}$'s 

(3) $\overline{x+y}$ 

(4) $\underline{x+y}$ 

(5) $A(\underline{s}n)$ 

(6) $\frac{\overline{\mathrm{PR}}}{\overline{\mathrm{OR}}}$ 

(This is a fraction.)

(7) $P(\overline{x}) = \overline{P(x)}$ 

Instructions: Treat the three examples of unit vectors in problem #2 as displayed mathematical material.

PRACTICE 12B

Carets and Bars as Modifiers

1. Unit vectors can be denoted with normal vector notation, i or $\vec{\imath}$, or with standard unit vector notation, $\hat{\mathbf{i}}$, spoken "i-hat".

2. Here are examples of unit vectors in various coordinate systems.

   Cartesian coordinate system: $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, $\hat{\mathbf{z}}$

   Cylindrical coordinate system: $\hat{\boldsymbol{\rho}}$, $\hat{\boldsymbol{\phi}}$, $\hat{\mathbf{z}}$

   Spherical coordinate system: $\hat{\mathbf{r}}$, $\hat{\boldsymbol{\theta}}$, $\hat{\boldsymbol{\phi}}$

3. $1.142857\overline{142857}$

4. $\overline{5}$, $\underline{3}$

5. $\frac{7}{15} = .4\overline{6}$

6. $2 \cdot 3 = \overline{2} \cdot 3 = \overline{2 \cdot 3}$

7. $\overline{\mathrm{PQ}}$, $\overline{\mathrm{x}'}$, $\overline{\mathrm{R}''\mathrm{S}''}$

8. $\overline{\mathbf{s}}$, $\overline{\alpha}$, $\overline{m}'$

9. $\mathrm{m}\overline{\mathrm{BC}} = \mathrm{a}$

10. $\overline{\mathrm{C}} = 100 \times 1000$

11. $F = 2\pi\overline{r}l$

12. $\mathrm{g}(\overline{\mathrm{x}}\,\overline{\mathrm{y}}) = \mathrm{g}(\overline{\mathrm{xy}})$

13. $\mathrm{P}\left(\overline{\mathrm{a} + \mathrm{bi}}\right) = \overline{0} = 0$

14. $3.14\underline{159}$

15. $\hat{\overline{x}}_i = 0.5\left(\overline{x}_i + \underline{x}_i\right)$

12.6 Other Symbols as Modifiers: Apply the five-step rule for the construction of simple modified expressions when the following symbols are used as modifiers.

12.6.1 Dot

Dot	•	⠄

In print, the recurrence of one digit in a decimal numeral may be indicated by one dot over the recurring digit.

⋙ $\dot{4}$ ⠼⠲⠲

"4" is the expression being modified. It is modified "directly over" with a single dot.

⋙ $\frac{1}{6} = 0.1\dot{6}6\ldots$ ⠼⠁⠌⠖⠨⠄ ⠨⠅ ⠼⠁⠋⠁⠃⠃⠃⠲⠲⠲ ⠠⠠⠠

The last "6" is the expression being modified. It is modified "directly over" with a single dot.

The recurrence of one or more digits in a decimal numeral may be indicated by one dot over each recurring digit. In braille *only one dot* is used as a modifier. The dot is placed after the last modified digit in the recurring sequence.

⋙ $1.\dot{3}7\dot{5}$ ⠼⠁⠲⠒⠛⠢

In print, a single dot is shown over each of the three digits 3 7 and 5. In braille, only one dot is used as a modifier. The multipurpose indicator and termination indicator clearly show what is included within the modified expression.

Example 12.6-1 Expressed as a fraction, $0.\dot{9}0\dot{9} = \frac{1}{11}$.

⠠⠑⠭⠏⠗⠑⠎⠎⠑⠙ ⠁⠎ ⠁ ⠋⠗⠁⠉⠞⠊⠕⠝⠂ ⠼⠚⠲⠊⠚⠊ ⠨⠅ ⠼⠁⠂⠁⠁⠲

In print, a single dot is shown over the last digit 0 and a single dot is shown over the last digit 9.

When one or more dots occur over or under a single letter or numeral, the symbol for the dot is used as many times as necessary to conform with the print text.

⋙ $\dddot{x}$ ⠠⠭⠄⠄⠄

⋙ $\underset{\cdot\cdot}{u}$ ⠥⠄⠄

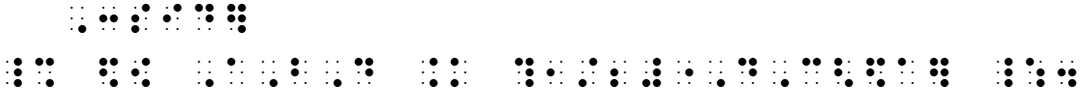
Example 12.6-2 Prove $\ddot{x} = \frac{d^2x}{dt^2}$

⠠⠏⠗⠕⠧⠑ ⠠⠭⠄⠄ ⠨⠅ ⠙⠭⠘⠼⠃ ⠌ ⠙⠞⠘⠼⠃

12.6.2 Arc

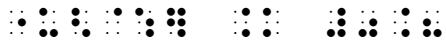
Arc Concave Upward	⌒	⠸⠒

Example 12.6-3 Consider $\angle ABD = \frac{1}{2}\overset{\frown}{DC}$.

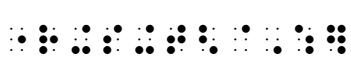


12.6.3 Tilde

Single Tilde	~	⠈⠔
Extended Tilde	~~	⠈⠔⠈⠔

⋙ $\tilde{u} = 0.8$ 

⋙ $\widetilde{A \cup B}$ 

⋙ $\overset{\sim\sim}{r+s+t}$ 

Remember, an extended tilde has more than one peak. The next example is a single tilde even though it is covering several items.

⋙ $\widetilde{r+s}$ 

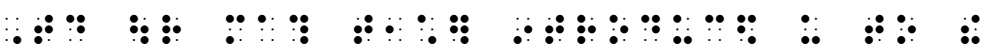
12.6.4 Question Mark: When the question mark is not functioning as a sign of omission, the punctuation mark is brailled. In mathematical context, a punctuation indicator precedes the question mark to prevent it from being misread as the numeral 8.

Question Mark	**?**	⠸⠢

⋙ $\overset{?}{=}$ 

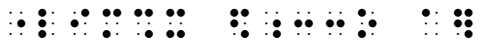
"=" ⠶ is the expression being modified. It is modified "directly over" ⠒ with a question mark ⠸⠢

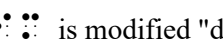
Example 12.6-4 Today our math teacher introduced us to the paradox $0.\overline{9} \overset{?}{=} 1$.

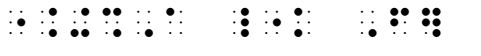




12.7 Expressions as Modifiers: When a modifier is, itself, a mathematical expression, follow Nemeth Code rules regarding spacing and use of indicators. Each example below contains a comparison sign in the modifier: an arrow, the "inclusion" symbol, a "less than" sign, and an "equals" sign. A space is required before and after a sign of comparison. English letter indicators are not needed when a letter immediately precedes or follows a sign of comparison or when a letter touches an indicator; a numeric indicator is required when a numeral follows a space. The termination indicator ⠸⠄ marks the end of the modifier.

⋙ $\lim_{x \searrow a}$ 

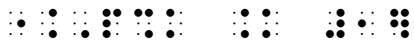
"lim" ⠍⠊⠍ is modified "directly under" ⠰ with the expression $x \searrow a$ ⠰ 

⋙ $\underset{A \subset F}{\cup}$ 

Fill in the blanks: "⋃" ___ is modified "directly under" ____ with the expression $A \subset F$ _____

⋙ $\underset{i < j}{\Sigma}$ 

Sigma is modified "directly under" with the inequality i ＜ j

⋙ $\underset{k = 1}{\prod}$ 

Fill in the blanks: ____ is modified "directly ____" with the equality _____

Reminder: A numeric indicator is required when a numeral follows a space.

Greek capital letters Sigma and Pi are often encountered in this notation.

Sigma	
Pi	

12.7.1 Binomial Coefficient: A binomial coefficient is written as two expressions, one atop the other, enclosed in parentheses. For example, "n choose q" is written like this:

$\binom{n}{q}$

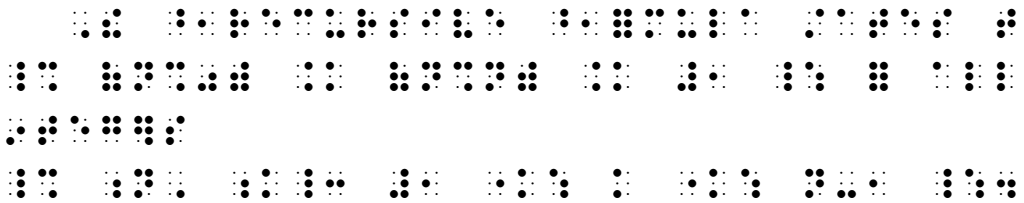
The binomial coefficient does not follow the five-step rule for modified expressions. The opening parenthesis is followed by the upper expression; the directly-under indicator is placed next, followed by the lower expression. The closing parenthesis ends the expression.

Directly-under indicator	⠨

⋙ $\binom{n}{q}$ ⠨⠷ ⠝ ⠨⠆ ⠟ ⠨⠾

⋙ $\binom{n-1}{k-1}$ ⠨⠷ ⠝ ⠤ ⠁ ⠨⠆ ⠅ ⠤ ⠁ ⠨⠾

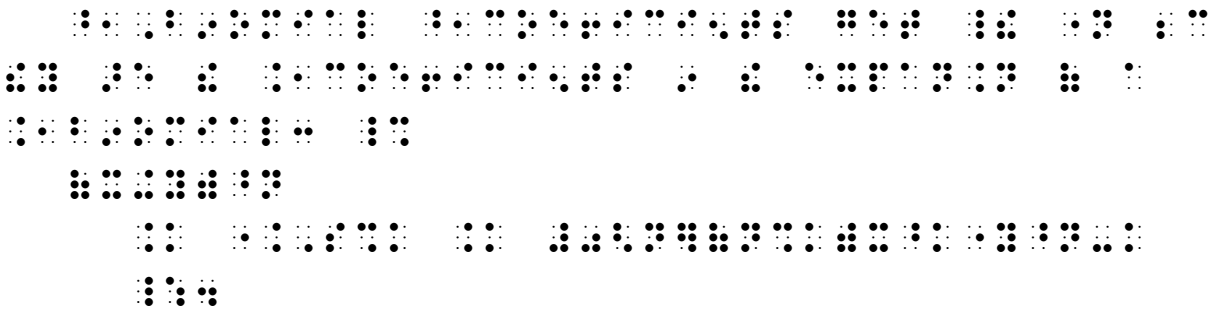
Example 12.7-1 The **recursive formula** states that $\binom{n}{0} = \binom{n}{n} = 1$ for all integers $n, k : 1 \leq k \leq n - 1$.



Reminder from 6.7.16.a – A colon that means "such that" is brailled unspaced from the letter it follows, is preceded by a punctuation indicator, and then is followed by a space. This spacing is applied regardless of the spacing shown in print.

Example 12.7-2 **Binomial coefficients** get their name because they are the *coefficients* in the expansion of a *binomial*:

$$(x + y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}.$$



The linked expression is too long to fit on one line. A new line begins with the equals sign placed in the runover position (cell 5 is the runover position for an expression displayed to narrative). The equals sign in the modifier is not a logical division site because it would disrupt the modifier.

PRACTICE 12C

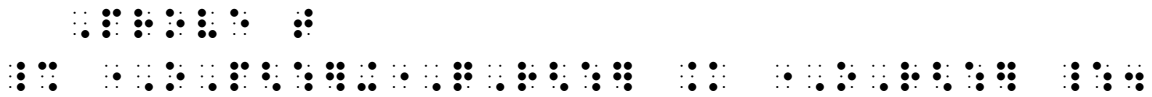
1. In Figure 7.3, if $\overset{\frown}{AB} = \overset{\frown}{CD}$ in circle O, then $\angle AOB = \angle BOC$.
2. $x \stackrel{?}{=} y$ means "does x equal y?"
3. $\sum_{d|n}$ (where d|n means "d divides n").
4. $\binom{t}{p} = R_t^p$
5. $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$ for all integers $n \geq 0$.
6. Does $(\tilde{x}, \tilde{y})$ mean $\tilde{x}$ and $\tilde{y}$?
7. More modified expressions:

   a. $.249\dot{9}$ b. $2.431\dot{3}\dot{1}$ c. $\dot{x}\ddot{y} - \dot{y}\ddot{x}$ d. $\underset{\cdot}{a} + \underset{\cdots}{a} = ?$

   e. $f \to \tilde{f}$ f. $\widetilde{x + y}$ g. $\overbrace{(A \cap B \cap C)}$

12.8 Spacing with Modified Expressions: The spacing before and after an entire modified expression is subject to the spacing rules for the symbols preceding or following it.

Example 12.8-1 Prove that $\overline{\mathrm{OP}} + \overline{\mathrm{QR}} = \overline{\mathrm{OR}}$.



There is no space before or after the operation (plus) sign. There is a space before and after the comparison (equals) sign.

When the modifier is wider than the modified symbol, the print copy will insert extra space to clarify what exactly is being modified. In the next example, only the Sigma is modified, not the letter *a*. The space between the Sigma and the *a* clarifies the extent of the modifier, i < j. In braille, however, indicators define the extent of a modifier. There is no need for the space in braille.

$\Sigma_{i<j}\, a_{ij}$ 

Sigma is modified "directly under" with the inequality $\mathrm{i} < \mathrm{j}$.
The termination indicator signals the completion of the modifier.

PRACTICE 12D

Spacing with Modified Expressions

A. The probability of the event A, written P(A), is defined as

$$\mathrm{P(A)} = \sum_{\wedge} \mathrm{f(x)}$$

where $\sum_{\wedge} \mathrm{f(x)}$ means sum f(x) over those values x_i that are in A.

B. $\underset{\alpha \in A}{\times} \mathrm{A}_{\alpha}$

C. $\underset{i>j}{\prod} (x_i - x_j)$

D. $\overline{7} + 2 \stackrel{?}{=} \overline{7 + 2}$

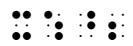
Modified Expressions and Superscripts / Subscripts

12.9 Modified Expression on the Baseline: A modifier and its related expression must be placed on the same level of writing.

12.9.1 Superscript/Subscript After the Modification: When a modifier affects only the expression and not the superscript or subscript, the modified expression terminates before the superscript (or subscript) is brailled.

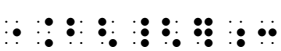
⋙ $\overline{P_1Q}^{\,2} + \overline{QP_2}^{\,2}$ 

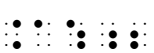
A termination indicator precedes each superscript.

⋙ $\overline{x}^{\,2}$ 

The contracted form of "bar over" does not have an explicit termination indicator. The superscript follows the "bar" symbol, unspaced.

Note that, when a numeric subscript follows a modified letter, a subscript indicator is required. This rule applies whether the letter is modified by the five-step rule or by the contracted form.

⋙ $\hat{\beta}_3$ 

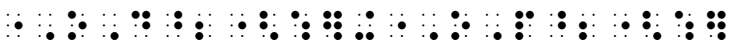
⋙ $\overline{\alpha}_2$ 

⋙ $\underline{\mathrm{N}}_1$ 

12.9.2 Superscript/Subscript Within the Modification: When a modifier affects both an expression on the baseline of writing and its superscript or subscript, the baseline indicator is placed *before* the directly-over (or the directly-under) indicator. This assures that the expression as a whole appears on the same level of writing.

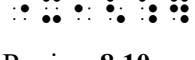
⋙ $\overline{x^2}$ 

The five-step method is used because the modified expression (x^2) is not a single letter. The first dot 5 begins the modified expression. The second dot 5 is a baseline indicator following the superscript "2".

⋙ $\overline{OD^2} + \overline{OP^2}$ 

The first dot 5 begins the first modified expression. The second dot 5 is a baseline indicator following the first superscript "2". Similarly, the third dot 5 begins the second modified expression and the fourth dot 5 is a baseline indicator following the second superscript.

Note that when a numeric subscript which does not require a subscript indicator is part of the modified expression, the subscript indicator is not used.

⋙ $\overline{x_1}$ 

Review **8.10** regarding nonuse of the subscript level indicator.

12.9.3 Binomial Coefficient: Although a binomial coefficient is not technically a "modified expression," notice how this rule applies.

$$\gg \begin{pmatrix} a_x \\ b_y \end{pmatrix}$$



The baseline indicator precedes the "directly under" indicator. This keeps that indicator on the same level of writing as the letters "a" and "b".

PRACTICE 12E

Superscripts and Subscripts

A) $\overline{\mathrm{AB}}^{\,2} + \overline{\mathrm{BC}}^{\,2}$

B) $\overline{\mathrm{A}} = [\,\overline{a}_i]$

C) $\sqrt{\ddot{x}^2 + \ddot{y}^2}$

D) $\underline{\mathrm{Z}}^{\circ}$

E) If $\overline{a}_1 = 72$, find $\overline{a}_7$.

F) Draw $\overrightarrow{\mathrm{P_1P_2}}$ if $\mathrm{P_1}$ is the point (1, 3) and $\mathrm{P_2}$ is the point (2, -1).

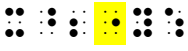
G) $\left(\overline{3^{-1}}\right) \in \mathrm{P}$

H) $\overline{x_1} + \overline{y_1}$

12.9.4 Modified Expression on the Baseline That Follows a Superscript or a Subscript:

When a modified expression written on the baseline of writing immediately follows a superscript or a subscript, several details must be considered in order to determine the necessity of level indicators. Because braille dot 5 has several meanings besides that of baseline indicator (hence the name "multi-purpose indicator") mindful use of that symbol is required in order to give the reader the correct information. Several examples will illustrate. Each "dot 5" is highlighted in the examples.

If a level indicator is needed for the superscript or subscript, the baseline indicator is brailled before starting the modified expression.

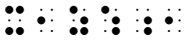
⋙ $x^2\overline{y}$ 

This dot 5 is a baseline indicator, following the superscript 2.
The contracted form of the "bar over" does not use a dot 5.

⋙ $\overline{x}_n\overline{z}_m$ 

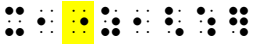
This dot 5 is a baseline indicator, following the subscript n.
The contracted form of the "bar over" does not use a dot 5.

If a subscript indicator is not needed, a baseline indicator is not needed before starting the modified expression.

⋙ $x_1\overline{z}_1$ 

Each 1 is printed at the subscript level.

There are no dot 5s in this example. A baseline indicator is not needed when a subscript indicator is not used. (No subscript indicator is needed for the first subscript "1" because it is a numeral that is a right-subscript to a letter.) The contracted form of the "bar over" does not use a dot 5.

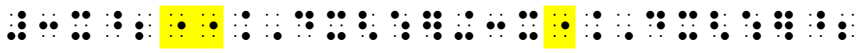
⋙ $x_1\overline{z_1}$ 

Each 1 is printed at the subscript level.

This dot 5 is the start of the modified expression $\overline{z_1}$. A baseline indicator is not needed when a subscript indicator is not used. (No subscript indicator is needed for either subscript because each is a numeral that is a right-subscript to a letter.)

Two indicators may be needed. First, a baseline indicator (dot 5) is used to terminate the effect of the superscript or subscript level indicator. Next, a multipurpose indicator (dot 5) is required to begin the five-step modification.

⋙ $3x^2\overline{\Delta x} + 3x\overline{\Delta x}^{\,2}$



x^2 requires a superscript indicator and the five-step rule is used for the modified expression, so two dot 5s are needed. The baseline indicator is immediately followed by a dot 5 beginning the modified expression $\overline{\Delta x}$.

⋙ $a_k\overline{b_1 \ldots b_p}$



a_k requires a subscript indicator and the five-step rule is used for the modified expression, so two dot 5s are needed. The baseline indicator is immediately followed by a dot 5 beginning the modified expression $\overline{b_1 \ldots b_p}$. Finally, the directly-over indicator is preceded by a baseline indicator to assure that the entire modified expression is brailled on the same level.

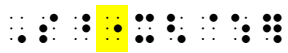
⋙ $a_i \prod_{j \neq i} (A - r_j I)$



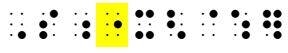
a_i requires a subscript indicator and the five-step rule is used for the modified expression, so two dot 5s are needed. The baseline indicator is immediately followed by a dot 5 beginning the modified expression $\prod_{j \neq i}$

The last dot 5 is a baseline indicator following the subscript "j".

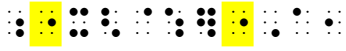
12.10 Modified Expression <u>Within</u> a Superscript or Subscript: Recall that a modifier and its related expression must be placed on the same level of writing. When a modified expression occurs as a superscript or subscript, or as the first part of a superscript or subscript, the level indicator is brailled first, followed by the multipurpose indicator which begins the modified expression. This assures that the expression as a whole appears on the same level of writing.

⋙ $S^{\tilde{x}}$ 

This dot 5 begins the modified expression $\tilde{x}$ (which is in the superscript position). It will not be misread as a baseline indicator because nothing comes between it and the superscript indicator.

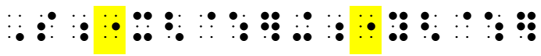
⋙ $S_{\tilde{x}}$ 

This dot 5 begins the modified expression $\tilde{x}$ (which is in the subscript position). It will not be misread as a baseline indicator because nothing comes between it and the subscript indicator.

⋙ $\tilde{x}A_1$ 

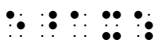
The first dot 5 begins the modified expression $\tilde{x}$ (which is a left-subscript to the letter A). The second dot 5 is a baseline indicator.

If the modified expression occurs in the middle or at the end of the superscript or subscript, the appropriate level indicator must be repeated before the multipurpose indicator to show continuation of the same level of writing. This assures that the multipurpose indicator will not be misread as a baseline indicator.

⋙ $S_{\tilde{x}+\tilde{y}}$ 

The first dot 5 begins the modified expression $\tilde{x}$. The second dot 5 begins the modified expression $\tilde{y}$. To assure that the second dot 5 is not read as a baseline indicator, the subscript level is restated before the dot 5.

Since the multipurpose indicator is absent in the contracted form of "bar over" or "bar under", the level continues with certainty. It is not necessary to repeat the level indicator when the contracted form is used.

⋙ $e^{a\overline{x}}$ 

PRACTICE 12F

More About Superscripts and Subscripts

(1) $a_0\overline{x}^{\,n} + a_1\overline{x}^{\,n-1}$

(2) $W = \frac{2}{3}\pi r^3 \underline{w}\left(h + \frac{3}{8}r\right)$

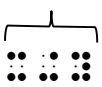
(3) $S^{\tilde{x}}$ and $S^{\tilde{x}+\tilde{y}}$

(4) $D_{\overline{x}}$ or $D_{\overline{x}+\overline{y}}$

(5) $3_{\overline{x}} - 2_{\overline{x}}$

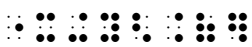
(6) $_{\overline{n}}A_{\,1}$

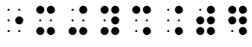
12.11 Horizontal Grouping Signs as Modifiers: It is recommended that a horizontal grouping sign be drawn as a tactile graphic as this method more accurately conveys the meaning of the symbol.

⋙ $\overbrace{x+y}$ 

However, a horizontal grouping sign may be represented in braille symbols as a modifier over or under a mathematical expression. In such a case, the entire modified expression is transcribed in accordance with the five-step

rule. The opening sign of grouping is used when the modifier is above; the closing sign of grouping is used when the modifier is below.

⋙ $\overbrace{x+y}$  *A brace is printed above* $x+y$.

⋙ $\underbrace{x+y}$  *A brace is printed below* $x+y$.

Modified Signs of Comparison

12.12 Definition: A modified sign of comparison consists of a simple sign of comparison, such as the equals sign or the tilde, modified by a caret, dot, triangle, question mark, vertical bar, or any symbol except another sign of comparison. (When a simple sign of comparison occurs above or below another simple sign of comparison the combination is transcribed as a sign of comparison compounded vertically. See **6.8** for a review of that construction.)

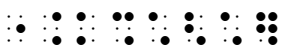
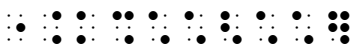
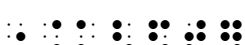
12.13 Transcription: A modified sign of comparison as defined above is transcribed in accordance with the five-step rule for modified expressions.

In addition to the caret and inverted caret seen earlier in this lesson, you may also encounter a left- or right-pointing caret in a modified sign of comparison. Do not confuse these two symbols with the "less than" and "greater than" comparison signs. Ask an expert if context does not clarify the identity of this symbol.

Left-Pointing Caret	<	
Right-Pointing Caret	>	

The following list contains the modified equals signs most commonly used.

Modified Equals Sign

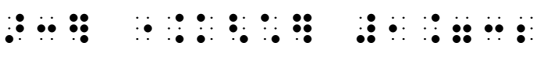
Caret Over	$\overset{\wedge}{=}$	
Caret Under ("is projective to")	$\underset{\wedge}{=}$	
Inverted Caret Over	$\overset{\vee}{=}$	
Left-Pointing Caret Over	$\overset{<}{=}$	
Right-Pointing Caret Over	$\overset{>}{=}$	
Dot Over ("is approximately equal to")	$\dot{=}$	
Dot Over and Dot Under	$\doteqdot$	
Two Dots Over and Two Dots Under	$\ddot{=}$	
Hollow Dot Over ("is equal in degrees to")	$\overset{\circ}{=}$	
Equilateral Triangle Over	$\overset{\triangle}{=}$	

Question Mark Over 

Question Mark Under 

Vertical Bar Over 

We apologize for the blurry images in this lesson. We hope to provide better graphics in future editions.

⋙ $\sqrt{3} \doteq 1.732$ 

⋙ $5(3) + 2(-2) \stackrel{?}{=} 11$ 

If a symbol is encountered which does not appear on this list, construct a symbol in accordance with the same principles. A created symbol should be listed on the Special Symbols page.

⋙ $A \stackrel{m}{=} B$ 

The remainder of the list provided in the code contains other modified comparison signs most commonly used.

Modified Tilde

Dot Under 

Dot Over 

Modified Horizontal Bar

Caret Over 

Caret Under ("is perspective to") 

Dot under

Note: If the horizontal bar is modified by a dot *over* it, the combination is a modified sign of operation ("minus with dot over" signifying "proper difference"). The five-step rule is not used for this symbol. See **6.4.8.**

Instructions: Use the five-step rule to show the horizontal grouping sign in the last item.

PRACTICE 12G

Modified Signs of Comparison and More

1. $A \doteq 3.14r^2$
2. $\angle b \triangleq \frac{1}{2}\overset{\frown}{\mathrm{EB}}$
3. The symbol $\triangleq$ is used to make a definition.
4. $x \dot{\sim} \mathcal{N}(0,1)$
5. $x^n = \underbrace{x \cdot x \cdot x \cdot \ldots \cdot x}$

Expressions with More Than One Modifier

12.14 Modifiers of Higher Order: When two or more modifiers occur one above the other and apply *to exactly the same expression*, the second, third, etc. modifiers are "modifiers of higher order." In such cases, the directly-over indicator or the directly-under indicator is doubled, tripled, etc., before each modifier to indicate its position. The termination indicator is used only after the last modifier shown.

Directly-over indicators	
Second order	⠘⠘
Third order	⠘⠘⠘
Directly-under indicators	
Second order	⠨⠨
Third order	⠨⠨⠨
Termination indicator	⠸⠸

⋙ $\overset{a = 3}{\overline{x + y}}$ ⠘⠘⠶⠭⠀⠽⠻⠀⠘⠶⠀⠼⠉⠻⠸⠸ ⠀ ⠘⠘⠶ ⠀ ⠘⠶⠶⠶

Analysis: The expression "x + y" is modified "directly over" ⠘⠶ with a horizontal bar which, in turn, is modified "directly over" ⠘⠘⠶ with the expression "a = 3".

⋙ $\underset{\substack{a = 3 \\ b = 2}}{\underline{x + y}}$ ⠨⠨⠶⠭⠀⠽⠻⠀⠨⠶⠀⠼⠉⠻⠀⠨⠨⠶⠼⠃⠻⠸⠸ ⠀ ⠨⠨⠶ ⠀ ⠨⠶⠶⠶

Analysis: The expression "x + y" is modified "directly under" ⠨⠨⠶ with a horizontal bar which, in turn, is modified "directly under" ⠨⠨⠨⠨⠶ with the expression "a = 3" which is modified "directly under" ⠨⠨⠨⠨⠨⠶ with the expression "b = 2".

⋙ $\underset{\substack{i,\ j,\ k \\ i < j < k}}{\Sigma}$ ⠨⠨⠶⠎⠊⠛⠍⠁⠀⠊⠀⠶⠀⠚⠀⠶⠀⠅⠀⠨⠨⠶⠶⠶ ⠀ ⠨⠨⠶ ⠀ ⠨⠶⠶⠶

Analysis: The Sigma is modified "directly under" ⠨⠶ with "i, j, k" which, in turn, is modified "directly under" ⠨⠨⠶⠶ with the expression "i < j < k".

Review the rules regarding use of the English letter indicator in **Lesson 4**. An English letter indicator is needed for the first letter "j" because it is a "single letter" according to the definition in the Nemeth Code. The other letters are either touching indicators or are preceded or followed by a sign of comparison, so an English letter indicator is not needed.

12.14.1 Parallel Horizontal Bars: When two or more parallel horizontal bars *are the same length and apply to exactly the same expression*, they are treated as a single modifier. In such cases, the directly-over or the directly-under indicator is used only once, and the symbol for the bar is used as many times as necessary to correspond to the print text.

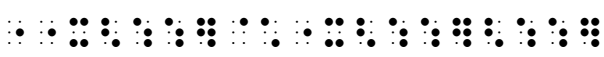
⋙ $\bar{\bar{x}}$ 

⋙ $(\bar{\bar{A}})$ 

⋙ $\underline{\underline{x}}$ 

Attention: Two parallel horizontal bars must not be misinterpreted as the equals sign, and three parallel horizontal bars must not be misinterpreted as the identity sign. The symbol's meaning can be determined by reading the surrounding text for context. See **6.8** ("Signs of Comparison Compounded Vertically") to review equals sign printed above or below a sign of comparison.

12.15 Individual Modifiers: When two or more modifiers do not apply to exactly the same symbols but *cover different portions of the same expression*, the longer modification encloses the entire modified expression. Within the long expression, each inner expression is modified individually.

⋙ $\overline{\bar{x} \times \bar{x}}$ 

Analysis: In the simulated braille below, the longer modification indicators and symbols are highlighted; the inner modified expressions are underlined.



The five-step rule and the contracted form of transcription may be used together.

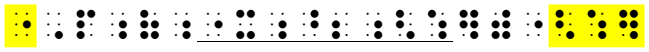
⋙ $\overline{(\bar{a}A + \bar{c}C)}$ 

Analysis: In the simulated braille below, the longer modification indicators and symbols are highlighted; the inner modified expressions are underlined.



⋙ $\overline{P_{(\overline{x^2})}}$ 

Analysis: In the simulated braille below, the longer modification indicators and symbols are highlighted; the inner modified expression is underlined.



Reminder: The subscript level indicator must be repeated before the modified expression indicator so the dot 5 is not misread as a baseline indicator.

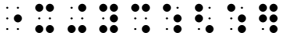
12.16 Simultaneous Modifiers: When an expression is simultaneously modified *both above and below*, the modifier below is brailled first and the modifier above is brailled second. The termination indicator is used only at the end of the entire modification.

⋙ $\sum_{3}^{7}$ 

⋙ $\prod_{k=2}^{6} a_k$ 

Recall from **12.8** that, when the modifier (in this case, k = 2) is wider than the modified symbol (in this case, Pi, Π), the print copy may need to insert extra space to clarify what exactly is being modified. The two letters (Pi and "a") are unspaced in braille because they are part of one math expression.

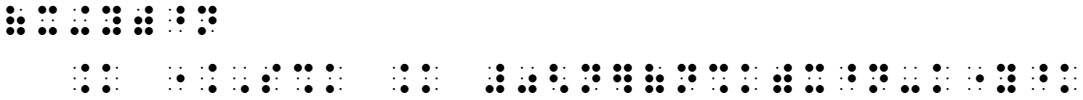
⋙ $\underline{\overline{x}}$ 

⋙ $\underline{\overline{x+y}}$ 

More Examples

As you study these constructions, note the use/nonuse of the English letter indicator and the numeric indicator. All simbraille is in Nemeth Code. Switch indicators are omitted.

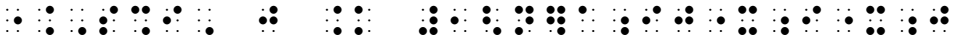
Example 12.16-1 $(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k}y^k$



Example 12.16-2 $\sum_{k=1}^{n}(2k) = n(n+1)$



Example 12.16-3 $\sum_{i,\,j=1}^{n} a_{ij}x_i x_j$



Example 12.16-4 $\sum_{x_1=0}^{1} \sum_{x_2=0}^{1}$



Example 12.16-5 $\bigcup_{r=1}^{n} A_r = A_1 \cup A_2 \ldots$



Example 12.16-6 $\overline{\overline{\prod_{x \in a} c'x}}$



Example 12.16-7 $\cfrac{\cfrac{b=2}{x+y}}{a=3}$



Example 12.16-8 $\sum_{i=1}^{6} \overline{P_{i-1}P_i}$



PRACTICE 12H

Expressions with More Than One Modifier

1. $\overline{\overline{9} \cdot \overline{3}} = \overline{\overline{9 \cdot 3}}$
2. $\overline{\overline{A^n}}$
3. $\overline{A \cap \overline{B \cap \overline{C}}}$
4. $\overline{a_{\overline{n}} + b_{\overline{p}}}$
5. $\overline{x_{\overline{a}} + y^{\overline{n}}}$
6. $\underline{\overline{\mathrm{N}}}$
7. $\sum_{i=1}^{k} \sum_{j=1}^{k}$
8. $\prod_{\substack{j=1 \\ j \neq k}}^{n}$

FORMAL PROOF

12.17 Definition: A proof is a valid argument that establishes the truth of a mathematical statement. A progressive sequence of sentences leads to the conclusion. In a *formal proof,* every step of the argument is shown and each step is supported by a definition or by a previously-proven statement.

12.17.1 Spacing and Margins: A line is left blank before the beginning and after the end of the entire formal proof. The narrative portion of the formal proof begins in cell 3 and its runovers begin in cell 1. If the narrative is in the form of a labeled statement, follow the format directives outlined in **Lesson 11**.

12.17.2 Auxiliary Captions: If the proof contains auxiliary captions such as *Given, Prove,* or *Conclusion*, etc., such captions begin a new paragraph in cell 3 with runovers in cell 1. A line is not skipped above a caption. If a caption is printed in a nonregular typeface, UEB typeform indicators are applied in accordance with the print text. Print capitalization is maintained.

12.17.3 Step-Number Format: When a formal proof is printed in two columns, the layout is changed to "list" format in the braille transcription as follows.

- A line is left blank before the beginning of the list.
- The first item from the left column is placed in cell 1, with any runovers beginning in cell 3. The item must be labeled with an identifier as described below.
- Its matching item from the right column is placed on the next line, also beginning in cell 1, with any runovers beginning in cell 3. This item must also be labeled with an identifier as described below.
- A line is left blank after the end of the list.

Step Identifiers: Typically, the print copy includes the column headings "Statements" and "Reasons". The steps might be consecutively numbered. If this is the case, the letter "S" for *Statement* and "R" for *Reason* is placed immediately after the appropriate step number: "1S", 1R"; "2S", 2R"; etc. Identifiers are transcribed in either code (UEB or Nemeth) according to code-switching guidelines established in Lesson 3. If steps are not numbered in print, identifiers are added to the braille transcription in the manner stated above. If other column headings appear in print, the transcriber chooses an appropriate letter to represent each heading. If no column headings appear in print, use "S" for the left-column items and "R" for the right-column items.

A transcriber's note or a note on the Transcriber's Notes page is included to explain the change in format and to specify the meaning of the letters used to represent the headings. Follow guidelines for the Transcriber's Notes page as presented in *Braille Formats*. For example, "Formal proofs printed in columns headed "Statements" and "Reasons" are brailled as follows: An S or R is added to the step number to show the column in which the step appears. Each step from the Statements column is immediately followed by the corresponding step from the Reasons column."

Example 12.17-1

Theorem 2. All right angles are equal.
Given: ∠ ABC and ∠ DEF are right angles.
Prove: ∠ABC equals ∠DEF.
Proof:

Statements	Reasons
1. ∠ ABC and ∠ DEF are right angles.	1. Given.
2. ∠ ABC = 90°; ∠ DEF = 90°.	2. A right angle contains 90 degrees.
3. ∠ ABC = ∠ DEF.	3. Transitivity postulate.

7 (blank line)

8 ⠀⠀⠀

PRACTICE 12I

Instructions: Braille a Transcriber's Notes page that might appear in a volume which contains the proof shown above. Refer to *Braille Formats* for the structure of a Transcriber's Notes page.

A statement citing the codebook is required in every braille volume that uses the Nemeth Code. In the first paragraph, state the title and edition of the Nemeth codebook as well as any Updates. Something like this:

> Mathematical content is transcribed according to *The Nemeth Braille Code for Mathematics and Science Notation, 1972 Revision,* 2007-2015 Updates including the *Guidance for Transcription Using the Nemeth Code within UEB Contexts.*

In the second paragraph, explain the step-number format as described in **12.17.3**.

Now transcribe this algebraic proof.

PRACTICE 12J

Given: $3x = 7 - \frac{1}{2}x$

Prove: $x = 2$

STEP	REASON
1. $3x = 7 - \frac{1}{2}x$	1. GIVEN
2. $6x = 14 - x$	2. Multiplication Property
3. $7x = 14$	3. Addition Property
4. $x = 2$	4. Division Property

Lesson 10 looked at spatially-arranged addition problems. We now examine multiplication problems, which share some of the same rules.

SPATIAL ARRANGEMENT WITH MULTIPLICATION

The parts of a spatial multiplication problem are labeled below. Note that the multiplication cross is the standard sign used in vertically-arranged multiplication problems.

```
  2     multiplicand
× 3     multiplier
-----   separation line
  6     product
```

Review of Format for Spatial Arrangements

1. A blank line is required above and below a spatial arrangement. (10.22)
2. The numeric indicator is not used. (There are some exceptions.) (10.17)
3. An identifier associated with a spatial arrangement is positioned according to rules applied to the specific topic. (10.27)
4. Side-by-side arrangement is allowed according to certain spacing rules. (10.27.1)
5. If a spatial arrangement is brailled on lines 1-2, any symbol of the arrangement cannot fall within three blank spaces of the first symbol of the print page number on line 1. If a spatial arrangement is brailled on lines 24-25, any symbol of the arrangement cannot fall within three blank spaces of the first symbol of the braille page number on line 25. (10.27.2)

12.18 Alignment: In a spatial arrangement for multiplication, the multiplier and multiplicand must be aligned the same way as in the print copy. Any associated symbols such as dollar signs, commas, and decimal points correspond to the print placement.

12.19 Placement of Multiplication Symbol: The multiplication sign must immediately precede the multiplier, regardless of print placement. The multiplication sign is not always present. If there is no symbol, examine the surrounding text to determine that this is indeed a multiplication problem. Then apply alignment rules for multiplication.

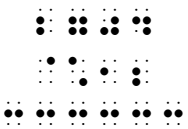
12.20 Separation Line: The separation line extends one cell to the left and to the right of the longest entry appearing above or below it. If there is more than one separation line in a given arrangement/problem, each must be the same length regardless of the way it is printed.

Separation Line (varying in length) ⠒⠒ ⠒⠒ ⠒⠒ ⠒⠒ ⠒⠒ ⠒⠒

Examples Code switch indicators are not shown in the examples in this section.

Example 12.20-1

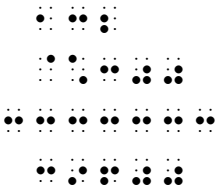
2704
× 12



Alignment: Digits in the multiplier and multiplicand (lines 1 and 2) are vertically aligned the same way as in the print copy. *Placement of Multiplication Symbol:* The multiplication cross is unspaced from the multiplier (line 2) even though this symbol is printed further to the left. *Separation Line:* The separation line extends one cell to the left and to the right of the longest entry appearing above or below it.

Example 12.20-2

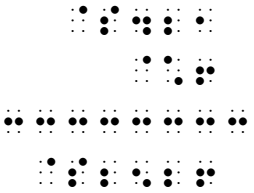
132
× 300
39600



Alignment: Digits in the multiplier and multiplicand (lines 1 and 2) and in the product (line 4) are vertically aligned the same way as in the print copy. *Placement of Multiplication Symbol:* The multiplication cross is unspaced from the multiplier (line 2) even though this symbol is printed further to the left. *Separation Line:* The separation line extends one cell to the left and to the right of the longest entry appearing above or below it.

Example 12.20-3

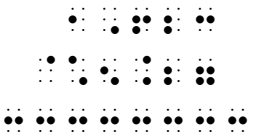
$421
× 6
$2526



Spacing and Alignment: The dollar signs correspond to the print placement.

Example 12.20-4

1,623
× 5.27



Spacing and Alignment: The commas and decimal points correspond to the print placement.

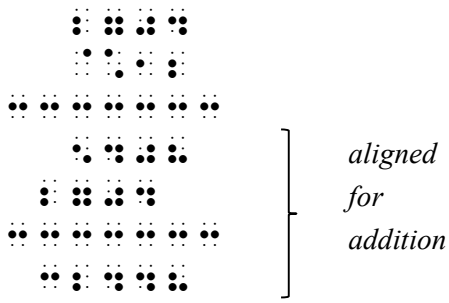
Alignment of Partial Products

12.21 Partial Products: When partial products are shown in a sample problem, note that the final answer (the product) is obtained by *adding* the partial products. Hence, partial products and the final product (the answer) must be aligned for addition.

```
  2704      (multiplicand)
× 12        (multiplier)
  5408      partial product
 2704       partial product      } an addition problem
 32448      product
```

Example 12.21-1

```
  2704
× 12
  5408
 2704
 32448
```



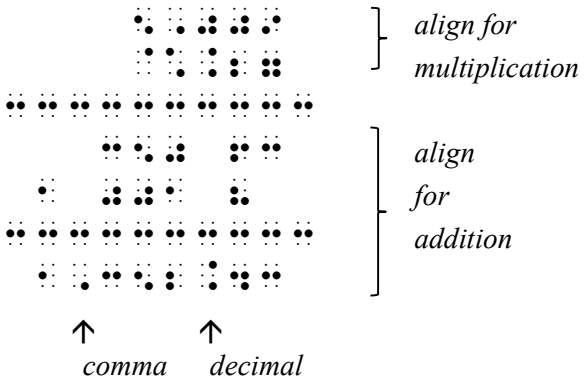
Alignment of Partial Products and Final Product: Lines 4 and 5 constitute the partial products. Alignment follows print. The final product (line 7) is aligned by place value according to the rules of addition.

Separation Lines: In braille, all separation lines are the same length and in the same cells even though they may not appear this way in print.

12.21.1 Spacing: If the product contains a comma or a decimal point, the corresponding cells in the partial products above it are left blank. No blank cells are inserted in the separation lines.

Example 12.21-2

```
  5,009
  ×.27
  35063
 10018
1,352.43
```



Instructions: Use side-by-side layout, leaving one blank space between the end of one separation line and the beginning of the next.. Review this format in **Lesson 10.** Include Nemeth Code switch indicators in your transcription.

PRACTICE 12K

```
  $98        $33        12.12        7,165
 × 100      × 200      × 15.3       ×  85

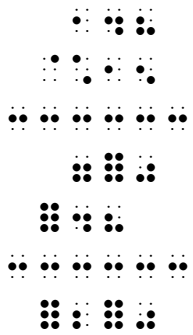
  $98        $33         12.12        7,165
 × 100      × 200       ×  15.3      ×   85
$9,800     $6,600         3636       35 825
                         6060       573 20
                        1212        609,025
                        185.436
```

12.22 Omissions: In order to maintain necessary alignment when an arrangement contains omissions, the long dash, the ellipsis, or shape indicators are not used. *Only the general omission symbol is used*. The same number of omission symbols as shown in print should be brailled.

General Omission Symbol	⠻

Example 12.22-1

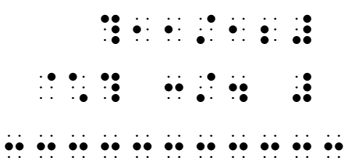
```
 148
×15
 7□0
□48
□2□0
```



12.23 Fractions and Mixed Numbers: In a multiplication problem which contains fractions and mixed numbers, the terms, indicators, and place values are aligned vertically.

Example 12.23-1

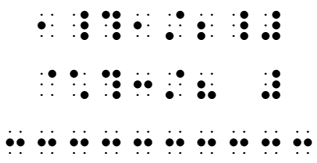
$$\begin{array}{r} \frac{11}{12} \\ \times \frac{3}{4} \\ \hline \end{array}$$



The fraction indicators and the fraction lines are vertically aligned. Numerators and denominators touch the fraction line.

Example 12.23-2

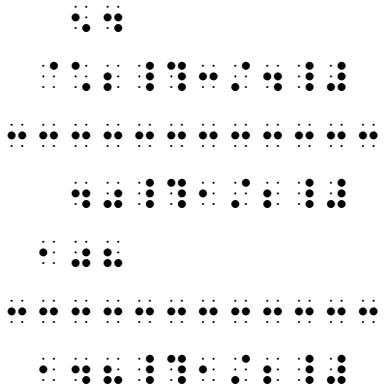
$$\begin{array}{r} 1\frac{1}{2} \\ \times \frac{3}{8} \\ \hline \end{array}$$



The fraction indicators align, including the fractional part of the mixed number. The multiplication cross touches the multiplier.

Example 12.23-3

$$\begin{array}{r} 54 \\ \times 2\frac{3}{4} \\ \hline 40\frac{1}{2} \\ 108 \\ \hline 148\frac{1}{2} \end{array}$$

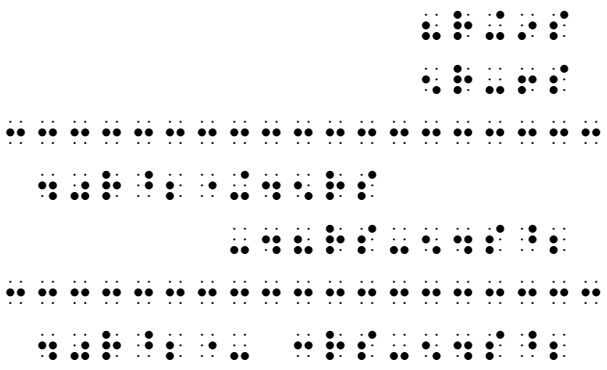


Place values and indicators are aligned throughout.

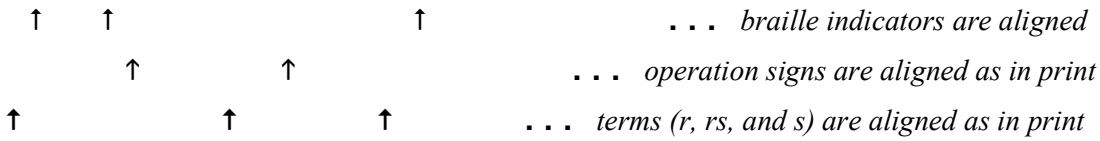
12.24 Polynomials: In a multiplication problem which contains polynomials, the terms and indicators are aligned vertically in the partial products and final product. Above the first separation line, the multiplicand and multiplier are aligned as in the print copy.

Example 12.24-1

$$\begin{array}{r} 8r+9s \\ 5r-6s \\ \hline 40r^2+45rs \\ \quad -48rs-54s^2 \\ \hline 40r^2-\ 3rs-54s^2 \end{array}$$



aligned for addition



... *braille indicators are aligned*

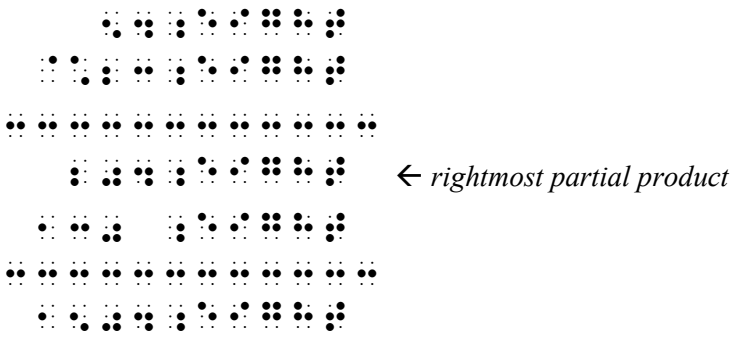
... *operation signs are aligned as in print*

... *terms (r, rs, and s) are aligned as in print*

12.25 Subscripts Denoting Nondecimal Bases: In arrangements which show multiplication of non-decimal bases, the subscript indicators are vertically aligned. The rightmost partial product sets the location of this alignment. This may differ from the spacing shown in the print copy.

Example 12.25-1

$$\begin{array}{r} 54_{\text{eight}} \\ \times\ 23_{\text{eight}} \\ \hline 204_{\text{eight}} \\ 130_{\text{eight}} \\ \hline 1504_{\text{eight}} \end{array}$$



← *rightmost partial product*

↑ *subscript indicators aligned*

12.26 Carried Numbers with Multiplication: If carried numbers are shown, follow the rules for carried numbers with addition. (**10.28**) A line of carried number indicators the same length as the separation line is inserted between the carried numbers and the first line of the multiplication problem. A carried number line is brailled whether or not the line appears in the print copy. Carried numbers should appear in the same columnar position as in print.

Carried Number Indicator (varying in length)	⠈⠈ ⠈⠈ ⠈⠈ ⠈⠈ ⠈⠈ ⠈⠈

Example 12.26-1

```
  1 15
 5,319
   × 6
------
31,914
```

⠀⠁ ⠀⠁⠃
⠈⠈ ⠈⠈ ⠈⠈ ⠈⠈ ⠈⠈ ⠈⠈ ⠈⠈ ⠈⠈ ⠈⠈
⠁⠃ ⠁⠁ ⠈⠁ ⠁⠃
⠀⠀ ⠀⠃ ⠃⠃
⠈⠈ ⠈⠈ ⠈⠈ ⠈⠈ ⠈⠈ ⠈⠈ ⠈⠈ ⠈⠈ ⠈⠈
⠈⠈ ⠈⠁ ⠁⠁ ⠁⠃ ⠁⠁ ⠈⠁

12.27 Placement of Identifiers with Spatial Multiplication: An identifier, if present, is placed on the first line of the multiplication problem (the multiplicand) regardless of its placement in print. If there are carried numbers, the identifier is still placed on the line with the multiplicand.

Example 12.27-1

```
1.  19        2.  15
                  319
   × 6            × 6
------        ------
   54            1914
```

(blank line)

⠀⠁ ⠀⠁
⠈⠈ ⠈⠈ ⠈⠈ ⠈⠈ ⠈⠈ ⠈⠈
⠁⠁ ⠈⠁ ⠁⠃ ⠁⠁ ⠈⠁ ⠁⠃
⠀⠃ ⠃⠃ ⠀⠃ ⠃⠃
⠈⠈ ⠈⠈ ⠈⠈ ⠈⠈ ⠈⠈ ⠈⠈
⠁⠃ ⠁⠁ ⠁⠃ ⠁⠁

(blank line)

Reminders: One blank space comes between the last symbol in the identifier and the symbol furthest left in the overall arrangement, including separation lines. No symbol of one spatial arrangement or its identifier may be less than three cells distant from any symbol on any line of a neighboring arrangement or its identifier, except at the ends of separation lines.

Instructions: Use side-by-side layout. Be sure that three blank cells come between any symbol on any line of one spatial arrangement and any symbol on any line of a neighboring arrangement (separation lines excluded). Include Nemeth Code switch indicators in your transcription.

PRACTICE 12L

1) $\begin{array}{r} \frac{1}{2} \\ \times \frac{15}{16} \\ \hline \end{array}$

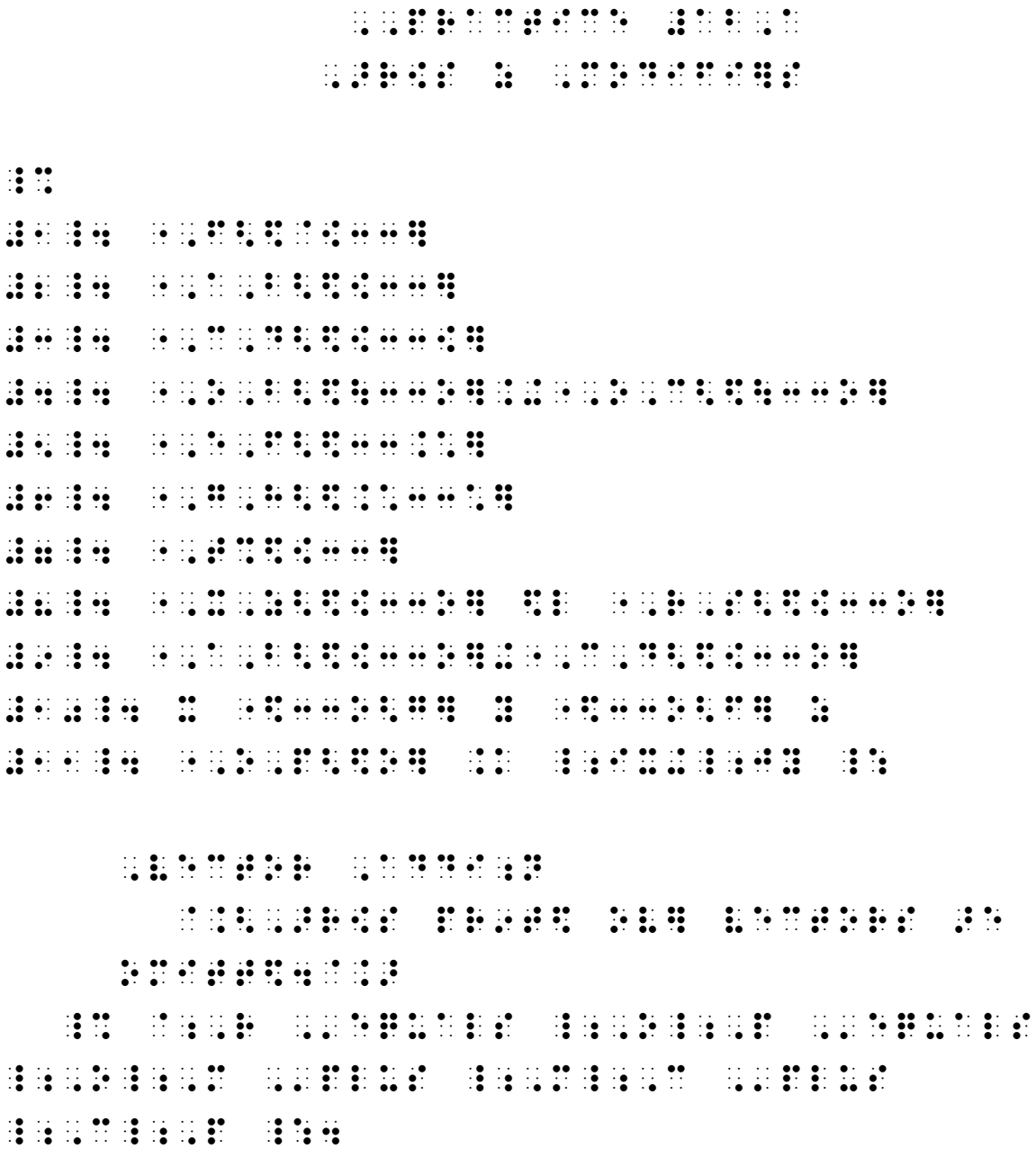
2) $\begin{array}{r} 9\frac{3}{4} \\ \times 4\frac{7}{12} \\ \hline \end{array}$

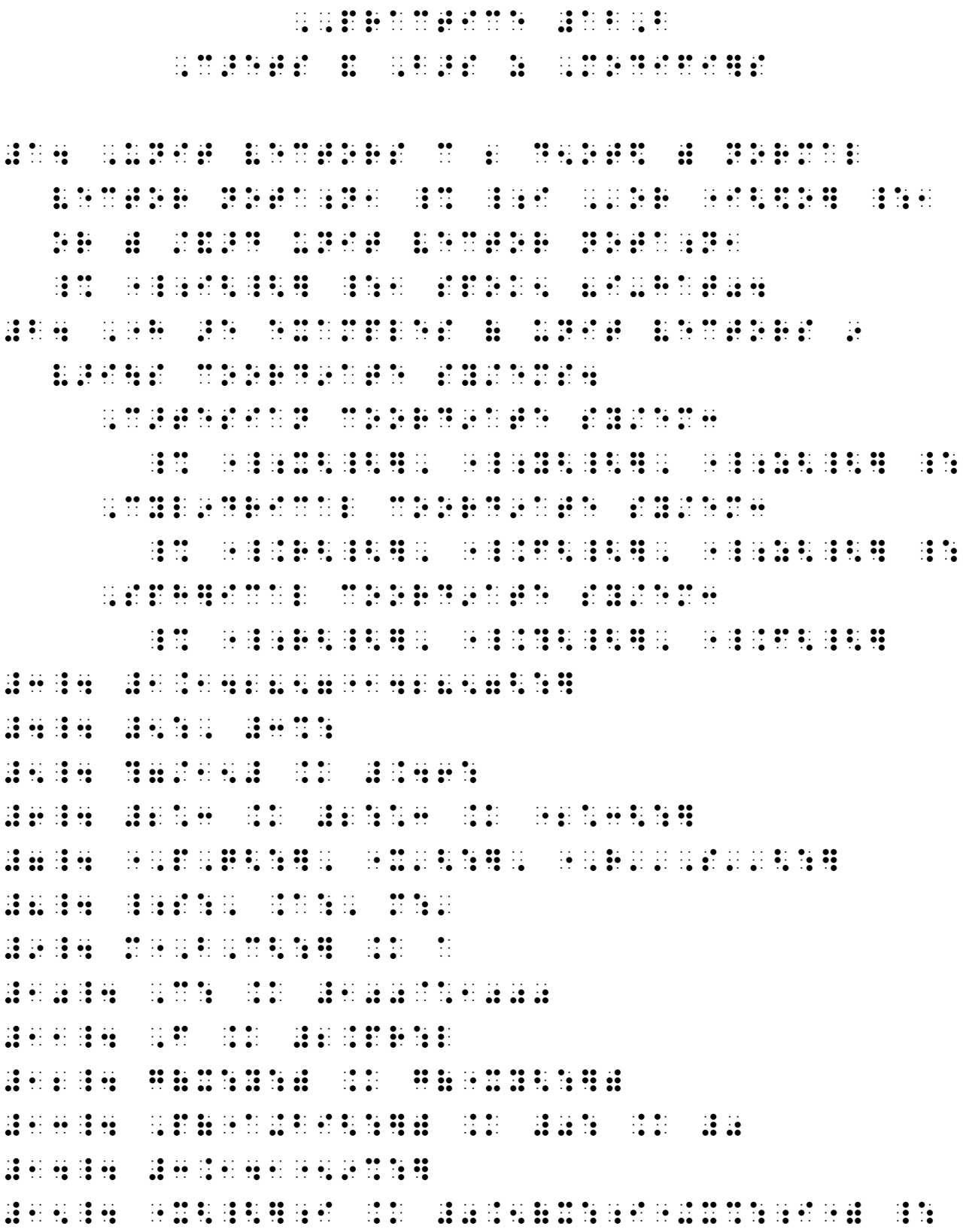
3) $\begin{array}{r} 33 \\ 999 \\ \times\ 4 \\ \hline 3,996 \end{array}$

4) $\begin{array}{r} 3p+6q \\ 11p-2q \\ \hline 33p^2+66pq \\ -\ 6pq-12q^2 \\ \hline 33p^2+60pq-12q^2 \end{array}$

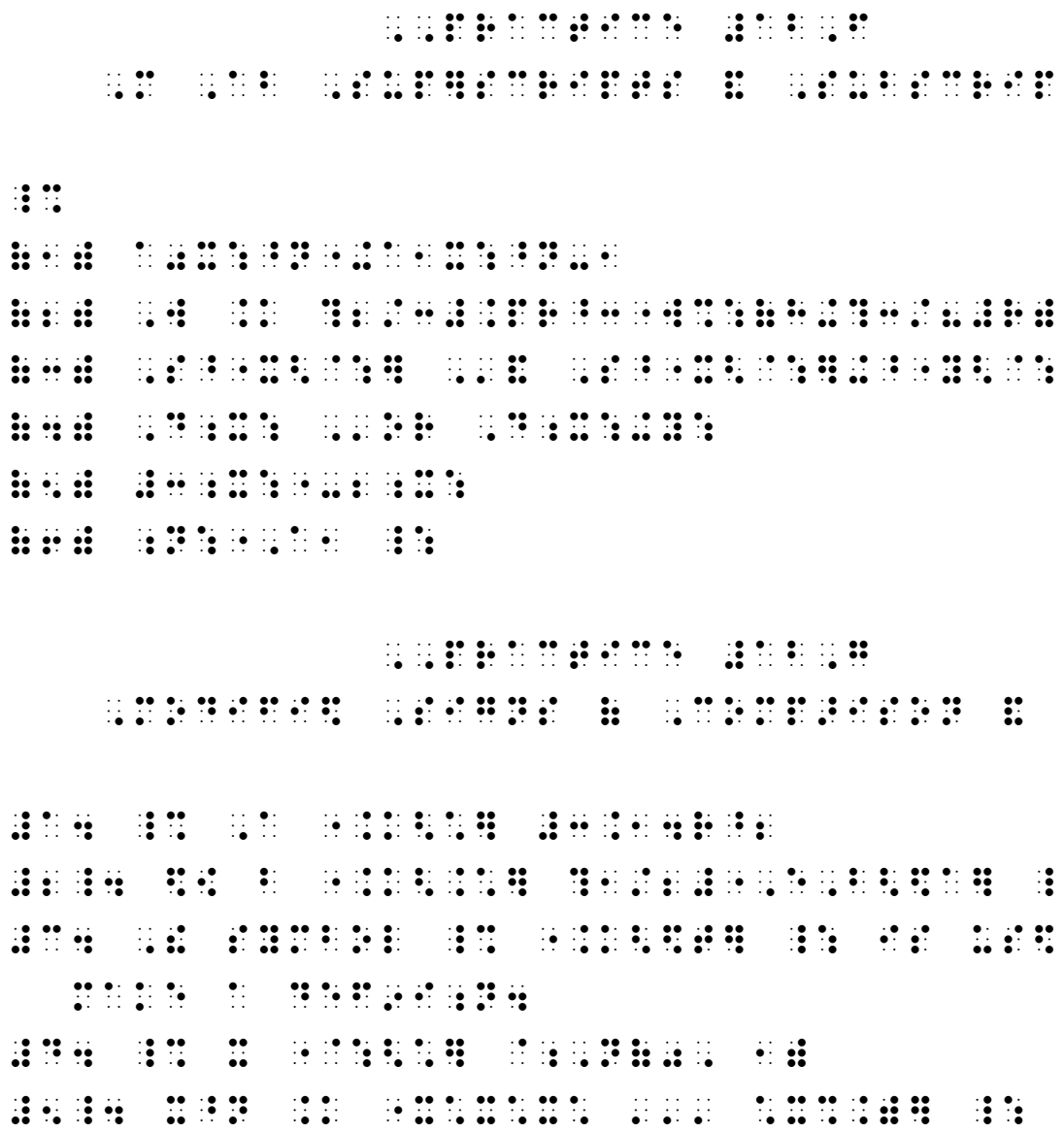
For further practice, see Appendix A—Reading Practice.

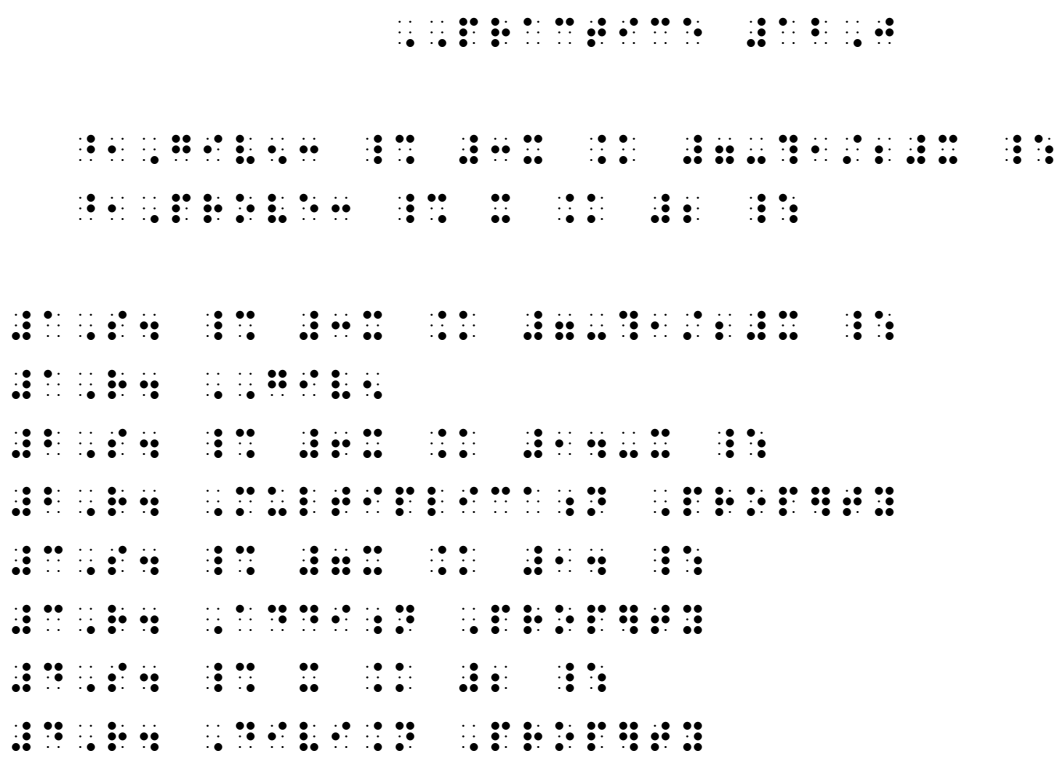
ANSWERS TO PRACTICE MATERIAL



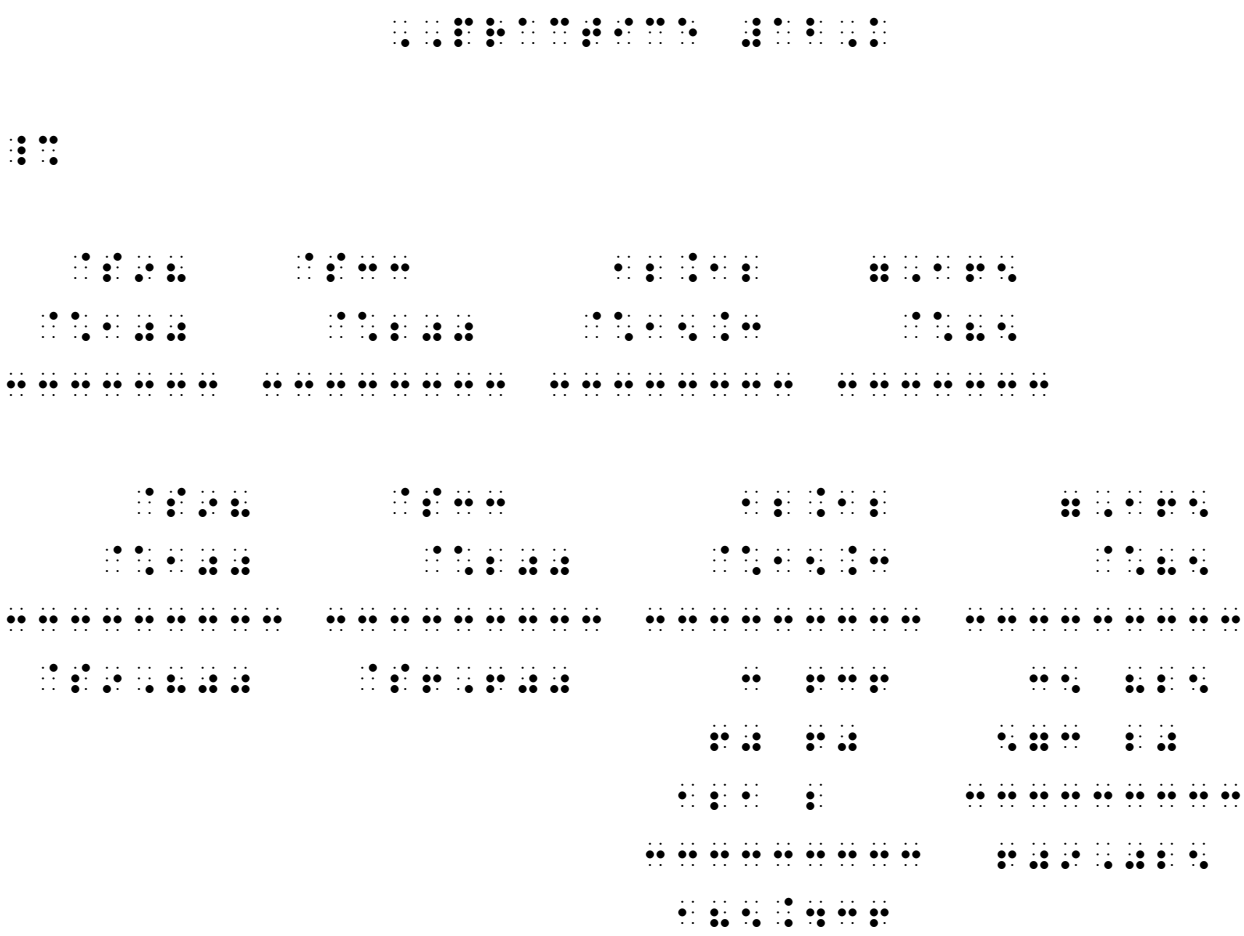


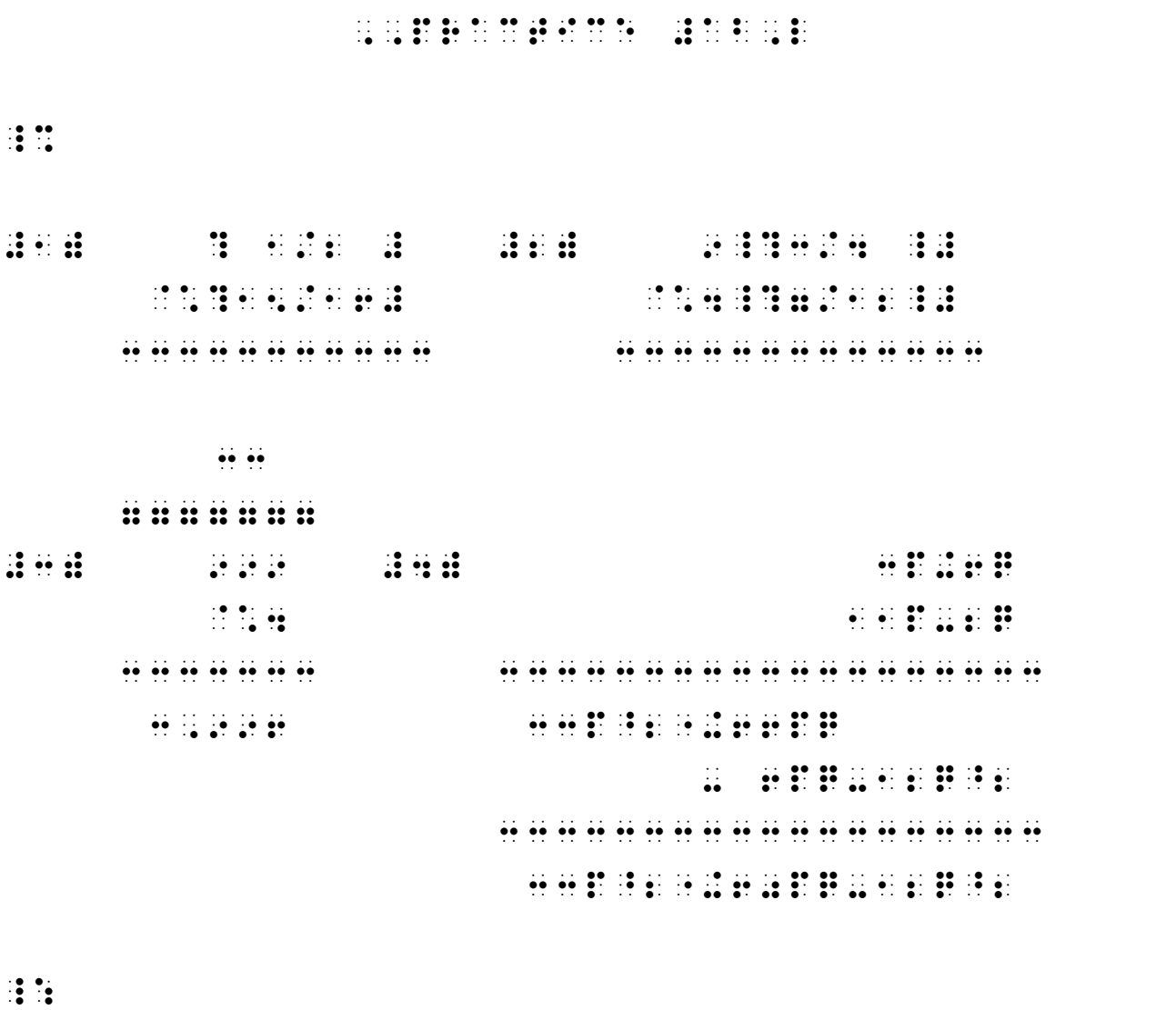
Note: The opening Nemeth Code indicator that starts line 15 may be placed at the end of line 14 instead. This transcription maintains the pattern shown on lines 11 and 13.





It is acceptable not to terminate Nemeth Code on line 3, using the single-word switch for ***Prove****. However, all step numbers are to be brailled in UEB.*





EXERCISE 12

Exercise 12 will be available when this course is finished being written and is no longer "Provisional".

Proceed to Lesson 13.

LESSON 13

Read about this PROVISIONAL EDITION in the front matter to this book.
Check the NFB website periodically for updates to this lesson.

- *MISCELLANEOUS SYMBOLS*
  - *Unspaced Miscellaneous Symbols*
  - *Spaced Miscellaneous Symbols*
  - *Spacing with the Angstrom Unit and Tally Marks*
- *SUPERPOSED SIGNS*
- *AMBIGUOUS SIGNS*
- *MULTIPURPOSE INDICATOR*

FORMAT
Division Problems
Spatial Arrangements With Division

MISCELLANEOUS SYMBOLS

In this lesson, symbols are grouped according to spacing rules: unspaced, spaced, and those with special rules. Within each section, the symbols are presented in alphabetical order.

Unspaced Miscellaneous Symbols

13.1 Spacing Rules for Unspaced Symbols: No space is left between the symbols listed below and any other symbol or quantity to which they apply, regardless of print spacing idiosyncrasies. *However*, a space must be left between these symbols and a word, an abbreviation, a sign of comparison, or other symbol which specifically requires a space before or after it.

13.1.1 Caret

Caret	^	⠈⠢

In addition to the caret being used as a modifier above or below a mathematical expression as shown in **Lesson 12,** it may be used as a place indicator with the decimal point to show the position to which the decimal point has been moved. Used as a place indicator, it is treated as a numeric symbol.

⠸⠸ .37^688 ⠼⠲⠉⠛⠈⠢⠓⠓⠓

Example 13.1-1 The caret (^) shows the place to which the decimal is moved: .37^688

⠼⠲⠉⠛⠈⠢⠓⠓⠓

13.1.1.a Use of the UEB Caret: The caret may be brailled more than one way within a document. The distinction is based on meaning. In a technical context, use the Nemeth Code caret. In nontechnical context, use the UEB caret ⠈⠢ to indicate the insertion of literary material, or the UEB circumflex ⠘⠢ to indicate a modified letter or accent. In UEB context, follow UEB rules for spacing of the caret or circumflex symbol.

13.1.2 Crossed Letters

Crossed d	đ	⠨⠙
Crossed h	ħ	⠨⠓
Crossed lambda	ƛ	⠨⠇⠆
Crossed R	℞	⠨⠗⠆

⋙ 2đv ⠼⠃⠨⠙⠧

⋙ 2πħv = ϕe ⠼⠃⠏⠨⠓⠧⠀⠨⠑

⋙ 3ƛ ... nƛ ⠼⠉⠨⠇⠀⠀⠀⠝⠨⠇

⋙ n = 3℞ ⠝⠀⠀⠼⠉⠨⠗

Since the "crossed d" is the same symbol as the UEB transcriber-defined shape indicator (often used to define an icon), if the UEB symbol is also used in the volume, both symbols should be listed on the Special Symbols page. Here is a sample.

⠠⠠⠨⠙⠀⠠⠠⠨⠓⠀⠠⠠⠨⠇⠆⠀⠠⠠⠨⠗⠆
⠀⠀⠀⠨⠙⠀⠀⠨⠓⠀⠀⠨⠇⠆⠀⠀⠨⠗⠆

⠠⠠⠠

⠠⠠⠨⠙ ⠠⠠⠨⠓ ⠠⠠⠨⠇⠆ ⠠⠠⠨⠗⠆ ⠠⠠⠨⠙ ⠠⠠⠨⠓ ⠠⠠⠨⠇⠆ ⠠⠠⠨⠗⠆
⠠⠠⠨⠙ ⠠⠠⠨⠓ ⠠⠠⠨⠇⠆ ⠠⠠⠨⠗⠆ ⠠⠠⠨⠙ ⠠⠠⠨⠓ ⠠⠠⠨⠇⠆ ⠠⠠⠨⠗⠆
⠠⠠⠨⠙ ⠠⠠⠨⠓ ⠠⠠⠨⠇⠆ ⠠⠠⠨⠗⠆ ⠠⠠⠨⠙ ⠠⠠⠨⠓ ⠠⠠⠨⠇⠆ ⠠⠠⠨⠗⠆
⠠⠠⠨⠙ ⠠⠠⠨⠓ ⠠⠠⠨⠇⠆ ⠠⠠⠨⠗⠆

13.1.3 Del

Del (nabla, gradient)	∇	⠸⠸

When the inverted triangle is used as a sign of omission, it is spaced according to the rules presented in **11.27**. In all other cases it is spaced in accordance with **13.1**.

⋙ s∇t+t∇s ⠎⠸⠸⠞⠬⠞⠸⠸⠎

⋙ $\nabla(r^n)$ ⠸⠸⠐⠣⠗⠰⠝⠐⠜

Example 13.1-2 The symbol "∇" is called "del" as in $\mathbf{T} \cdot \nabla\ (r_1 + r_2)\ = 0$.

⠸⠸ ⠠⠞⠐⠣⠸⠸⠠⠞⠐⠜ ⠸⠸ ⠠⠞⠐⠣⠠⠗⠂⠬⠠⠗⠆⠐⠜ ⠸⠸ ⠠⠞⠐⠣⠸⠸⠠⠞⠐⠜ ⠠⠞⠐⠣⠠⠗⠂⠬⠠⠗⠆⠐⠜
⠠⠞⠐⠣⠸⠸⠠⠞⠐⠜ ⠸⠸ ⠠⠞⠐⠣⠠⠗⠂⠬⠠⠗⠆⠐⠜

13.1.4 Derivative and Partial Derivative (round d): Derivatives are commonly notated as d*x* and d*y*. (You may also encounter d*t,* d*u,* d*v,* d*z,* etc.) Consider d*x* and d*y* as symbols meaning "the derivative of x" and "the derivative of y". In print it is quite common that d*x* and d*y* are spaced within a mathematical expression. In braille, the space is omitted.

⋙ $(1 + 4xy)\mathrm{d}x\ \mathrm{d}y$ ⠐⠣⠼⠁⠬⠼⠙⠭⠽⠐⠜⠙⠭⠙⠽

The partial derivative is represented by the following symbol.

Partial Derivative	∂	⠸⠸

⋙ $\frac{\partial f}{\partial x}$ ⠸⠸⠍⠋⠳⠸⠸⠍⠭

⋙ $\partial/\partial y(\partial b/\partial x)$ ⠸⠸⠳⠸⠸⠽⠐⠣⠸⠸⠳⠃⠳⠸⠸⠳⠭⠐⠜

⋙ $\frac{\partial h}{\partial u} = \frac{\partial f}{\partial x}\ \frac{\partial x}{\partial u}$

⠸⠸⠍⠓⠳⠸⠸⠍⠥ ⠶ ⠸⠸⠍⠋⠳⠸⠸⠍⠭ ⠸⠸⠍⠭⠳⠸⠸⠍⠥

Example 13.1-3 ***Geometry*** The partial derivative of *V* with respect to *r* is $\frac{\partial V}{\partial r} = \frac{2\pi rh}{3}$.

⠠⠧ ⠸⠸⠍⠠⠧⠳⠸⠸⠍⠗ ⠶ ⠼⠃⠨⠏⠗⠓⠳⠼⠉ ⠠⠠⠧ ⠠⠠⠗ ⠠⠠⠓
⠠⠠⠧ ⠠⠠⠗ ⠠⠠⠓ ⠠⠠⠧ ⠠⠠⠗ ⠠⠠⠓

13.1.5 Empty Set (null set, void set)

Empty Set (null set, void set)		
Represented by Zero With a Slanted		
or Vertical Bar Through It	∅ or ⌽	
Represented by Facing Braces	{ }	

The print symbols ∅ and ⌽ used to denote the empty set must not be mistaken for the Greek uncapitalized phi (φ or ϕ) which they resemble.

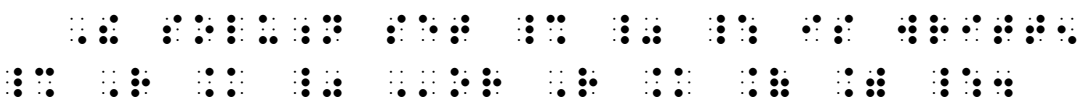
⋙ $A \cap \emptyset = \emptyset$ 

⋙ $\emptyset \subseteq A$ 

When facing braces are used to denote the empty set, one space is left between the braces.

⋙ $S \cup \{\ \}$ 

Example 13.1-4 The solution set ∅ is written $R = \emptyset$ or $R = \{\ \}$.



13.1.6 Factorial Sign

Factorial Sign	!	

⋙ $n!$

⋙ ${}_5C_1 = \frac{5!}{1!4!}\,n!$ 

Example 13.1-5

Poisson Probability People enter a line for the *Jack Rabbit Coaster* at the rate of 4 per minute. The following formula can be used to determine the probability that x people will arrive within the next minute.

$$P(x) = \frac{4^x e^{-4}}{x!}$$

where

$$x! = x \cdot (x-1) \cdot (x-2) \cdot \ldots \cdot 3 \cdot 2 \cdot 1.$$

Determine the probability that $x = 5$ people will arrive within the next minute.

1 ⠠⠠⠠

13.1.7 Infinity

Infinity	∞	⠈⠔⠿

⋙ $a - (+\infty) = -\infty$ ⠁⠀⠤⠀⠐⠣⠬⠿⠐⠜⠀⠤⠀⠤⠿

⋙ $n \to \infty$ ⠝⠀⠳⠒⠀⠈⠔⠿

⋙ 1^{∞} ⠼⠁⠘⠔⠿

⋙ $\sum_{n=-\infty}^{\infty} f(n)$ ⠠⠎⠢⠤⠔⠿⠢⠔⠿⠀⠋⠣⠝⠜

Example 13.1-6

Interval	Inequality
(a, +∞)	x > a
[a, +∞)	x ≥ a
(−∞, a)	x < a
(−∞, a]	x ≤ a

⠈⠁⠀⠐⠣⠁⠂⠀⠬⠿⠐⠜ ⠈⠁⠀⠐⠣⠁⠂⠀⠬⠿⠐⠜

⠈⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒ ⠈⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒

⠳⠒

⠨⠂⠄ ⠄⠂⠀⠀⠨⠂ ⠄

⠨⠂⠄ ⠄⠂⠀⠀⠨⠂ ⠄

⠨⠂⠄ ⠄⠂⠀⠀⠨⠂ ⠄

⠳⠆ ⠨⠂ ⠄

13.1.8 Integral

Single Integral	$\int$	⠮
Double Integral	$\iint$	⠮⠮
Triple Integral	$\iiint$	⠮⠮⠮

⋙ $\int(\mathrm{d}u + \mathrm{d}v) = \int \mathrm{d}u + \int \mathrm{d}v$ ⠮⠐⠣⠙⠥⠬⠙⠧⠐⠜⠀⠮⠙⠥⠬⠮⠙⠧

⋙ $V = \iiint \mathrm{d}x\ \mathrm{d}y\ \mathrm{d}y$ ⠠⠧⠀⠒⠀⠮⠮⠮⠙⠭⠀⠙⠽⠀⠙⠽

Reminder: There are no spaces brailled before and after the terms of the expression (dx and dy) even though they are spaced in print.

Superscripts and subscripts are printed just to the right of the integral symbol. The subscript is brailled first. A subscript indicator is required for numeric subscripts.

⋙ $\int_1^3 \frac{\mathrm{d}x}{x^2-2x+5}$

⠮⠡⠂⠀⠭⠘⠆⠤⠆⠭⠬⠢⠙⠭⠀⠡⠆⠻

⋙ $\int_1^2 \int_0^{x-1} y\ \mathrm{d}y\ \mathrm{d}x$ ⠮⠡⠂⠀⠮⠡⠴⠀⠽⠀⠙⠽⠀⠙⠭

Modifiers (symbols printed directly above and/or directly below the integral symbol) are transcribed according to the five-step rule for the transcription of modified expressions.

⋙ $\int_0^\infty f(x)dx$ ⠮⠡⠴⠀⠋⠣⠭⠜⠙⠭

The single integral sign is modified directly below with the numeral zero and directly above with an infinity symbol.

⋙ $\iint_Q f(x, y)\ dy\ dx$ ⠮⠮⠡⠠⠟⠀⠋⠣⠭⠂⠽⠜⠀⠙⠽⠀⠙⠭

The double integral sign is modified directly below with the letter "Q".

We apologize for the blurry images in this lesson. We hope to provide better graphics in future editions.

CHALLENGE PROBLEM

$$\iint_R 6xy^2 \, dA = \int_1^2 \int_2^4 6xy^2 \, dx \, dy$$

$$= \int_1^2 (3x^2y^2)\Big|_2^4 \, dy$$

$$= \int_1^2 36y^2 \, dy$$

$$= 12y^3|_1^2$$

$$= 84$$

Can you explain the format, spacing, and indicators in the transcription?

⠸⠂⠸⠂⠸⠂

13.1.9 Quantifiers

Existential Quantifier		
There Exists, for Some	∃ or 𝟃	⠈⠢
There Exists Uniquely for Exactly One	∃| or 𝟃|	⠈⠢⠳
Universal Quantifier		
For All, For Each, For Every	∀ or 𝞨	⠈⠄

⋙ $\exists x,\ x < \frac{1}{n}$ ⠈⠢⠭⠂ ⠭ ⠨⠅ ⠹⠨⠅⠼⠁⠳

⋙ $(\exists_x \in A)$ ⠈⠢⠣⠭⠈⠔⠁⠜ ⠨⠔ ⠨⠁⠜

Example 13.1-7 $\exists|_v \,|\, v = -v$ means "there exists exactly one *v* such that *v* equals –*v*."

⠈⠢ ⠨⠁⠧⠂⠭ ⠔ ⠁ ⠨⠅ ⠁⠃ ⠁⠧⠂ ⠁⠝⠙ ⠨⠁⠧⠂ ⠁⠝⠙ ⠨⠁⠧⠂⠭
⠨⠁⠧⠂⠭⠂⠽ ⠨⠅ ⠁⠃ ⠁⠧⠂ ⠔ ⠁ ⠨⠅ ⠁⠃ ⠁⠧⠂⠭
⠈⠢ ⠁⠃ ⠨⠁⠧⠂⠽

*The second vertical bar is a sign of comparison(meaning"such that" as understood from the narrative) and is spaced accordingly. Review **6.7.16**.*

⋙ $(\forall_x)$ ⠈⠄⠣⠭⠈⠔⠁⠜

⋙ $\forall_x \forall_y -\frac{y-x}{x+y}$ ⠈⠄⠭⠈⠄⠽ ⠤⠆⠽⠤⠭⠒⠭⠤⠽

Example 13.1-8 Assuming the domain of variable *s* is ℝ, is the following statement true?

$(\forall_s)(s \cdot 0 = 0)$

⠈⠢⠣⠎⠈⠔⠁⠜ ⠨⠅ ⠠⠗⠂ ⠔ ⠎ ⠨⠅ ⠎⠄⠼⠚⠂ ⠁⠝⠙ ⠨⠅
⠠⠗⠂ ⠔ ⠁ ⠨⠅ ⠁⠃ ⠁⠧⠂ ⠁⠝⠙ ⠨⠁⠧⠂ ⠁⠝⠙ ⠨⠁⠧⠂⠭
⠈⠢ ⠎ ⠨⠅ ⠠⠗⠂ ⠁⠝⠙ ⠎⠄⠼⠚⠂ ⠨⠅ ⠼⠚⠂

*A transcriber's note would explain the substitution of the script typeform indicator for the double-struck letter R. See **Lesson 7**.*

Instructions: Except for the headings, use Nemeth Code throughout.

PRACTICE 13A

Unspaced Miscellaneous Symbols

1.2^

(v > ϕħ)

ℝ: 24 grams

‖▽f(a)‖

$\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y}$

{ } ∩ { }

$\binom{n}{r} = \frac{n!}{(n-r)!\,r!}$

$-\infty < x < \infty$

$f'(x) = 0$ or ∞

$\int_a^b f(x)dx = F(x)\Big]_a^b$

$\int_{x=a}^{x=b} f(t)dt$

$(\exists x)(\exists y)[x + y = 85]$

$\exists|_x$

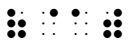
$\forall_x \in A$

Spaced Miscellaneous Symbols

13.2 Spacing Rules for Spaced Symbols: A space is left *before and after* the symbols listed below. However, no space is left between these symbols and a mark which applies to them such as a sign of grouping, a braille indicator, or a punctuation mark.

13.2.1 "At" Sign in Mathematical Context

At Sign	@	

⋙ (@) 

Example 13.2-1 15 pencils @45¢ = $6.75.

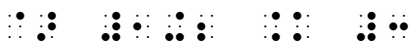


Switching Decision: The problem is not "45¢ = $6.75 " therefore "15 pencils @" is part of the problem and is brailled in Nemeth Code. Spacing: In the print copy there is no space following the "at" symbol. In braille, a space is inserted both before and after this symbol.

13.2.2 Check Mark: Within Nemeth Code material, rather than switching out of Nemeth Code to braille a UEB transcriber-defined symbol, the following Nemeth Code symbol may be used.

Check Mark	✓	

A space is left before and after a single check mark.

⋙ ✓ 1 + 2 = 3 

A sequence of two or more check marks is written unspaced, but the combination as a whole is preceded and followed by a space.

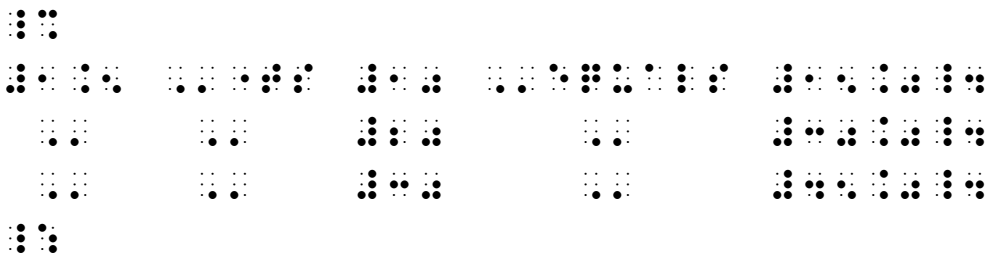
⋙ ✓✓ 2 + 2 = 4 

13.2.3 Ditto Marks: A ditto mark is centered beneath the material to which it applies. It is separated from any expression which precedes or follows it by at least one space. In Nemeth Code, the ditto mark shown below is used.

Ditto Marks	"	

Example 13.2-2

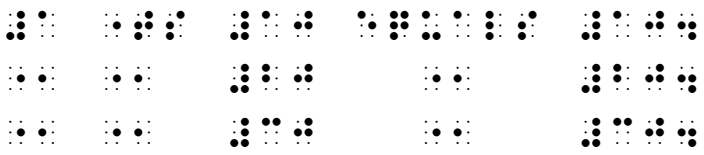
1.5 times 10 equals 15.0.
" " 20 " 30.0.
" " 30 " 45.0.



In UEB contexts, the UEB ditto mark symbol is used, but Nemeth Code format is applied: center the ditto under the word or item to which it applies.

Example 13.2-3

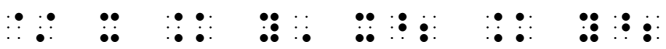
1 times 10 equals 10.
" " 20 " 20.
" " 30 " 30.



13.2.4 Since (because)

Since (because)	∵

➢ ∵ $x = y, x^2 = y^2$



➢ (∵) RS = RT



No space is left between the symbol and grouping signs which apply to it.

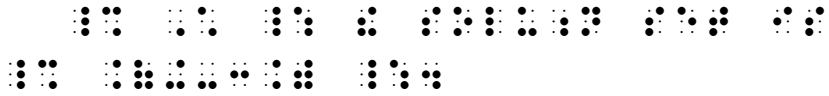
13.2.5 Therefore

Therefore		
Normal	∴	
Negated (it does not follow that)	∴	

⋙ ∴ CM ⊥ AB 

⋙ ∴R = S 

Example 13.2-4 ∴ the solution set is {±3}.

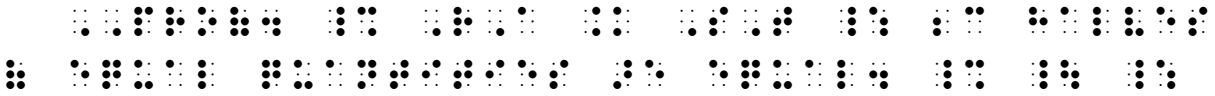


13.2.6 Boldface Vertical Bar (end of proof)

A single boldface vertical bar may be used in print to mark the end of a proof. The two-cell symbol shown below is brailled, preceded and followed by a space. Note that the bar may be printed at the right margin.

Boldface Vertical Bar		

Example 13.2-5 **Proof.** RA = ST because halves of equal quantities are equal. |



Instructions: In Item 4, follow Nemeth Code rules regarding centering of ditto marks. Insert guide dots as needed according to *Braille Formats*.

PRACTICE 13B

Spaced Miscellaneous Symbols

1. Su bought 25 boxes of tissue for her classroom. Priced @99¢, can she pay with only one $20 bill?

   $25 \times \$.99 = \$24.75$
   ✓ $\$24.75 > \$20$

   Answer: No. Su needs more than $20 to buy the tissues.

2. $\because 8x + 3y = 15$, substituting 0 for x gives $8(0) + 3y = 15$, or $3y = 15$. $\therefore y = 5$.

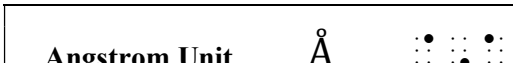
3. The end of proof mark "∎" is used in mathematics to denote the end of a proof, in place of the traditional abbreviation "Q.E.D." for the Latin phrase "**quod erat demonstrandum**", meaning "which was to be shown".

4. Discuss the following table in your group.

BTB	**ACR**	**pH range**
5.55×10^{-5}	2.57×10^{-1}	1.00-6.39
4.75 "	2.33 "	1.49-12.01
5.30 "	3.24 "	0.99-12.31
4.44 "	2.06 "	1.17-12.01

Spacing with the Angstrom Unit and Tally Marks

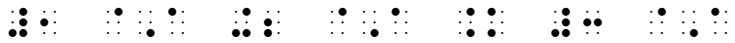
13.3 Angstrom Unit: The angstrom unit is a mathematical symbol, but it is spaced according to the rules of abbreviations.

Angstrom Unit	Å	



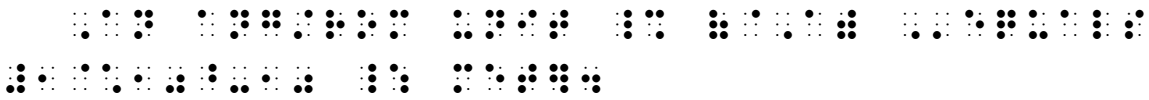
A space is left before and after the angstrom unit even when it is preceded or followed by a sign of operation.

Example 13.3-1 $1Å + 2Å = 3Å$



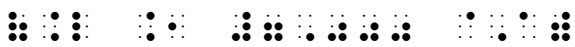
A space is not left between the angstrom unit and a sign of grouping, a horizontal or diagonal fraction line, a braille indicator, or a punctuation mark which applies to it.

Example 13.3-2 An angstrom unit $(Å)$ equals 1×10^{-10} meter.



The angstrom unit is unspaced from the opening and closing parentheses.

Example 13.3-3 $(\lambda > 7{,}000Å)$



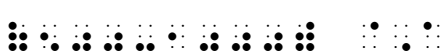
A space precedes the angstrom unit, but it is unspaced from the closing parenthesis.

Example 13.3-4 $\frac{(2)(2Å)}{4\times10^{6}Å}$



A space precedes each angstrom unit, but the first angstrom unit is unspaced from the closing parenthesis and the second angstrom unit is unspaced from the closing fraction indicator.

Example 13.3-5 $(500 - 1000)Å$



The closing parentheses does not apply to the angstrom unit, therefore the symbol is preceded by a space.

13.3.1 Punctuation: Although the Angstrom unit is spaced like an abbreviation, it is a mathematical symbol and therefore must be punctuated mathematically when the punctuation falls within the Nemeth Code switches.

⋙ 1Å.

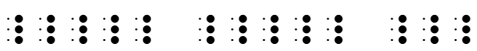
The angstrom unit is unspaced from the punctuation which follows it.
(Assume Nemeth Code continues after this period.)

13.4 Tally Mark

Tally Mark	|	

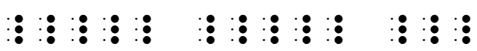
13.4.1 Grouping: If tally marks are grouped in print they are similarly grouped in braille, with one blank cell between groupings.

⋙ ||||| ||||| |||



When a group of tally marks is crossed through, the cross-tally is treated as an additional (456) tally mark which is added to its group.

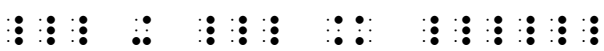
⋙ 𝍸 𝍸 |||



In the print copy, each group of five tally marks appears a long diagonal stroke superposed on four vertical strokes.

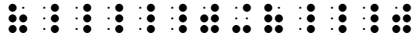
13.4.2 Spacing: One space is left before and after a group of tally marks even when the group is preceded or followed by a sign of operation.

⋙ ||| + ||| = ||||||



No space is left between a group of tallies and a sign of grouping or a braille indicator applying to it.

⋙ (|||||) + (|||)



⋙ (||| + |||)

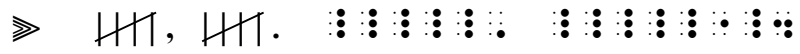


⋙ 𝍸²



When the combination of symbols cannot be accommodated on one braille line, transition to another braille line may take the place of the required space.

13.4.3 Punctuation: When a tally mark is followed by punctuation requiring the punctuation indicator, a multipurpose indicator (dot 5) is placed between the tally mark and the punctuation indicator to avoid misreading the identical symbols. No space is left between a group of tallies and a punctuation mark applying to it.



*See **Lesson 6** regarding spacing with the crosshatch symbol # and the diagonal slash /.*

PRACTICE 13C

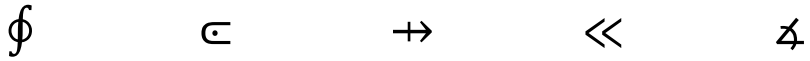
I. Tally marks ||||||| equal the number 11_{six}. What numeral does ||||||||| equal in base six?

II. Does ||| + ||| = 𝍸| ?

III. 5550Å = $555m_{\mu}$ or 0.555 micron. (m_{μ} is a millimicron.)

SUPERPOSED SIGNS

13.5 Definition and Analysis: Superposed signs are signs which are printed one upon another so that one sign extends beyond the boundary of the other. Contrast this with "shapes with interior modification" presented in **Lesson 11**, where one symbol is printed inside the boundaries of the other. Here are some examples of superposed signs.



In order to transcribe a superposed sign, the basic sign and the superposed sign need to be determined because the basic sign is brailled first. The following order of preference is used as a guide. A symbol lower on the list is regarded as being superposed upon a symbol higher on the list.

— Integral sign
— Signs of operation
— Horizontal and vertical bars
— Signs of shape
— Signs of comparison
— Signs not listed above

Here is an analysis the first three print examples shown above.

∮ The basic sign is an integral sign; the superposed sign is a sign of shape (circle).

⊂ The basic sign is a sign of operation (dot); the superposed sign is a sign of comparison (inclusion).

↛ The basic sign is a vertical bar; the superposed sign is a sign of comparison (arrow).

If two signs belong to the same category, the superposition may be represented in either order, provided the same order is followed consistently throughout the transcription. Here is an analysis the last two print examples shown above.

≪ Both signs belong to the same category – signs of comparison (nested "less than" signs).

∡ Both signs belong to the same category – signs of shape (an angle and an arc).

13.6 Transcription of Superposed Signs: The components of a superposed sign are unspaced and are transcribed in the following order:

— The symbol for the basic sign is written first.
— The superposition indicator is written next.
— The symbol for the superposed sign is written third.
— The termination indicator is written last.

Superposition Indicator	⠘
Termination Indicator	⠸

13.6.1 Integral Modified by Superposition: The most common integrals with superposed symbols are listed below. Unlisted "integrals modified by superposition" are transcribed in accordance with the rules for superposed signs. Spacing and punctuation follow the same rules as for unmodified integral signs (see **13.1.8**).

Integral with superposed circle	∮	⠘⠮⠘⠸⠮
Integral with superposed infinity	∯	⠘⠮⠘⠶⠮
Integral with superposed rectangle	⨍	⠘⠮⠘⠻⠮
Integral with superposed square	⨏	⠘⠮⠘⠼⠮

⋙ $\oint \frac{\mathrm{d}q}{T} < 0$ ⠘⠮⠘⠸⠮⠙⠟⠳⠞⠀⠀⠠⠀⠀⠀⠀⠴

⋙ $\oint P\ \mathrm{d}x + Q\ \mathrm{d}y$ ⠘⠮⠘⠸⠮⠏⠀⠙⠭⠀⠬⠀⠟⠀⠙⠽

⋙ $\oint f(x,y)\ \mathrm{d}y$ ⠘⠮⠘⠶⠮⠋⠣⠭⠂⠽⠜⠀⠙⠽

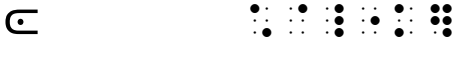
13.6.2 Signs of Operation Modified by Superposition: The most common symbols use the multiplication dot as the basic symbol. Three examples are shown below.

Dot

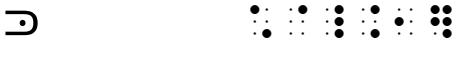
... between bars of equals sign

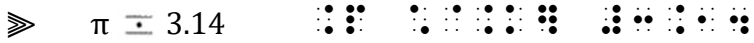


... within inclusion sign



... within reverse inclusion sign



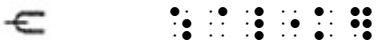


Unlisted "signs of operation modified by superposition" are transcribed in accordance with the rules for superposed signs.

13.6.3 Horizontal and Vertical Bars Modified by Superposition: The most common symbols are shown below.

Horizontal Bar

... through inclusion sign



... through reverse inclusion sign



Vertical Bar

... through shaft of right-pointing arrow



.. through shaft of left-pointing arrow

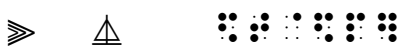


Unlisted "bars modified by superposition" are transcribed in accordance with the rules for superposed signs.



"Horizontal bar" is higher on the list than a sign of shape (the circle).

13.6.4 Signs of Shape Modified by Superposition



"Triangle" is a sign of shape; "perpendicular to" is a sign of comparison.

Signs of shape modified by superposition are spaced and punctuated as other signs of shape. (See **Lesson 11**).

When both signs are signs of shape, the superposition may be represented in either order, provided the same order is followed consistently throughout the transcription.



*This arc shape extends beyond the boundary of the angle shape, making this "a shape modified by superposition". Compare this symbol to the "angle with interior arc" (**Lesson 11**) which has a different braille form.*

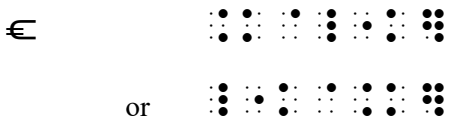
Note: When the print copy uses either "angle with interior arc" symbol throughout the text to simply mean "angle", the simple braille shape symbol for "angle"  may be used. A transcriber's note is required to inform the reader of the substitution.



13.6.5 Signs of Comparison Modified by Superposition: When both signs are signs of comparison, the superposition may be represented in either order, provided the same order is followed consistently throughout the transcription.

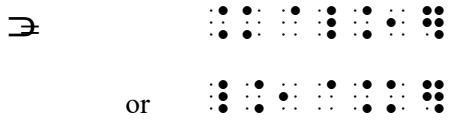
Two Signs of Comparison

Equals Sign Through Inclusion Sign



or

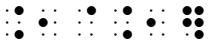
Equals Sign Through Reverse Inclusion Sign



or

Nested Greater Than Signs (means "is large compared with")

... with straight sides ≫

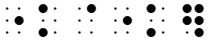


... with curved sides ⪢



Nested Less Than Signs (means "is small compared with")

... with straight sides ≪



... with curved sides ⪡



Spacing and punctuation follow the same rules as for any other sign of comparison (see **6.6**).

13.6.6 Symbols That Are Not Superposed Signs: Negated symbols are not transcribed as superposed signs. See **Lesson 6** and **Lesson 11**. Also note that the print practice of showing a group of five tally marks as a long diagonal stroke superposed on four vertical strokes does not follow the Nemeth Code rules for superposed signs. Review section **13.4** above.

Instructions: See **Lesson 11** to review symbols with interior shapes and **Lessons 6**, **11**, and **13** regarding negated symbols.

PRACTICE 13D

Superposed Signs	Symbols with Interior Shapes	Negated Relations
$\oint$	⊚	/∴
⋓	⊡	≠
⊖	⊖	∦
⋬C		⋦
$5 \ll y$		⋦
R ⊃ s	⑬	≠
Q ↠ R	⍟	⁒

AMBIGUOUS SIGNS

13.7 Context: It is often easy to mistake the print symbols for the letters o, O, and the numeral "zero", or letters l, i, I, and the numeral "one". Beyond these obvious examples, there are certain print signs which look similar to other print signs. The braille symbols may be altogether different. Brailling the wrong symbol will give the reader false information. In order to assure your transcription is correct, search the surrounding context to determine the meaning of the ambiguous sign. Magnification may help you identify the symbol. If you are still unsure, seek help from someone knowledgeable in the math or science topic who can correctly identify the print symbol. Some examples include ...

ϕ Greek letter phi	or	⦰ null set
	or	∅ canceled numeral zero
	or	0 zero in certain fonts
	or	⦰ horizontal bar with superposed circle
	or	θ Greek letter theta
α Greek letter alpha	or	∝ "varies as"
	or	*a* English letter "a"
ε Greek letter epsilon	or	Ɛ "membership"
ν Greek letter nu	or	*v* English letter "vee"
Δ Greek letter Delta	or	△ triangle shape
	or	⩓ logical product with underbar
< "less than"	or	⟨ opening angle bracket
	or	< left-pointing caret
> "greater than"	or	⟩ closing angle bracket
	or	> right-pointing caret
|| two vertical bars	or	|| "is parallel to"
^ mathematical caret	or	⋀ logical product
	or	^ literary (UEB) caret or circumflex
√ radical sign	or	✓ checkmark

¬ Is it a right-pointing arrow with lower-only, straight arrowhead

... or a long-division structure

... or the negation symbol (brailled using a transcriber-devised symbol)

13.7.1 Vertical Bar and Colon: The symbols that give transcribers the most trouble due to their ambiguity are the vertical bar and the colon. The vertical bar can be a grouping sign, an operation sign, a comparison sign, or an “end of proof” symbol. The colon can be a ratio symbol or a punctuation mark. You need to recognize the meaning of the sign in order to braille the proper symbol.

|	(vertical bar)		
	... as a grouping sign		
	... as a sign of operation		(means "is a factor of")
	... as a sign of comparison		(means "such that" or "given")
	... as an “end of proof” symbol		(printed in boldface)
:	(colon)		
	... as a mathematical symbol		(ratio, meaning "is to"; "such that", mapping notation, and other meanings)
	... as a punctuation mark		(preceded by a punctuation indicator where appropriate)

13.7.2 Spacing: Some signs use the same braille symbol but have different spacing rules depending on their meaning. You can't depend upon the print copy to show the spacing according to Nemeth Code rules so you need to recognize the meaning of the sign in order to apply proper spacing.

|		Is the vertical bar a grouping sign, an operation sign, or a comparison sign?
~		Is the tilde an operation sign ("not") or a comparison sign ("is related to" or "is similar to")?
'		Is it a punctuation sign (an apostrophe or single quotation mark) or is it a math symbol (prime sign)?
/		Is it mathematical (a fraction line) or a UEB solidus?
:		A spaced sign of comparison, or
		An unspaced symbol?

13.7.3 Uppercase Greek Letters: Some uppercase Greek letters are indistinguishable from English letters. Unless the text identifies the letter as Greek, you can safely assume it is an English letter.

13.7.4 Chemical Notation: Some signs have yet another meaning in chemical notation. For example, the following symbols can be certain types of chemical bonds:

= || ≡ — | / \ · : ::

Details can be found in the *Braille Code for Chemical Notation*.

MULTIPURPOSE INDICATOR

Multipurpose Indicator ⠨

13.8 Review: In addition to being a baseline indicator, braille dot 5 assumes several other functions in the Nemeth Code. Dot 5 is called the *multipurpose indicator* in the following situations already introduced in previous lessons:

— A multipurpose indicator is used between two unspaced signs to indicate that they are printed horizontally (side-by-side).

  – side-by-side signs of operation. See **6.2**.

  – side-by-side tildes. See **6.4.9**.

  – side-by-side signs of comparison. See **6.9**.

  – side-by-side arrows. See **10.14**.

  – side-by-side modifiers within a sign of shape. See **11.17**.

— A multipurpose indicator is used when a number follows a letter with no space between them. See **7.16**.

— A multipurpose indicator is used between a regular polygon representing a sign of operation and a numeral immediately following it. See **11.29**.

— A multipurpose indicator triggers the beginning of a modified expression. See **12.2**.

— A multipurpose indicator is placed between a tally mark and a following punctuation indicator to avoid misreading the similar symbols. See **13.4.3**.

13.9 Additional Uses of the Multipurpose Indicator

13.9.1 Letter Followed by a Decimal Point and a Numeral: When a letter on the baseline of writing is immediately followed by a decimal point and a numeral, a multipurpose indicator is placed between the letter and the decimal point to show that the decimal point and numeral are not subscripts to the letter.

⋙ x.4 ⠨⠲⠼⠙

13.9.2 Numeric Subscript Followed by a Numeral: A multipurpose indicator is used after a numeric subscript if the subscript is followed by a numeral on the baseline of writing.

⋙ $x_7 10$ ⠨⠭⠢⠼⠁⠚

13.9.3 Decimal Point Followed by a Nonnumeric Character: A multipurpose indicator is used after a decimal point if the symbol following the decimal point is something other than a numeral.

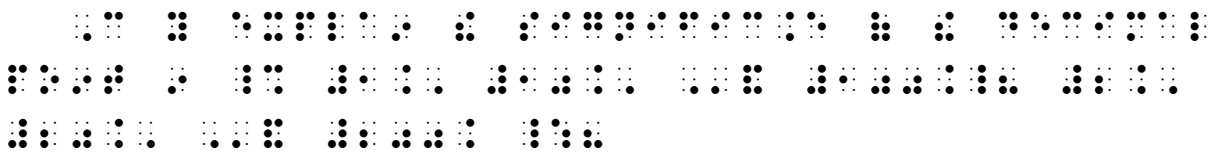
⋙ $0.a_1a_2a_3 \ldots a_n$ ⠨⠼⠚⠲⠁⠢⠁⠢⠁⠢ ⠄⠄⠄ ⠁⠢⠝

⋙ $\frac{1}{2} = .\underline{?}$ ⠨⠼⠁⠌⠃⠀⠨⠲⠸⠦

⋙ (5.) ⠨⠐⠣⠼⠑⠲⠐⠜

Note: When a decimal point is followed by a comma or the punctuation indicator, a multipurpose indicator is not needed.

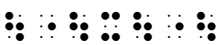
Example 13.9-1 Can you explain the significance of the decimal point in 1., 10., and 100.? 2., 20., and 200.?



13.9.4 Side-by-Side Vertical Bars: A multipurpose indicator is used between two unspaced vertical bars when the first bar is a closing sign of grouping, and the second bar is an opening sign of grouping.

⋙ $|x||y|$ 

A multipurpose indicator is used between two vertical bars which are nested grouping symbols (one bar is shorter and/or thicker than the other).

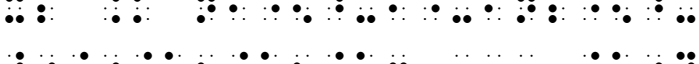
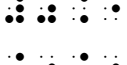
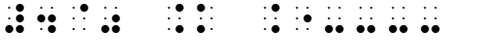
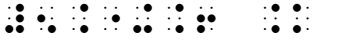
⋙ $\big||x|\big|$ 

This example shows only one outer closing bar.

⋙ $|x|\big|_{x=0}$ 

Instructions: Explain the function of the multipurpose indicator in each example.

PRACTICE 13E

1. x3, R10 
2. $120° + n320°$ 
3. $C = \pi 2r$ 
4. $\omega 2 = \omega \neq 2\omega$ 
5. $140\text{te}4\text{t}5_{12} + \text{e}5_{12}$ 
6. $\frac{\text{A}_0}{2} = \text{A}_0 2^{-0.05}$ 
7. $x_2 = n_1 5^{-1} - 1n_2 5^{-1}$ 
8. $0.\alpha_1\alpha_2\alpha_3 \ldots \alpha_n$ 
9. .% 
10. $4\% = .____$ 
11. $5. + .6 = 5.6$ 
12. $\|x\|\|y\|$ 
13. $\|\|x\|\|$ 

DIVISION PROBLEMS

13.10 Linear Representation: Lesson 1 introduced linear division problems using the symbol ÷ for "divided by", as in 15 ÷ 3 ("fifteen divided by three"). A division problem may also be printed using a curved, straight, or slanted line for the division sign, and a horizontal line printed above or below the divisor.

$$3\overline{)15}$$

When only a divisor and dividend are printed, and when the divisor and dividend are composed entirely of numerals with or without decimal points and/or commas, the problem is brailled as a linear construction. The print style is replicated by using one of the following symbols between the divisor and dividend. Note that the straight and slanted division signs are represented with the same braille symbol.

Division Signs:		
Curved Right	⠐⠜	)
Curved Left	⠸⠣	(
Straight or Slanted	⠸⠌	| or \ or /

In this linear format, only the division sign is brailled. The horizontal line printed above or below the dividend is not brailled. The numeric indicator is used where required according to other rules of the Nemeth Code. As with other operation signs, the division symbol is unspaced from the divisor and the dividend even if the print copy shows a space.

⋙ $3\overline{)15}$ ⠼⠉⠐⠜⠼⠁⠑

⋙ $3\,\underline{|\,15}$ ⠼⠉⠸⠌⠼⠁⠑

⋙ $\underline{110\,(}\,10$ ⠼⠁⠁⠸⠣⠼⠁⠚

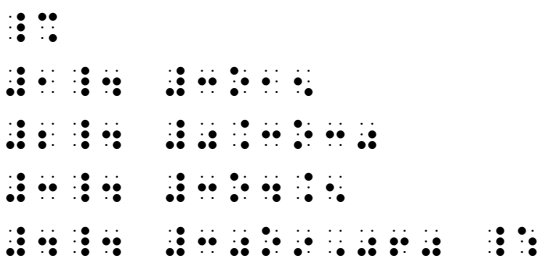
A horizontal line printed below the divisor also is disregarded.

⋙ $\underline{30}\,\overline{)90}$ ⠼⠉⠚⠐⠜⠼⠊⠚

Example 13.10-1 Show how to solve $9\overline{)216}$ on your calculator. Is there a remainder?

⠠⠎⠓⠕⠺ ⠓⠕⠺ ⠞⠕ ⠎⠕⠇⠧⠑ ⠠⠼⠊⠐⠜⠼⠃⠁⠋⠲ ⠠⠊⠎ ⠞⠓⠑⠗⠑ ⠁ ⠗⠑⠍⠁⠊⠝⠙⠑⠗⠦
⠠⠎⠓⠕⠺ ⠓⠕⠺ ⠞⠕ ⠎⠕⠇⠧⠑ ⠠⠼⠊⠐⠜⠼⠃⠁⠋ ⠕⠝ ⠽⠕⠥⠗ ⠉⠁⠇⠉⠥⠇⠁⠞⠕⠗⠲ ⠠⠊⠎ ⠞⠓⠑⠗⠑ ⠁ ⠗⠑⠍⠁⠊⠝⠙⠑⠗⠦

Example 13.10-2 1. $3\overline{)15}$ 2. $0.3\overline{)30}$ 3. $3\overline{)4.5}$ 4. $30\overline{)9{,}060}$



Reminders: When the print copy arranges nonspatial itemized material side-by-side across the page and there are no subdivisions, the braille format is changed so that all identifiers start in cell 1.To assure that identifiers each begin in the same cell, the opening Nemeth Code indicator is on the line before the first numbered item.

Instructions: Use appropriate margins for embedded, displayed, and itemized problems.

PRACTICE 13F

Converting Fractions to Decimals

To write the fraction 1/2 as a decimal you can divide "one point zero divided by two" or $2\overline{)1.0}$. Put away your calculator! Using mental math, it's easy! The answer is .5. Can you divide *these* fractions in your head?

1/5: $5\overline{)1.0}$

4/5: $5\overline{)4.0}$

9/10: $10\overline{)9.0}$

Practice

1. Solve these division problems.

   a. $8\overline{)128}$ b. $6\overline{)5{,}304}$

   c. $4\overline{)204}$ d. $9\overline{)837}$

SPATIAL ARRANGEMENT WITH DIVISION

13.11 Notation Devices: It will be helpful to refer to the parts by using the following standard terminology. In this example ("forty divided by ten equals 4"), "40" is the *dividend,* "10" is the *divisor,* and "4" is the *quotient*.

$$\begin{array}{r} \textit{quotient} \\ \textit{divisor}\,\overline{)\,\textit{dividend}} \end{array} \qquad \begin{array}{r} 4 \\ 10\overline{)\,40} \end{array}$$

Other print styles which can be found in various publications are illustrated below using the same numbers for dividend, divisor, and quotient. Each style includes a division sign (either straight, slanted, or curved) and a separation line. The quotient can be printed above, below, or next to the dividend.

$$\begin{array}{r} 4 \\ 10/\overline{40} \end{array} \qquad 10\overline{)\,40\,(4} \qquad \begin{array}{r} 4 \\ 10\,\lceil 40 \end{array} \qquad \begin{array}{l} 10\,\lfloor 40 \\ \quad\;\; 4 \end{array} \qquad \begin{array}{r} 4 \\ \underline{10\,\rceil 40} \end{array}$$

In a spatial transcription the separation line (the horizontal line between the dividend and the quotient) is brailled. The length of the line varies depending on the layout of the problem.

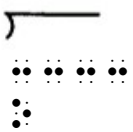
Separation Line	⠒⠒⠒⠒⠒	————	(varying in length)

The spatial division constructions given in the Nemeth Code are shown below. Notice the similarity between the print sign and the braille symbols.

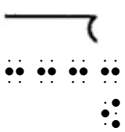
Separation Line Above the Dividend

Curved Division Sign

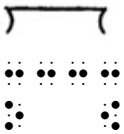
on Left



on Right

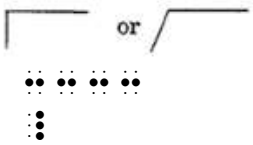


on Left and Right

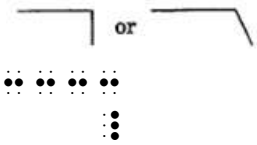


Straight or Slanted Division Sign

on Left



on Right



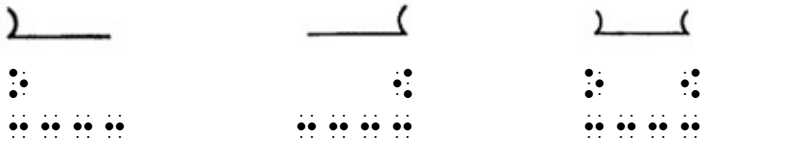
Straight Division Signs on Right and Left



Separation Line Below the Dividend

Curved Division Sign

on Left **on Right** **.on Left and Right**

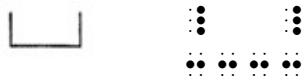


Straight or Slanted Division Sign

on Left **on Right**



Straight Division Signs on Left and Right



13.11.1 Length of the Separation Line: As shown above, the separation line which appears in a spatially-arranged division problem begins in the column containing a division symbol and ends in the column containing the other division symbol if the latter appears in print. When only one division symbol is printed, the separation line ends in a cell one column beyond the overall arrangement. Notice how this rule applies to the examples in the rest of this section.

Review of Format for Spatial Arrangements

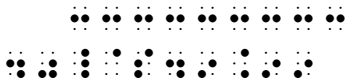
- A blank line is required above and below a spatial arrangement.
- The numeric indicator is not used. (There are some exceptions.)
- Side-by-side arrangement is allowed.
- An identifier associated with a spatial problem is positioned according to rules applying to the type of arrangement, not necessarily at the top line of the problem. (See 13.17 regarding spatial division.)

13.12 When a Spatial Arrangement is Required: When more than just the divisor and dividend are shown, or when the divisor and/or dividend consist of more than just numerals, a spatial (vertical) arrangement is required.

In this section, code switch indicators are not shown in the examples which contain no narrative.

13.12.1 More Than Just Numerals: The presence of the dollar sign in the dividend in the next example means a spatial arrangement must be brailled. *The required blank line before and after the arrangement is not shown in the isolated examples.*

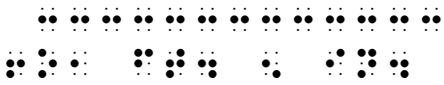
Example 13.12-1 $40 \overline{)\$49.99}$



Reminders: The division symbol is unspaced from the divisor and the dividend. The separation line begins in the cell directly over the division sign. The separation line ends one cell beyond the overall arrangement.

Because the next example contains abbreviations in the dividend, a spatial arrangement is required.

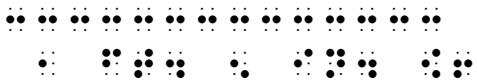
Example 13.12-2 $6 \overline{)\text{1 ft. 5 in.}}$



Reminder: The numeric indicator is not used in a spatially-arranged division problem.

13.12.1.a Spacing with Abbreviations: Spacing rules for abbreviations next to operation signs must be applied, as illustrated in this example where the dividend is printed to the left of the division symbol.

Example 13.12-3 $\overline{\text{1 ft. 5 in.}}\,(6$

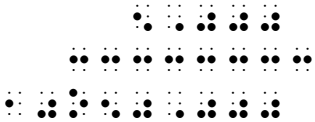


Reminder: A space is required between an abbreviation and a sign of operation.

13.12.2 A Quotient is Present: The quotient is aligned with the dividend (as shown in the print copy) unless it has been intentionally misaligned as an exercise for the student.

Example 13.12-4

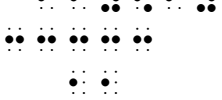
$$\begin{array}{r} 5{,}000 \\ 10 \overline{)\,50{,}000} \end{array}$$



The next example shows a different print arrangement of the division sign and separation line.

Example 13.12-5

$$\begin{array}{l} \underline{110\,(\,10} \\ 11 \end{array}$$

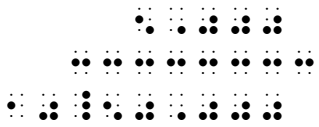


Reminders: One end of the separation line is directly aligned with the division sign; the separation line extends one cell beyond the overall arrangement.

In the next print style, the division sign is a straight line, the separation line is above the dividend, and a horizontal line is printed under the divisor. The line under the divisor is not shown in the braille transcription.

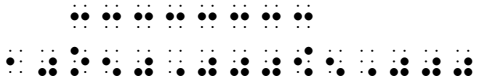
Example 13.12-6

```
   5,000
10|50,000
```



The same example is shown in different print style. Two division symbols are printed, and the quotient is printed to the right of the dividend rather than above the separation line. The separation line begins at the left division sign and ends at the right division sign.

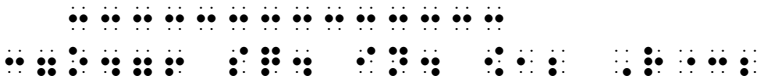
Example 13.12-7 10) 50,000 (5,000



Reminder: Because a quotient is shown, this problem is brailled as a spatial arrangement. The divisor, dividend, and quotient are brailled unspaced from the division symbols.

13.12.2.a A Quotient with a Remainder: When a remainder is printed next to the quotient, the capitalized or uncapitalized "r" (indicating "remainder") is preceded by a space. A multipurpose indicator (dot 5) separates the letter from the numeral following it.

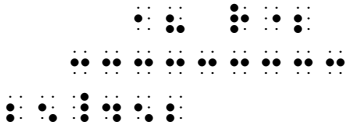
Example 13.12-8 37) 476 sq. in. (12 R32



When the quotient is arranged above the separation line, the line extends one cell to the right of the overall arrangement, including the remainder.

Example 13.12-9

```
  18 r2
25 | 452
```



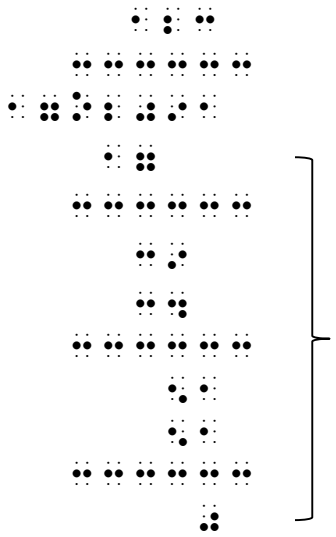
13.12.3 Long Division: In a long division problem, multiples are written below the dividend. A series of subtraction problems are performed, giving partial remainders and, finally, a remainder. The long division portion of the problem (bracketed below) is aligned for subtraction.

```
   123
17 ) 2091     dividend
  17          multiple (17×1)
  39          partial remainder
  34          multiple (17×2)        subtraction problems
  51          partial remainder
  51          multiple (17×3)
   0          remainder
```

In braille, the components of the problem are aligned in the same manner as they are aligned in the print copy. All separation lines must be the same length, regardless of their relative lengths in the print copy.

Example 13.12-10

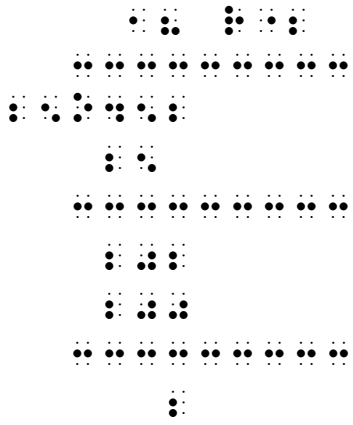
```
   123
17 ) 2091
  17
  39
  34
  51
  51
   0
```



aligned for subtraction

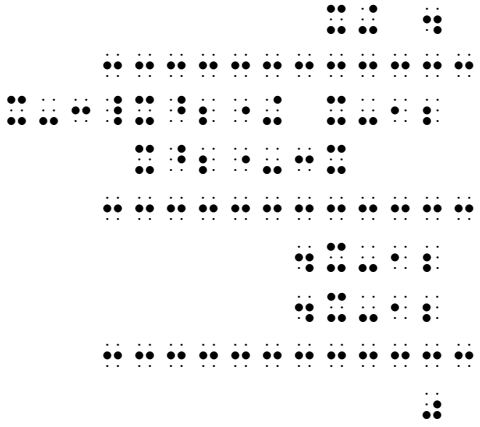
Example 13.12-11

```
  18 r2
25 ) 452
  25
  202
  200
    2
```



Example 13.12-12

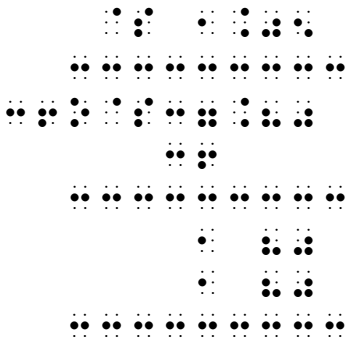
$$
\begin{array}{r}
x+4 \\
x-3 \overline{\big|\, x^2+x-12} \\
\underline{x^2-3x} \\
4x-12 \\
\underline{4x-12} \\
0
\end{array}
$$



When commas or decimal points occur in a dividend, corresponding blank cells should be left throughout the body of the division example, except in the separation lines.

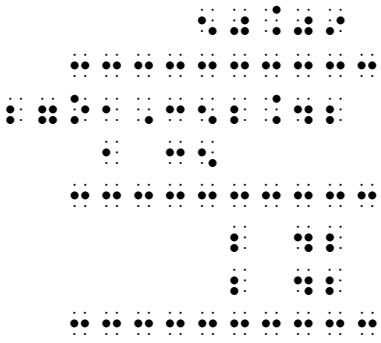
Example 13.12-13

$$
\begin{array}{r}
\$\ 1.05 \\
36 \overline{)\,\$37.80} \\
\underline{36} \\
1\,80 \\
\underline{1\,80}
\end{array}
$$



Example 13.12-14

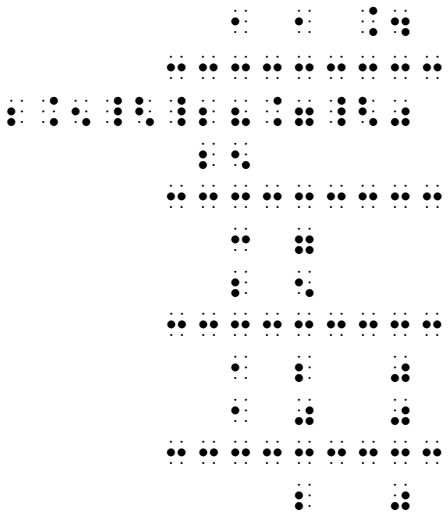
$$
\begin{array}{r}
50.09 \\
27 \overline{)\,1{,}352.42} \\
\underline{1\,35} \\
2\,42 \\
\underline{2\,42}
\end{array}
$$



If a caret occurs in a dividend, corresponding blank cells are left throughout the body of the division problem (except in the separation lines). Since two cells are required for the caret, the decimal point replacing the caret in the quotient is placed in the right-hand cell.

Example 13.12-15

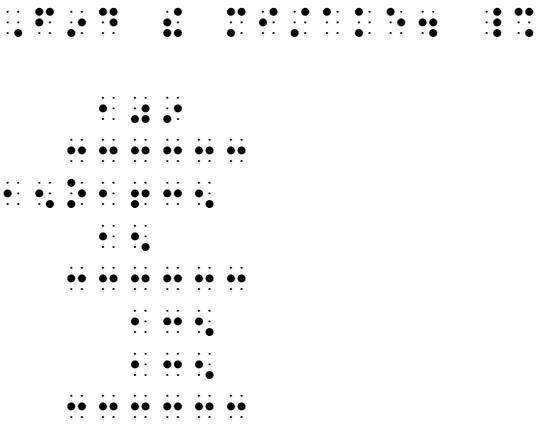
```
    11.4
2.5^ | 28.7^0
   25
   ---
    37
    25
   ---
    120
    100
   ----
     20
```



If the quotient has been intentionally misaligned as an exercise for the student, the same misalignment is shown in the transcription.

Example 13.12-16 Find the mistake.

```
     109
   -----
15 ) 1635
     15
     --
     135
     135
     ---
```

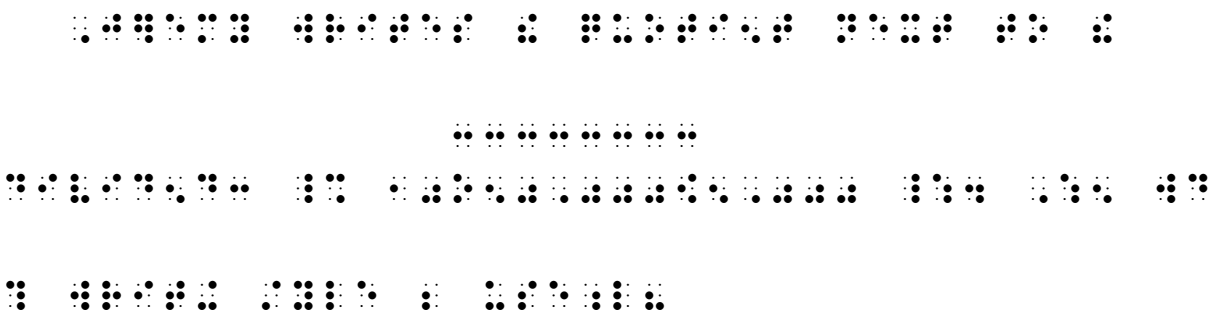




*The leftmost character in the spatial arrangement is placed in the appropriate cell for displayed material—in this case, cell 3. (See **Lesson 8** for details regarding margins for displayed mathematical expressions.) Review **10.38** regarding the placement of code switch indicators with displayed spatial arrangements.*

13.13 Blank Lines Required: A blank line must precede and follow a spatially-arranged problem, even when the problem is embedded within narrative.

Example 13.13-1 Jeremy writes the quotient next to the dividend: $10\overline{)50{,}000\,(5{,}000}$. When would this writing style be useful?



Instructions: Braille the first set of displayed problems as follows. Apply *Braille Formats* guidelines for a simple list in columns. The leftmost cell of column one is placed in cell 1; two blank cells come between the end of the longest item in one column and the left-hand margin of the next column. Guide dots are not used between unrelated columns. Maintain the same arrangement in braille (three columns) even though the fourth problem could fit next to the third problem. The last problem is displayed mathematical material.

PRACTICE 13G

Here are six other ways to write $\begin{array}{r}203\\ 11\overline{)2233}\end{array}$ or "2233 divided by 11 equals 203":

$$\begin{array}{r}203\\ \overline{2233\,(11}\end{array} \qquad \begin{array}{r}203\\ 11/\overline{2233}\end{array} \qquad \begin{array}{r}203\\ \overline{11|2233}\end{array}$$

$$\begin{array}{l}\underline{2233}|\,11\\ 203\end{array} \qquad \begin{array}{l}\underline{2233}(\,11\\ 203\end{array} \qquad \begin{array}{l}11\,)\,\underline{2233}\\ \quad\;\;203\end{array}$$

What is $11\overline{)2234}$? Use long division.

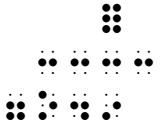
$$\begin{array}{r}203\text{ R1}\\ 11\overline{)2234}\\ \underline{22}\\ 34\\ \underline{33}\\ 1\end{array}$$

13.14 Omissions: When the arrangement contains omissions, in order to maintain necessary alignment *only the general omission symbol is used*. Other omission symbols such as the long dash, the ellipsis, or a sign of shape are not used, even when the omissions are shown using these symbols in the print copy.

General Omission Symbol	⠨

Example 13.14-1

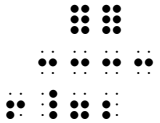
$$7\overline{)\,49}$$



The same number of omission symbols as shown in print should be used.

Example 13.14-2

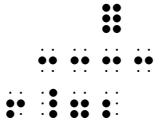
$$6\overline{)72}$$



A shaded or unshaded region that does not show a specific number of places is represented by a single general omission symbol, aligned above the rightmost character in the dividend.

Example 13.14-3

$$6\overline{)72}$$



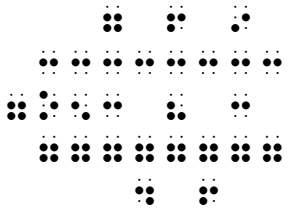
13.15 Carried Numbers in Long Division: Print copy does not have a standard layout for carried numbers in a division arrangement. When carried numbers are shown, regardless of print placement it is suggested that the transcriber arrange the carried numbers below the carried number indicator and to the left of the dividend digits to which they apply. A transcriber's note describing both the print and braille formats must be included.

Carried Number Indicator (varying in length)	⠨⠨⠨⠨⠨⠨

The carried number indicator is to be the same length as the separation line. Blank spaces are left in the dividend to allow separate columns for the carried numbers. No blank spaces are left in the carried number line.

Example 13.15-1

$$7\overline{)53_{4}8_{6}3}\qquad 7\ 6\ 9$$



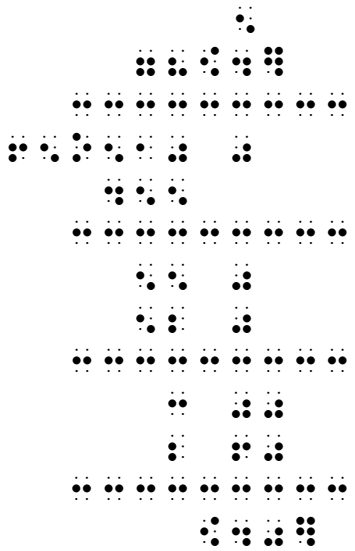
The transcriber's note relating to this example could say "In print the carried numbers appear below and to the left of the dividend digits to which they apply. In braille, the same alignment is followed. A carried number indicator is brailled below the dividend."

13.16 Cancellation in Long Division: If cancellation is shown in long division, the canceled material is enclosed in braille cancellation indicators. Blank cells are left where necessary for proper alignment.

Opening Cancellation Indicator	⠰
Closing Cancellation Indicator	⠆

Example 13.16-1

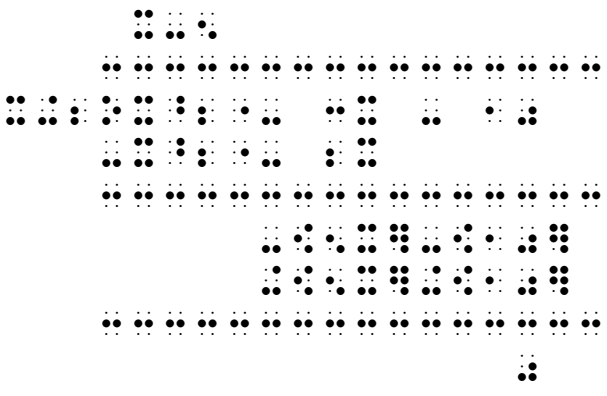
$$
\begin{array}{r}
5 \\
\cancel{784} \\
65\overline{)5100} \\
\underline{455} \\
550 \\
\underline{520} \\
300 \\
\underline{260} \\
\cancel{40}
\end{array}
$$



The material canceled in braille must exactly represent what is canceled in print.

Example 13.16-2

$$
\begin{array}{r}
x - 5 \\
x + 2\overline{)\ x^2 - 3x - 10} \\
\underline{-x^2 - 2x} \\
\cancel{-5x - 10} \\
\underline{\cancel{+5x + 10}} \\
0
\end{array}
$$



Notes

"x–5" is placed as in print, in the first cell available to the quotient.

The division sign is unspaced from the divisor.

The minus sign is brailled in the same column (cell) as the division sign.

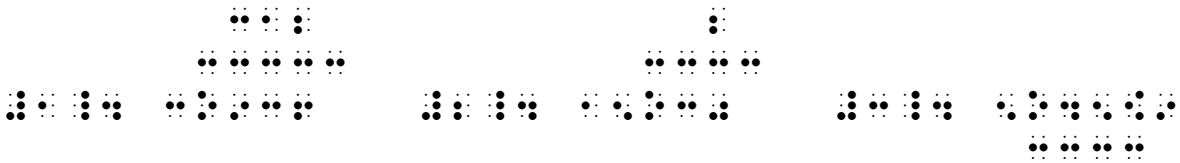
Space inserted above cancellation indicators.

Aligned for subtraction.

"0" is aligned by place value.

13.17 Placement of Identifiers with Spatial Division: An identifier, if present, is placed on the line with the dividend. One blank space must be left between the last symbol in the identifier and the symbol furthest left in the overall arrangement including separation lines. When arranged side-by-side across the width of the page, no symbol of one spatial arrangement or its identifier may be less than three cells distant from any symbol on any line of a neighboring arrangement or its identifier, except at the ends of separation lines.

Example 13.17-1 1. $3\overline{)936}^{312}$ 2. $15\overline{)30}^{2}$ 3. $5)\underline{45}(9$



Instructions: Include the required transcriber's note regarding the carried numbers.

PRACTICE 13H

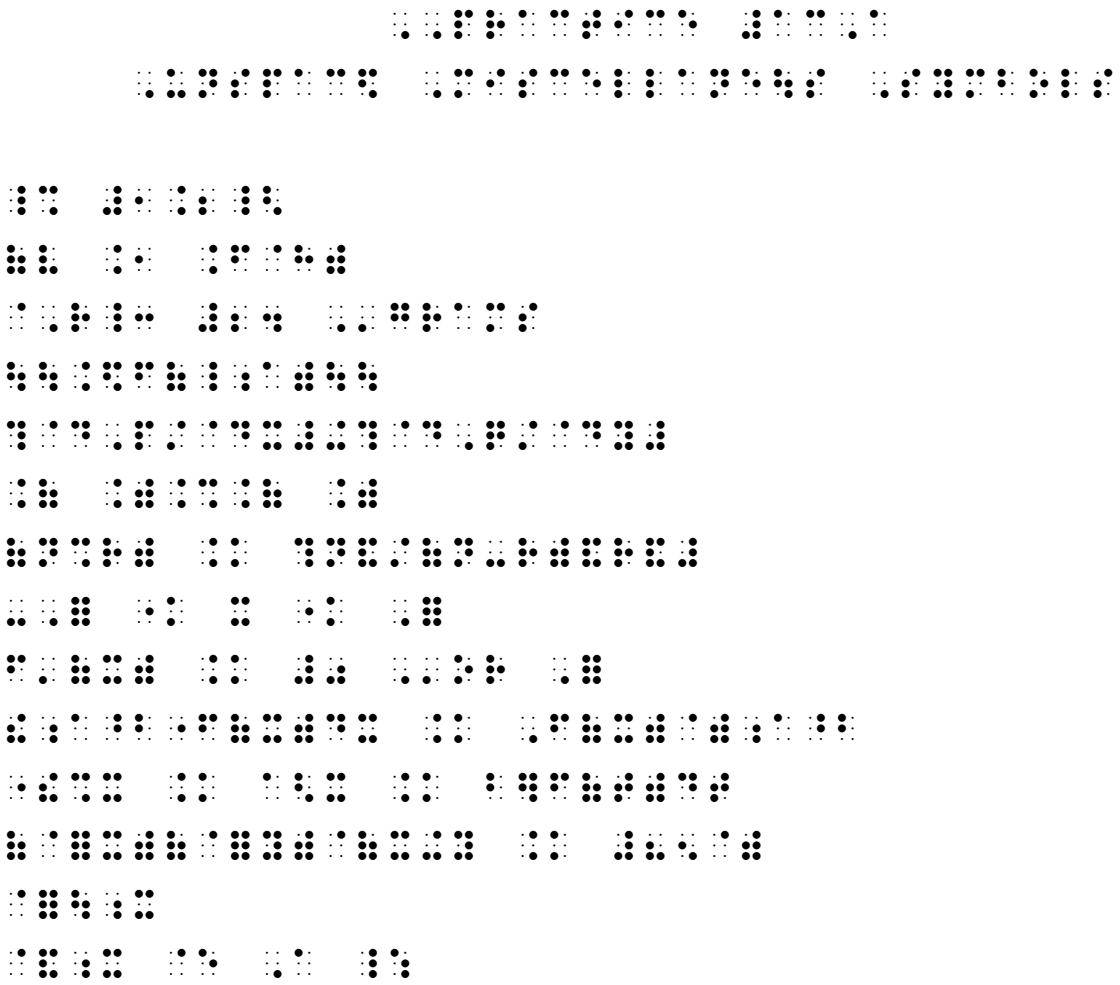
Fill in the boxes with the correct numeral.

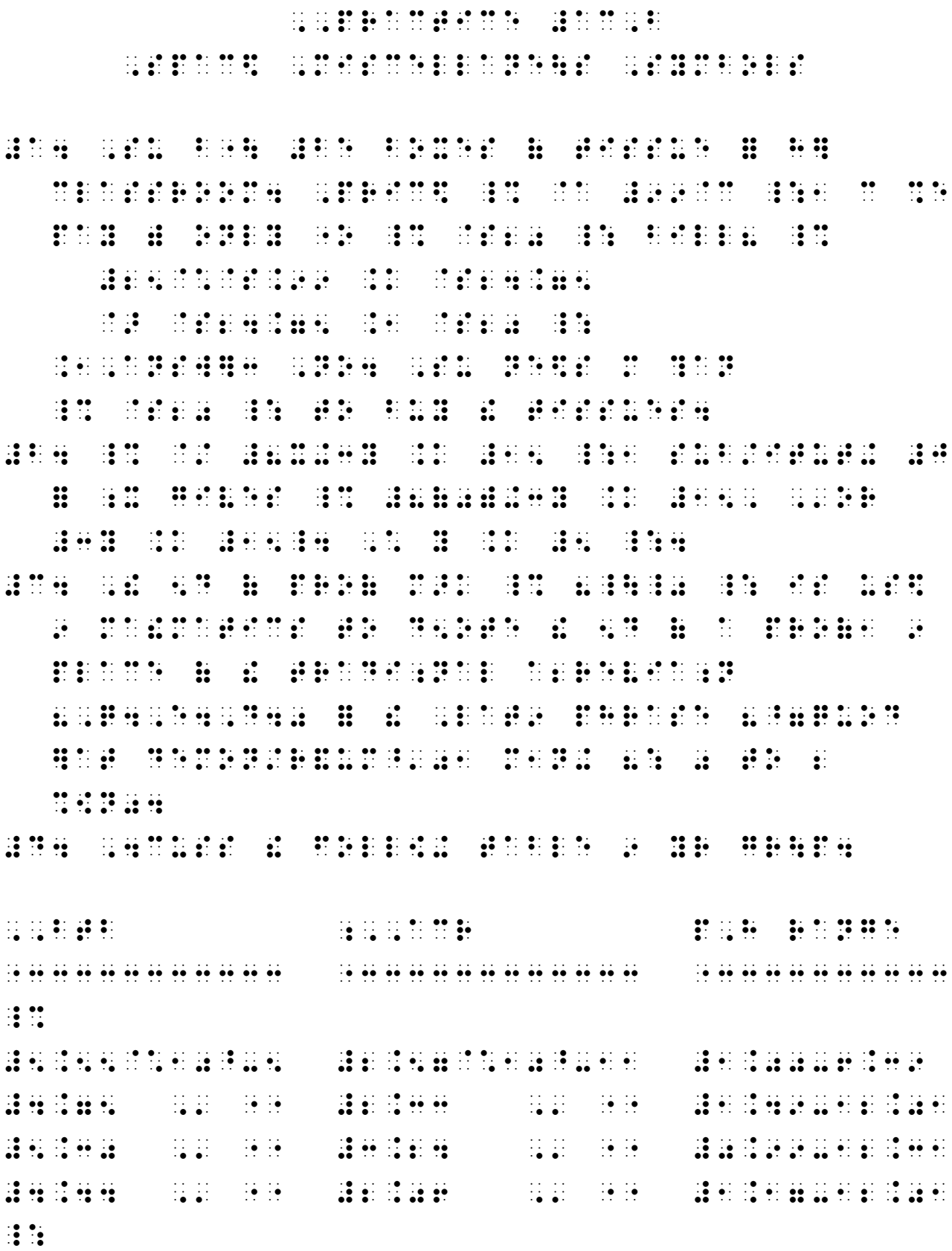
(a) $6\overline{)23^{5}5^{1}813}^{3\,9\,302\ \text{R}\,\square}$ (b) $25\overline{)80}^{3\frac{1}{5}}$ $\underline{75}$ $\square$ (c) $4\overline{)19}^{\square\ \text{R3}}$

Other print layouts showing division will be taught in the next lesson.

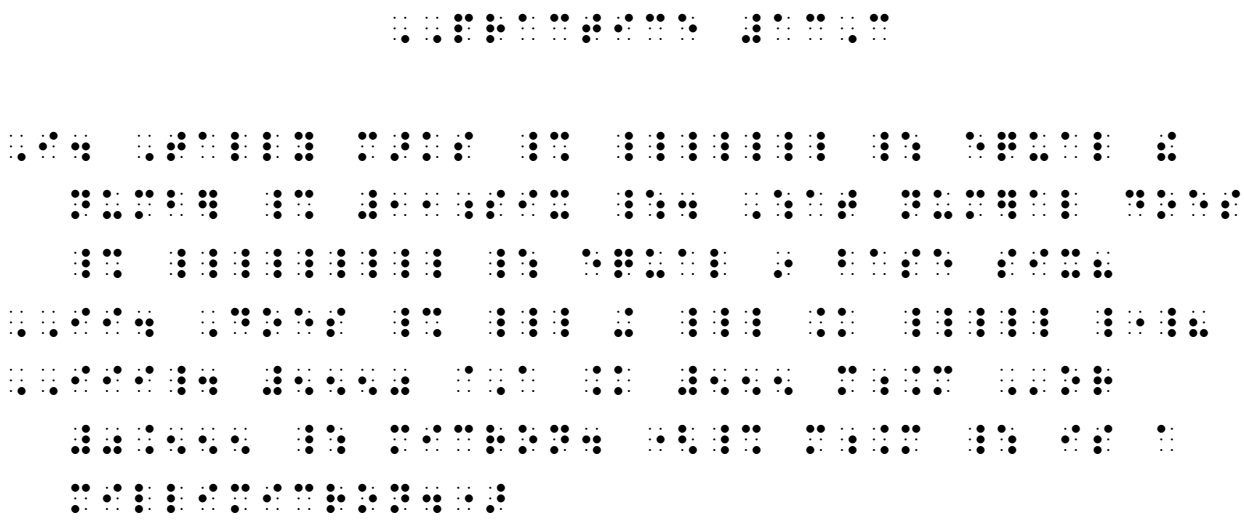
For further practice, see Appendix A—Reading Practice.

ANSWERS TO PRACTICE MATERIAL

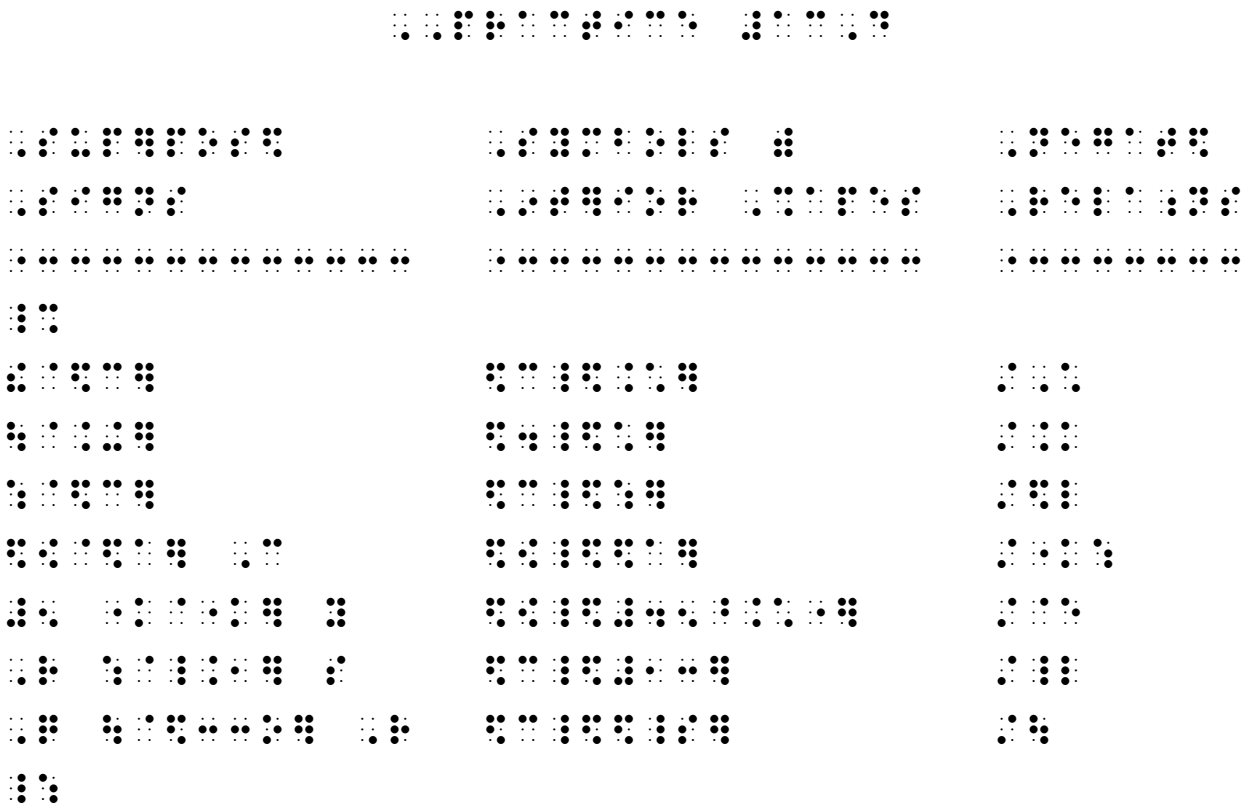




3. Contractions are used in the Latin words according to Braille Formats *1.16.3. The first column of a table begins in cell 1 (*Braille Formats*, Section 11).*



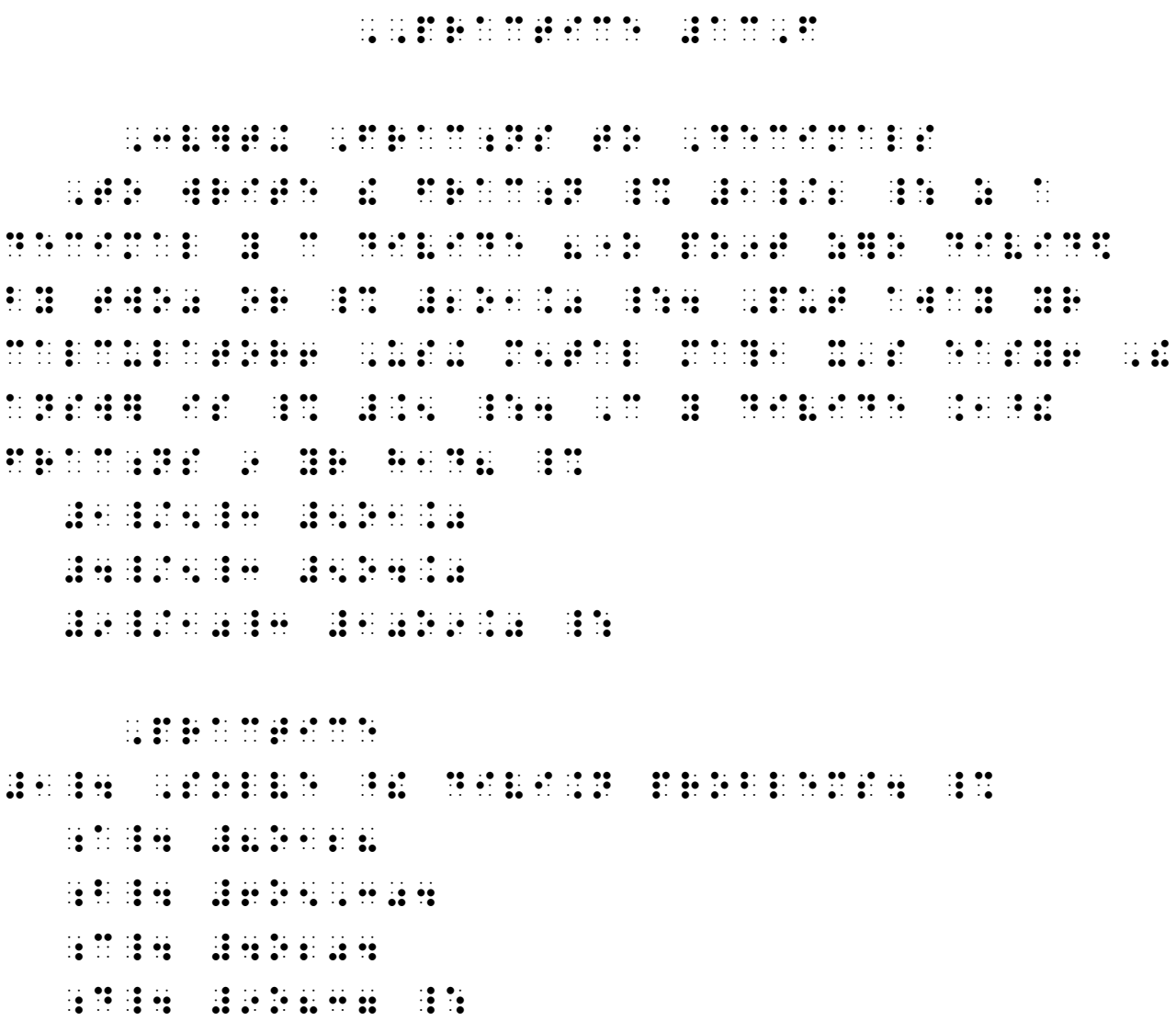
Switch decision: Preferred practice is to braille grouping symbols in the same code, if possible. In item III, the opening and closing parentheses are in UEB. The opening Nemeth Code indicator is placed immediately after the opening parentheses.



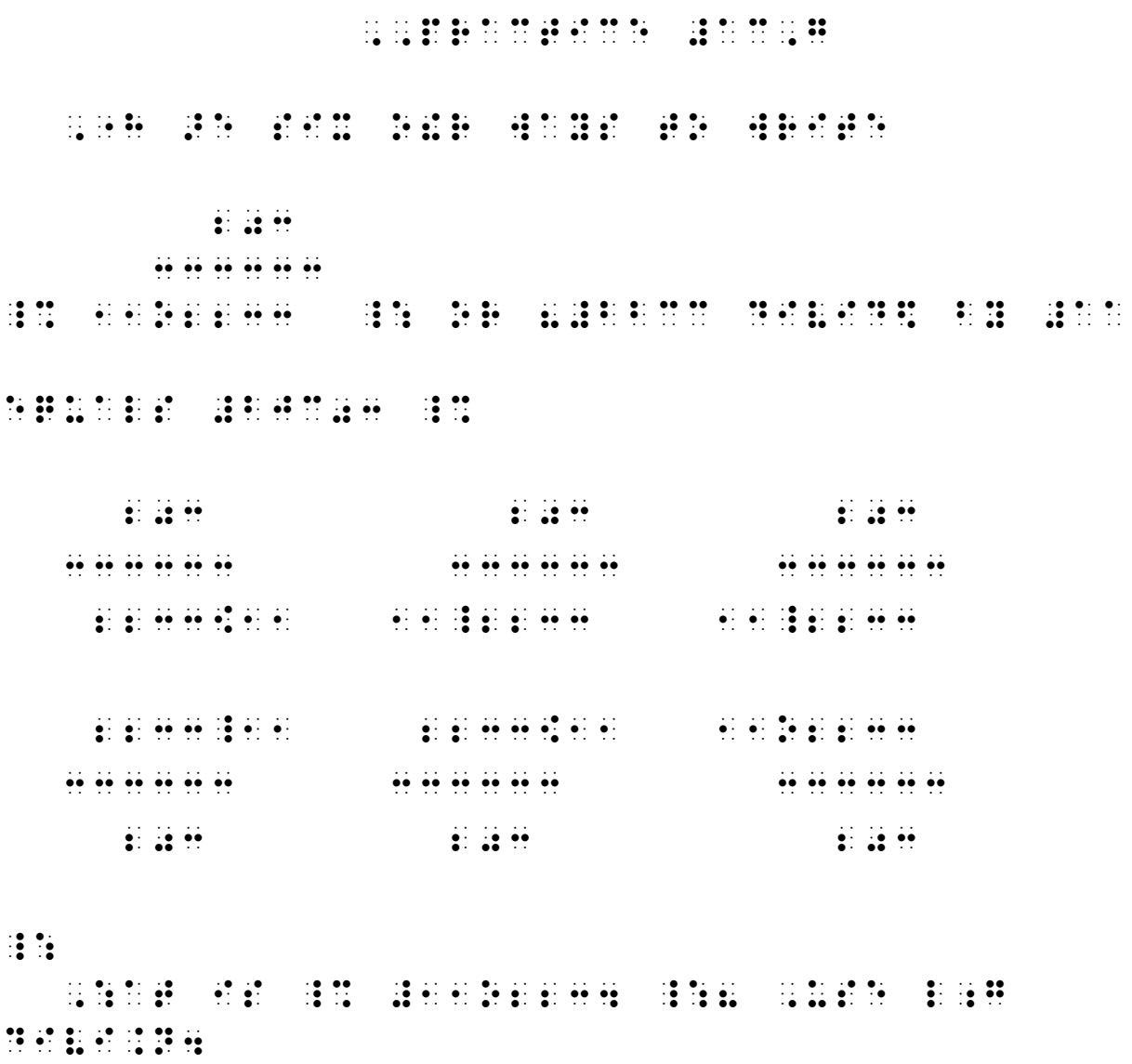
Guide dots are not used because the column entries are not related across the line.

PRACTICE 13E

1. A multipurpose indicator is used when a letter is followed by a numeral and they are both on the baseline of writing.
2. The first dot 5 is a baseline indicator because the plus sign is on the baseline and it follows a raised hollow dot. The second dot 5 is a multipurpose indicator which is needed to show that the numeral "3" is not a subscript to the letter "n".
3. The same rule applies to letters in any alphabet – a multipurpose indicator is needed to show that the numeral "2" is not a subscript to the Greek letter pi.
4. Same as #3 regarding Greek letter omega followed by numeral "2" in "ω2". Note that a multipurpose indicator is not needed for a letter following a numeral, as in " 2ω".
5. A baseline indicator precedes the plus sign, following the subscript "12". (The multipurpose indicator is not used following the "t" and "e" because they represent numerals in base 12.)
6. A multipurpose indicator is needed after the second numeric subscript "0" because the subscript is followed by a numeral on the baseline of writing ("2").
7. The function of the first and third dot 5 is the same as item 6. The second dot 5 is a baseline indicator which is needed for the minus sign following a superscript.
8. A multipurpose indicator is needed after the decimal point because the next symbol is not a numeral—it is the Greek letter "alpha".
9. A multipurpose indicator is needed after the decimal point because the next symbol is not a numeral—it is a percent sign.
10. A multipurpose indicator is needed after the decimal point because the next symbol is not a numeral—it is a long dash.
11. A multipurpose indicator is needed after the decimal point because the next symbol is not a numeral—it is a plus sign.
12. A multipurpose indicator is used between two unspaced vertical bar symbols (in this case, each is a "double vertical bar" symbol) when the first is a closing sign of grouping, and the second bar is an opening sign of grouping.
13. A multipurpose indicator is used between vertical bars which are nested grouping symbols. (The double bar symbols are shorter than the single bar symbols.)



Reminder: The subdivisions each begin a new line in braille. The rule for non-spatial material states that <u>all</u> subdivisions must fit on one line in order to braille them side-by-side.

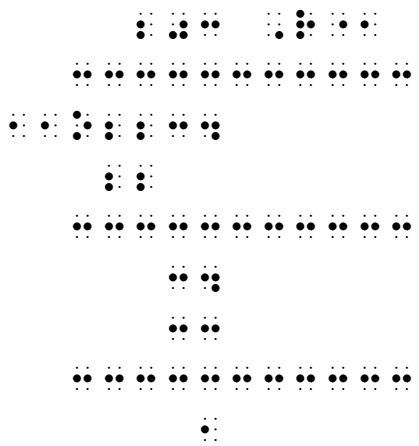


Lines 4 and 8 are blank because a spatial arrangement must be preceded and followed by a blank line, even when embedded within a narrative paragraph.

The Nemeth Code terminator is placed in cell 1 following the blank line after the completion of the spatially-arranged material.

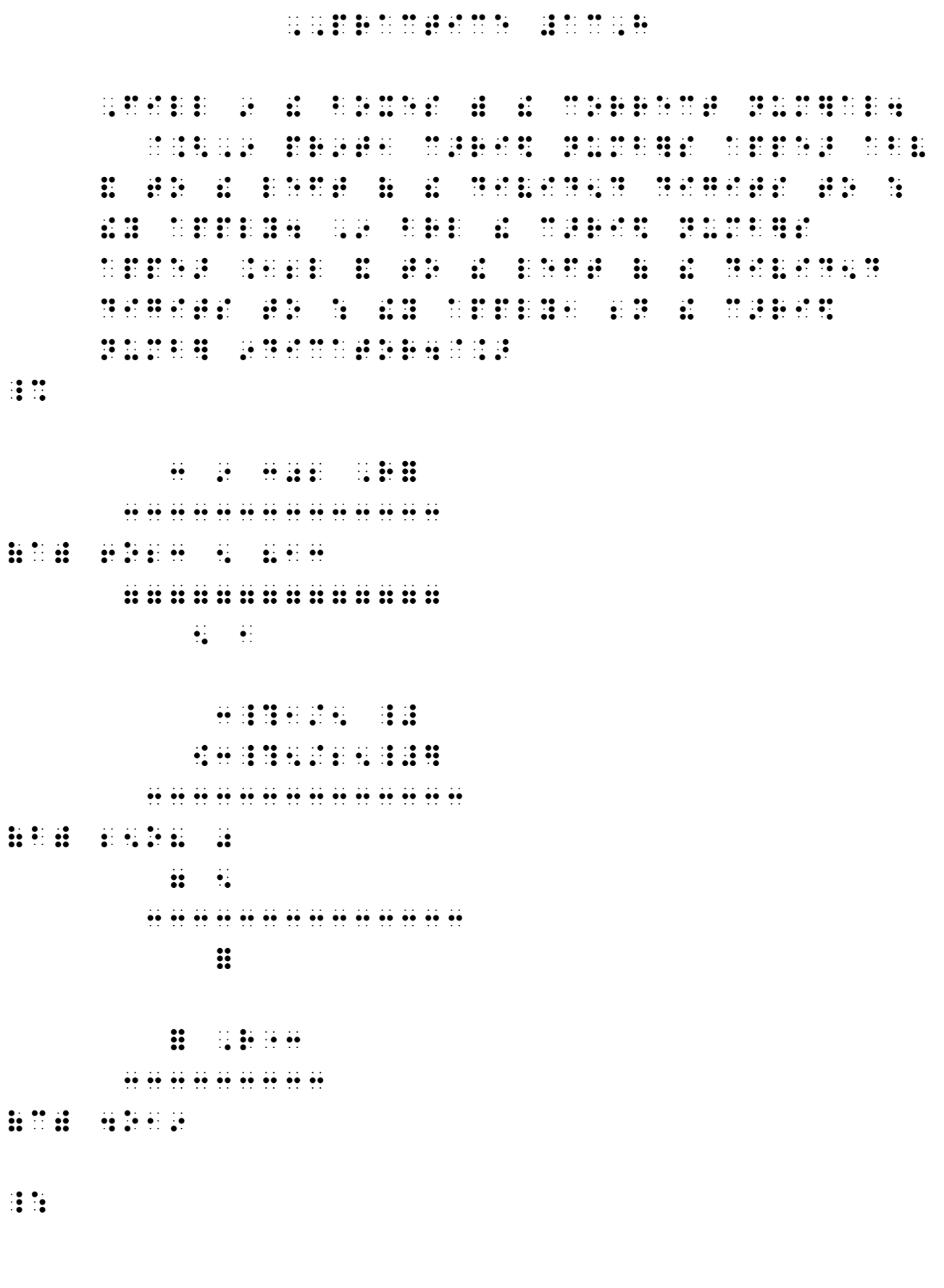
This practice continues on the next page.

⠸⠼



⠸⠳

The opening Nemeth Code indicator appears on the same page as the material to which it applies. The leftmost character in the problem (line 5, above) is placed in cell 3, the margin for material displayed to 3-1 narrative.



Notes to Practice 13H

The instructions begin in cell 5 and are followed by the required transcriber's note which reads as follows: In print, carried numbers appear above and to the left of the dividend digits to which they apply. In braille the carried numbers appear *below* and to the left of the dividend digits to which they apply, beneath the carried number indicator.

A blank line precedes each spatial problem. Because the general omission symbol is non-numeric (even though it may represent a numeral), a multipurpose indicator is not needed following the letter "R" in problem (a).

EXERCISE 13

Exercise 13 will be available when this course is finished being written and is no longer "Provisional".

Proceed to Lesson 14.

LESSON 14

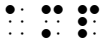
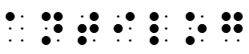
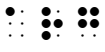
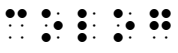
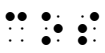
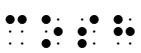
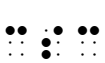
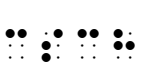
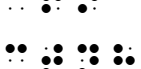
Read about this PROVISIONAL EDITION in the front matter to this book.
Check the NFB website periodically for updates to this lesson.

- *FUNCTION NAMES AND THEIR ABBREVIATIONS*
- *Division of Mathematical Expressions Between Braille Lines*

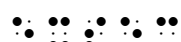
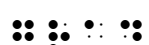
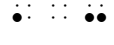
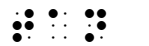
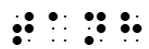
  Spatial Arrangements
  - *SQUARE ROOT DIVISION*
  - *OTHER PRINT LAYOUTS SHOWING DIVISION*

FUNCTION NAMES AND THEIR ABBREVIATIONS

14.1 Functions: The most common function names as well as their abbreviations are listed below. Function names or their abbreviations that do not appear in this list are subject to the same rules taught in this lesson.

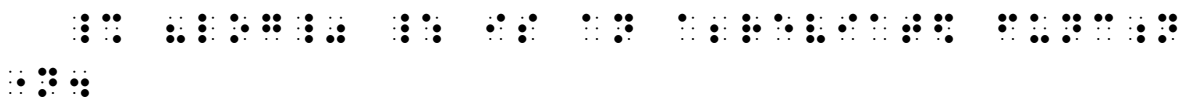
ABBREVIATED FUNCTION NAME	BRAILLE EQUIVALENT	UNABBREVIATED FUNCTION NAME
amp		amplitude
antilog		antilogarithm
arc		arc
arg		argument
colog		cologarithm
cos		cosine
cosh		hyperbolic cosine
cot		cotangent
coth		hyperbolic cotangent
covers		coversine
csc		cosecant
csch		hyperbolic cosecant
ctn		cotangent
ctnh		hyperbolic cotangent

(continued)

ABBREVIATED FUNCTION NAME	BRAILLE EQUIVALENT	UNABBREVIATED FUNCTION NAME
det		determinant
erf		error function
exp		exponential
exsec		exsecant
grad		gradient
hav		haversine
im		imaginary part
inf		infimum
lim		limit
$\overline{\text{lim}}$ or $\overline{\text{limit}}$		upper limit
$\underline{\text{lim}}$ or $\underline{\text{limit}}$		lower limit
ln		natural logarithm
log		logarithm
max		maximum
min		minimum
mod		modulo
re		real part
sec		secant
sech		hyperbolic secant
sin		sine
sinh		hyperbolic sine
sup		supremum
tan		tangent
tanh		hyperbolic tangent
vers		versine

14.1.1 Code Switching: Abbreviated function names are mathematical expressions and are transcribed following the rules of the Nemeth Code – they are not considered to be ordinary abbreviations.

Example 14.1-1 "log" is an abbreviated function name.



A punctuation indicator is required before the closing quotation mark in a mathematical expression.

Unabbreviated function names which are not associated with a math term are transcribed in UEB and appropriate contractions are used.

Example 14.1-2 "log" is the abbreviated function name for "logarithm".



When an unabbreviated function name is used in conjunction with an abbreviated function name, a switch to Nemeth Code is required.

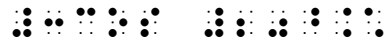
Example 14.1-3 What is the meaning of logsine?



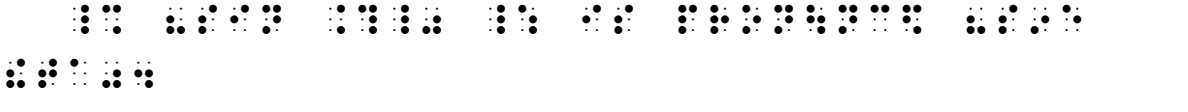
"log" is an abbreviated function name, so "logsine" requires a switch to Nemeth Code.

> For the remainder of this lesson, the term "function name" refers to both abbreviated and unabbreviated function names. If a distinction is necessary, the specific term will be used.

14.2 Spacing of Function Names: Within a mathematical expression, the following spacing rules are observed: no space is to come before a function name; a space is required after a function name. These spacing rules apply regardless of the spacing used in the print copy.

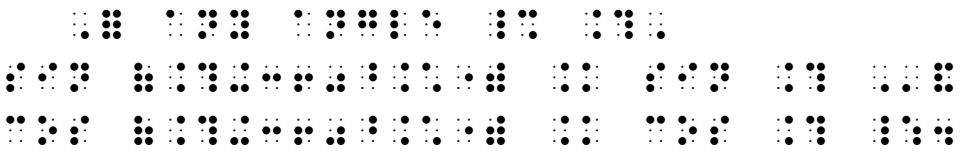
- 3 cos 20° 
- Sine 195° 
- sin θ 

Example 14.2-1 "sin θ" is pronounced "sine theta".



Other spacing rules of the Nemeth Code may take precedence.

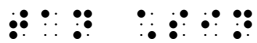
Example 14.2-2 For any angle θ, $\sin(\theta + 360°) = \sin\theta$ and $\cos(\theta + 360°) = \cos\theta$.



Within the expression, the space preceding "sin" and "cos" is present because a space is required following an equals sign. Note that each function name ("sin" and "cos") is followed by a space even though a space may not appear in print.

14.2.1 Spacing with Signs of Operation: Although a sign of operation is usually unspaced from the symbols which precede and follow it, the space required after a function name takes precedence.

⋙ $\tan \cdot \sin$



"tan" is followed by a space. (A space is not required before "sin".)

⋙ $1/\cos - \cos = \tan \cdot \sin$



Each function name is followed by a space. The only function name preceded by a space is the one following the equals sign ("tan") because a space is required following a sign of comparison.

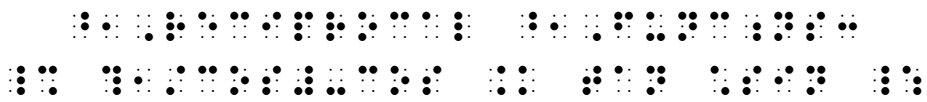
14.2.2 Spacing with Indicators: A space is not inserted between a function name and an indicator which applies to it.

⋙ $\frac{1}{\cos}$



There is no space between "cos" and the closing fraction indicator.

Example 14.2-3 **Reciprocal Functions:** $\frac{1}{\cos} - \cos = \tan \cdot \sin$



*Note that the "required space" following the first function name "cos" is not present here because the closing fraction indicator applies to it. The minus sign is unspaced from the closing fraction indicator because spacing before and after a fraction is subject to the spacing rules for the signs preceding and following the fraction. (Review **Lesson 9**.)*

Examples 14.2-4-14 Examine the spacing in the following examples. Also notice that mathematical letters are printed in italics, and abbreviated function names are in regular type.

(4) $\cos\theta = \frac{1}{\sin\theta}$



(5) $\sin(\theta + 90°) = \cos\theta$



(6) $y = 3\tan 2x$



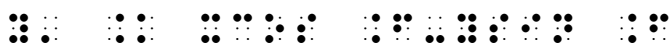
(7) $a^2 = 2ac\cos\beta + c^2$



(8) $\sin(\alpha + \beta) + \sin(\alpha - \beta)$



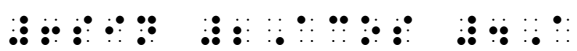
(9) $y' = x\cos\varphi - y\sin\varphi$



(10) $2\sqrt{x}\sin\sqrt{x} + 2\cos\sqrt{x} = C$



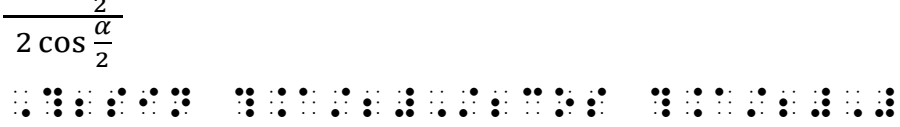
(11) $6\sin 2A\cos 4A$



(12) $\cos 203°\csc 203°$



(13) $\dfrac{2\sin\frac{\alpha}{2}}{2\cos\frac{\alpha}{2}}$



(14) $\ln\left|\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right| + C$

"ln" is the abbreviated function name for "natural logarithm". (Refer to the table of function names at the beginning of this lesson.)

PRACTICE 14A

(1) $\sin 30° \cos 45°$

(2) $\sin\theta / \cos\theta$

(3) $\sin 2\alpha = 2 \sin\alpha \cos\alpha$

(4) $\dfrac{\tan 90°}{\cot 90°}$

(5) $r[3\cos\theta + 4\sin\theta] = 5$

(6) $7(\cos 20° + i \sin 20°)$

(7) $\frac{1}{2}\ln|\sec 2t + \tan 2t| + C$

(8) $a \sin\frac{x}{a} \cdot \frac{1}{a} = \sin\frac{x}{a}$

14.3 Nonuse of the English Letter Indicator: The English letter indicator is not used with an English letter following a function name.

⋙ sin x 

The same rule applies to a letter combination that mimics a shortform, or to a Roman numeral following a function name. No ELI is used.

⋙ vers ab 

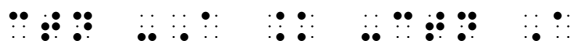
⋙ cot I 

Examples 14.3-1-6 Examine the English letters and the spacing in the following examples.

(1) $\sin x + y$



(2) $\mathrm{ctn} - \mathrm{A} = -\mathrm{ctn}\ \mathrm{A}$



(3) $y = 2\sin x + \sin 2x$



(4) $y = \sqrt{\cot x}$



(5) $\{\sin x \mid \sin x + 2 \leq +1\}$



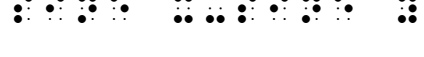
(6) $y = \ln|\tan x|$



14.4 Unabbreviated Function Names in Mathematical Context: When an unabbreviated function name occurs in mathematical context it is brailled in Nemeth Code, without contractions. "Mathematical context" includes being associated with a mathematical symbol ...

⋙ Sine I 

This unabbreviated function name ("Sine") is in Nemeth Code because it is associated with the Roman numeral "I".

⋙ $\sin e\, x - \sin e\, y$ 

... or when used in conjunction with an abbreviated function name.

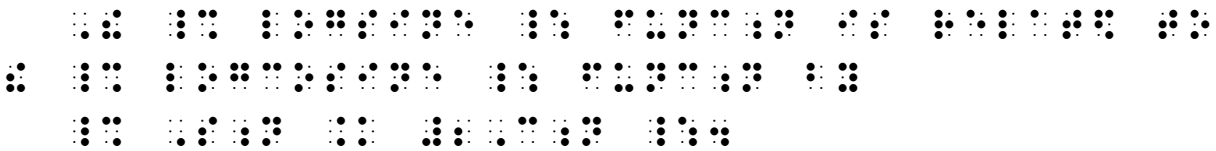
Example 14.4-1 What is the meaning of logsine?



Abbreviated function names are mathematical material. "log" is an abbreviated function name, therefore "logsine" requires a switch to Nemeth Code.

Example 14.4-2 The logsine function is related to the logcosine function by

$$S_n = 2C_n.$$

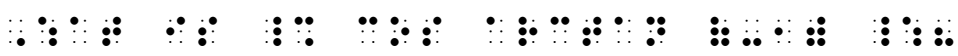


14.5 Consecutive Function Names: A space is required between two or more consecutive function names unless they are clearly unspaced in the print text. When there is doubt concerning the presence of a space between the function names or their abbreviations, a space should be left in the transcription.

⋙ $y = \arcsin x$



Example 14.5-1 What is $\cos \arctan(-1)$?



Examples 14.5-2-6 Study the following examples.

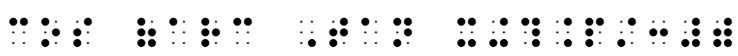
(2) $n = \log \sin 50°27'$



(3) $\cos\left[2 \operatorname{Arc\,csc}\left(-\frac{7}{5}\right)\right]$



(4) $\cos\left(\operatorname{arc\,Tan} x + \frac{\pi}{3}\right)$



(5) Arc Sine x



(6) $\operatorname{Arctan} x + \operatorname{Arccot} x = \frac{1}{2}\pi$



14.5.1 "Arc" in Context: "Arc" can be an unabbreviated function name as well as the abbreviated function name. Transcribe this function name in the braille code that the associated material is in.

⩾ arc θ 

The function name "arc" is brailled in Nemeth Code because it is associated with a math symbol.

Example 14.5-7 What is the arc sine function?



"sine" is unabbreviated (UEB), so "arc" is also brailled in UEB.

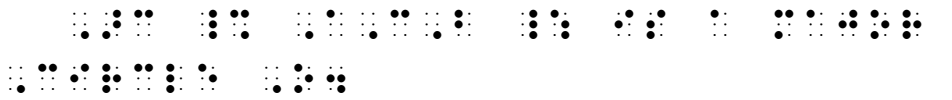
Example 14.5-8 What is the arc sin function?



"sin" is abbreviated (Nemeth Code), so "arc" is also brailled in Nemeth Code.

14.5.1.a Do Not Confuse: The word "arc" can also refer to a curved line.

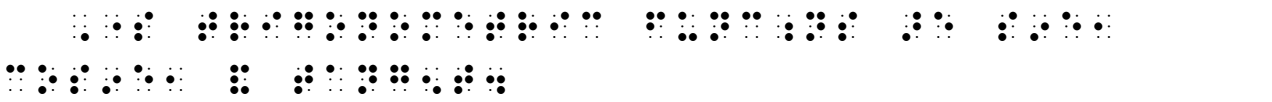
Example 14.5-9 Arc ACB is a major arc in Circle O.



"Arc" is not a function name in this context. It is referring to the curved line, "ACB".

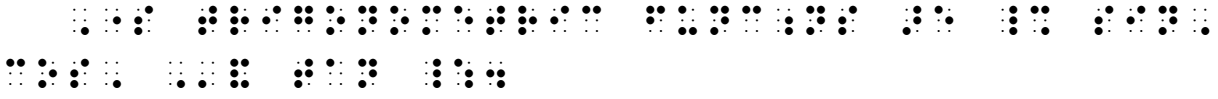
14.6 Punctuation: An abbreviated function name is a mathematical expression and is punctuated accordingly. An unabbreviated function name is punctuated in the mathematical or literary mode according to its context.

Example 14.6-1 Some trigonometric functions are sine, cosine, and tangent.



Unabbreviated function names are punctuated in literary mode, in UEB.

Example 14.6-2 Some trigonometric functions are sin, cos, and tan.



Abbreviated function names are punctuated in mathematical mode, in Nemeth Code.

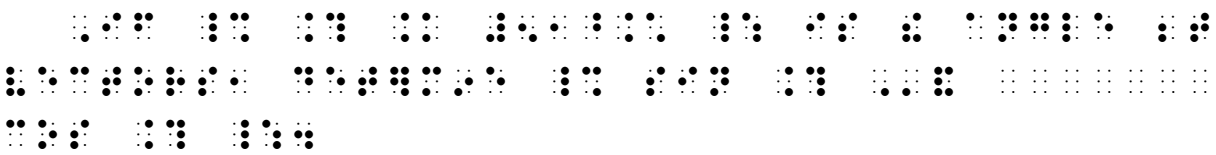
Example 14.6-3 "Arcsin" is the "inverse sine."



"Arcsin" is an abbreviated function name (math); "inverse sine" is an unabbreviated function name (narrative).

14.7 Keep Together: A function name and the sign which follows it is regarded as a single mathematical item and therefore should not be divided between braille lines.

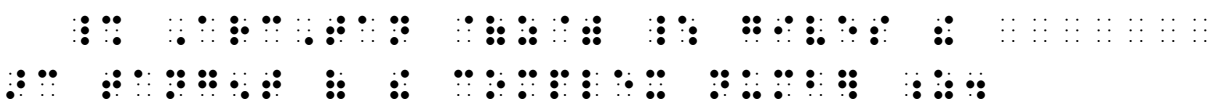
Example 14.7-1 If $\theta = 51°$ is the angle between vectors, determine sin θ and cos θ.



"cos θ" is not divided between lines even though "cos" fits on the previous line.

The rule applies even in UEB context, illustrated in this example of spaced consecutive function names.

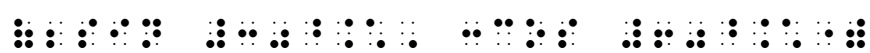
Example 14.7-2 ArcTan[z] gives the arc tangent of the complex number z.



"arc tangent" is not divided between lines even though "arc" fits on the previous line.

14.8 CLARIFICATION—Function Names in an Enclosed List: Although the numeric indicator is not used at the beginning of an item in an enclosed list, it must be used before a numeral (or decimal point and a numeral) following a function name.

⋙ (2sin 30°, 3cos 60°)



A function name and the item which follows it are regarded as a single item.

PRACTICE 14B

(A) $\sin x - \sin y$

(B) $2\sin x + 3\cos y$

(C) $\frac{1+\cos x}{\sin x} + \frac{\sin x}{1+\cos x}$

(D) The logarithm of sine 18° is written log sin 18°.

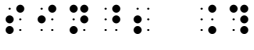
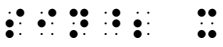
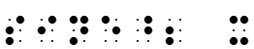
(E) $\cos 225° = -\sqrt{\frac{1+\cos 450°}{2}}$

(F) ArcTan[x, y] gives the arc tangent of $\frac{y}{x}$, taking into account which quadrant the point (x, y) is in.

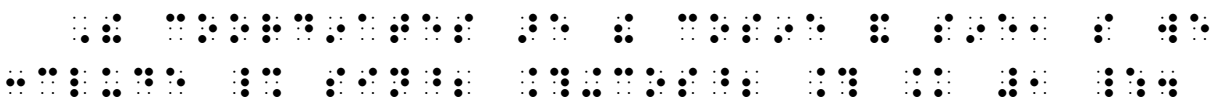
(G) The arc tangent of the complex number q is written "ArcTan[q]".

(H) Consider the ordered pair (cos .8000, 2 cos .8000).

14.9 Function Names and Superscripts/Subscripts: When a function name carries a superscript or a subscript, the required space follows the superscript or subscript. A letter, numeral, or other mathematical expression following this space assumes the same level as the function name. (In the following three examples, letters "θ" and "x" are printed on the baseline of writing and are unspaced.)

- $\sin^2\theta$ 
- $\sin^2 x$ 
- $\text{sine}^2 x$ 

Example 14.9-1 The coordinates are the cosine and sine, so we conclude $\sin^2\theta + \cos^2\theta = 1$.



The required space follows the superscript in sin² and cos². There is no space following the plus sign.

Example 14.9-2 Verify that $1 - \cos\frac{2\pi}{3} = 2\sin^2\frac{4\pi}{3}$.



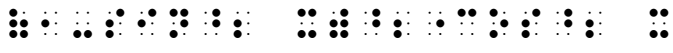
The required space follows "cos" and "sin²". There is no space following the minus sign.

Examples 14.9-3-6 Study the following examples.

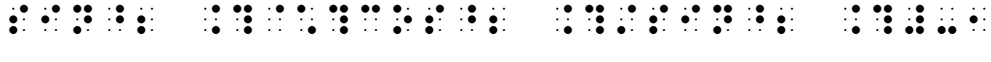
(3) $\sin^2 A + \cos^2(B + A)$



(4) $(1 - \sin^2 x)^2 \cos^2 x$



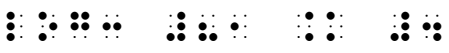
(5) $\sin^2\theta \times \frac{\cos^2\theta}{\sin^2\theta} - 1$



(6) $\dfrac{1 - \dfrac{\sin^2 x}{\cos^2 x}}{\sec^2 x}$



14.9.1 Use/Nonuse of the Subscript Indicator: The subscript indicator is not used when an abbreviated function name carries a numeric subscript on the first level below the baseline of writing.

⋙ $\log_3 81 = 4$ 

The numeral "3" is printed at the subscript level.

A subscript indicator is required in all other circumstances, including an unabbreviated function name which carries a numeric subscript.

⋙ $\text{logarithm}_3\ 81 = 4$ 

A subscript indicator is required because the subscript applies to a word (an unabbreviated function name is a word).

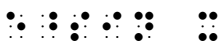
⋙ $\log_b N \cdot \log_a b$ 

A subscript indicator is required because each subscript is a letter.

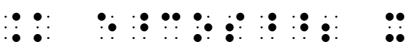
⋙ $\log_{2e} x = -1.4$ 

A subscript indicator is required because the subscript contains a letter.

14.9.2 Function Names Within a Superscript or a Subscript: When a function name occurs within a superscript or subscript, the required space following it maintains the level at which the function name appears. A restatement of the level indicator is not needed.

⋙ $e^{\sin x}$ 

"sin x" is in the superscript position. Because the item immediately following the space after a function name assumes the same level as its related function name, no level indicator is needed for "x".

⋙ $y = e^{\cos^2 x}$ 

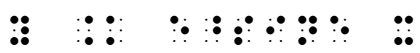
"cos² x" is in the superscript position. Because the item immediately following the space after a function name assumes the same level as its related function name, no level indicator is needed for "x".

Examples 14.9-7-15 Study the following examples.

(7) $y = e^{\sin x}$



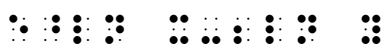
(8) $y = e^{\text{sine } x}$



(9) $y = (\sin x)^{\tan x}$



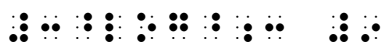
(10) $e^{\ln x - 2\ln y}$



(11) $a^{g(x)\log_a f(x)}$

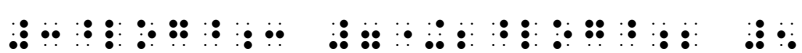


(12) $3^{\log_3 9}$



*Recall from **Lesson 8** that a subscript indicator is required in superscript and subscript combinations. The super/sub indicator shows a numeric subscript in the superscript position.*

(13) $3^{\log_3 7} + 2^{\log_2 5}$



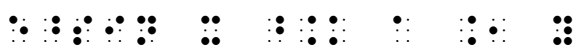
See note above.

(14) $a^{\log_a x} = x$



*Recall from **Lesson 8** that the space before a comparison sign returns the reader to the baseline.*

(15) $e^{\sin x = a} > y$

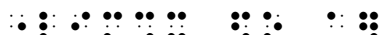


*Recall from **Lesson 8** that when a comparison sign occurs within a superscript, the level is restated before the comparison sign.*

PRACTICE 14C

1. $\log_n .125 = -.6$
2. $\text{antilog}_a\ x = N$
3. $\log_{.0543} x = -.7$
4. $\cot^{-1} x + \frac{\pi}{2} - \tan^{-1} x$
5. $\sin^2 90° + \cos^2 90° = 1$
6. $e^{x+\ln x}$
7. $e^{\sin x} + e^{\sin y}$
8. $2^{\sec x} = y$

14.10 Modifiers with Function Names: Modified function names are transcribed according to the five-step rule for the transcription of modified expressions introduced in **Lesson 12**.

⋙ $\lim_{x \to a}$ 

When a function name carries a modifier, the required space after the function name follows the termination of the modifier.

⋙ $\lim_{x \to a} f(x) = 1$ 

Examples 14.10-1-2 Study these additional examples.

(1) $\lim_{x \uparrow 4} (x-4)^{-1}$

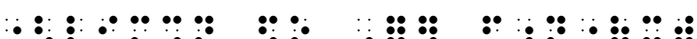


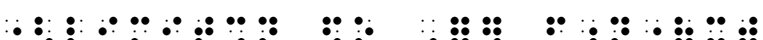
(2) $\lim_{\theta \to \theta_0} (\tan \theta)$

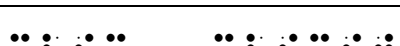


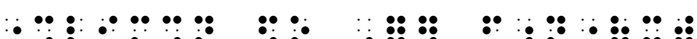
14.10.1 SPECIAL CASE—Upper Limit and Lower Limit: The symbols below denote "upper limit" or "lower limit". The horizontal bar directly over or under the function name "lim" or `limit" is <u>not</u> treated as a modifier.

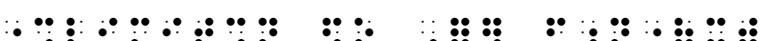
$\overline{\lim}$ or $\overline{\text{limit}}$		upper limit

⋙ $\overline{\lim_{n \to \infty}} f_n(x)$ 

⋙ $\overline{\lim_{n \to \infty}} f_n(x)$ 

$\underline{\lim}$ or $\underline{\text{limit}}$		lower limit

⋙ $\underline{\lim_{n \to \infty}} f_n(x)$ 

⋙ $\underline{\lim_{n \to \infty}} f_n(x)$ 

PRACTICE 14D

1. Find $\lim_{x \to 0.6} 2^{25x^2-10x-1}$.
2. Formulate a precise definition for $\lim_{x \downarrow -\infty} f(x) = L$.
3. If $\overline{\lim_{n\to\infty}}\ a_n = A$ and $\overline{\lim_{n\to\infty}}\ b_n = B$, must it be true that $\overline{\lim_{n\to\infty}}(a_n + b_n) = A + B$?
4. Find $\overline{\operatorname{limit}_{n\to\infty}}\ a_n$ when $a_n = (-1)_n$.
5. $\lim_{x\to 0} \csc x \ln(1+x)$

Division of Mathematical Expressions Between Braille Lines

14.11 Review: Recall from **Lesson 2** that a mathematical expression must not be divided between lines if it will fit on one braille line within the current margins. If there is insufficient space on the remainder of a line, the entire expression is brought down to the next line. In this lesson we will discuss what to do when a mathematical expression is too long to be contained within the current margins – that is, when a division is unavoidable. First, here is a review of "won't fit " topics already covered.

- **Nemeth Code Switch Indicators** (Lessons 2 and 4) If both switch indicators will not fit on the same line as the math expression, the opening Nemeth Code indicator may fall on the previous line. The Nemeth Code terminator and any related punctuation may be placed on the following line. **2.5.2, 4.7**
- **Long Numeral** (Lesson 2) A long numeral which cannot be contained on one braille line within the current margins is divided after a comma if a comma is present, and a hyphen is inserted. If the numeral does not contain a comma, the hyphen may be inserted after any digit. When a numeral is divided between braille lines, the numeric indicator is restated before the first digit of the numeral on the next line. Take another look at the two examples in **2.7.1.**
- **Enclosed List:** (Lesson 5) If an enclosed list will not fit on a single braille line, a runover line can begin following a comma used to separate the items. Take another look at the examples in **5.15.3.a.**
- **Linked Expressions** (Lesson 9) If a linked expression is too long to fit on one line, the expression continues on the next line, beginning with a sign of comparison. It is not necessary to divide at every comparison sign. The new line begins in the runover cell of the current format. **9.22**
- **Keystrokes** (Lesson 11) If items in a string of keyboard or calculator keystrokes will not fit on a single braille line, division may be made after any item in the keystroke string, but not within the keystroke. An example is shown in **11.25**.

14.12 The Concept of Logical Mathematical Units: When a math expression is too long to fit on one braille line, the expression is divided according to a priority list given in the Nemeth Code. When the rule is properly applied, a long or complicated mathematical expression will be disassembled into a series of logical mathematical units allowing the reader to mentally reassemble the expression. Conversely, a poorly divided expression will hinder the reader's understanding of the mathematics. Shrewd application of this rule can be properly rendered even if the transcriber is unfamiliar with the particular mathematics.

14.12.1 The Priority List: Mathematical expressions such as formulas, equations, etc., which cannot fit on one braille line within the boundaries of the margins used are to be divided between lines in the following order of preference:

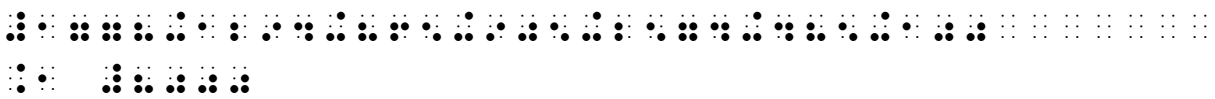
- before a comparison sign on the baseline of writing;
- before an operation sign on the baseline of writing;
- before a fraction line;
- before a baseline indicator;
- before a level indicator;
- between factors;
- after a termination indicator.

The examples in this section do not show code switch indicators. Assume mathematical context throughout. Full margins are shown for embedded material. Displayed material and subdivisions begin in cell 3 with runovers in cell 5. Reminder: A 38-cell line is used for the examples in this book.

14.13 PRIORITY #1—Before a Sign of Comparison: A division may be made before a sign of comparison as long as the sign is on the baseline of writing.

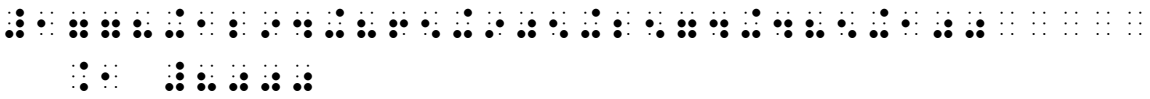
Example 14.13-1 $1778 + 1294 + 865 + 905 + 2574 + 485 + 100 > 8000$

Embedded:



The expression continues on the next line, beginning with the sign of comparison.

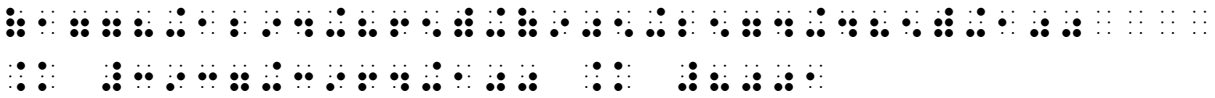
Displayed:



The new line begins in the runover cell of the current format.

Note that the print copy may divide after a comparison sign, but the braille transcription follows Nemeth Code rules and divides before the comparison sign.

Example 14.13-2 $(1778 + 1294 + 865) + (905 + 2574 + 485) + 100 =$
$3937 + 3964 + 100 = 8001$

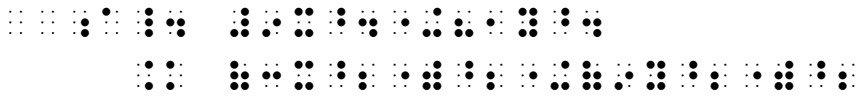


It is not necessary to divide at every comparison sign.

The transition to a new braille line made before a sign of comparison terminates the effect of any level indicator used on the line above, just as it would if the expression were not divided between lines.

Example 14.13-3 **a.** $9x^4 + 81y^4 = (3x^2)^2 + (9y^2)^2$

In a subdivision:

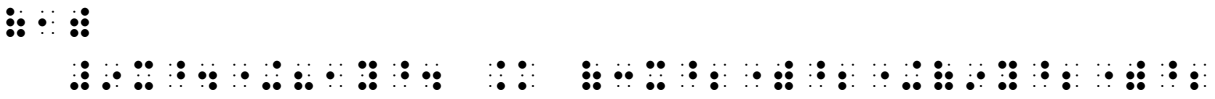


Continue the expression in the appropriate runover cell—cell 5, in this case.

14.13.1 Identifier May Stand Alone: Do not divide an expression if it will fit on one line. The identifier can stand alone on the previous line if the expression will not fit on the same line with the identifier.

Example 14.13-4 (1) $9x^4 + 81y^4 = (3x^2)^2 + (9y^2)^2$

In a main level entry starting in cell 1:



Begin the expression in the appropriate runover cell—cell 3, in this case.

Proper margins are used as runover sites are applied.

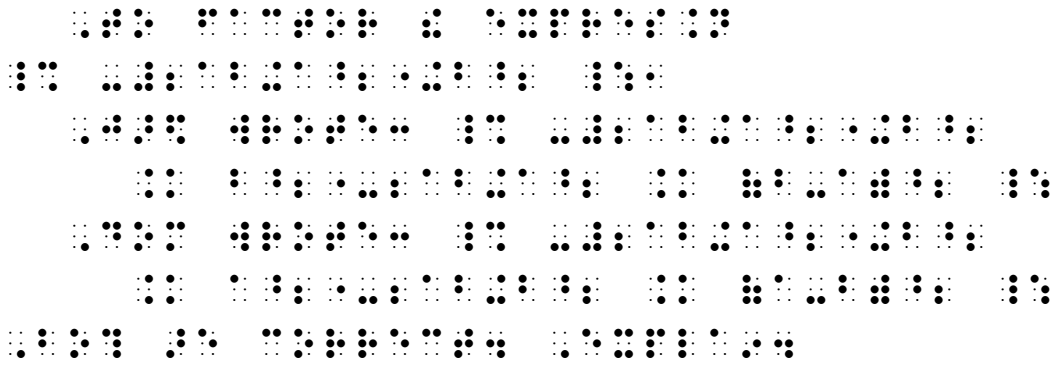
Example 14.13-5 To factor the expression $-2ab + a^2 + b^2$,

Jared wrote: $-2ab + a^2 + b^2 = b^2 - 2ab + a^2 = (b - a)^2$

Dom wrote: $-2ab + a^2 + b^2 = a^2 - 2ab + b^2 = (a - b)^2$

Both are correct. Explain.

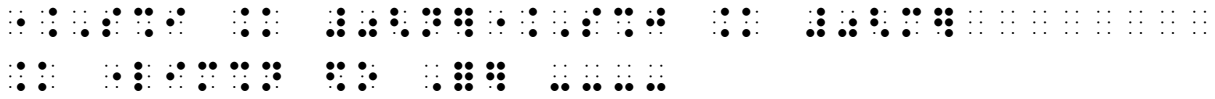
1
2
3
4
5
6
7



Lines 1-2: New paragraph begins in cell 3 with runovers in cell 1.
Lines 3-6: Two displayed expressions – each begins in cell 3, with runover in cell 5.
Line 7: Paragraph continues in the runover cell (cell 1).

14.13.2 *Logical Mathematical Units*—A comparison sign within a modifier, superscript, subscript, fraction, radical expression, etc. is not a logical division site.

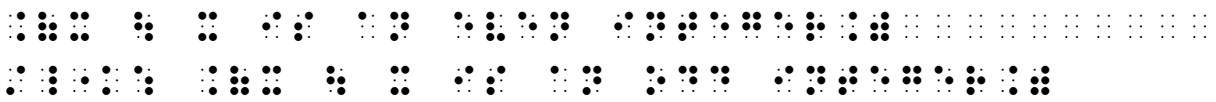
Example 14.13-6 $\sum_{i=0}^{n} \sum_{j=0}^{m} = \lim_{n \to \infty}$ ___



Only the comparison sign on the baseline is an appropriate division site.

Also note that material within mathematical grouping signs is a unit and should not be divided unless the entire grouping will not fit on one line.

Example 14.13-7 $\{x \mid x \text{ is an even integer}\} \nsubseteq \{x \mid x \text{ is an odd integer}\}$

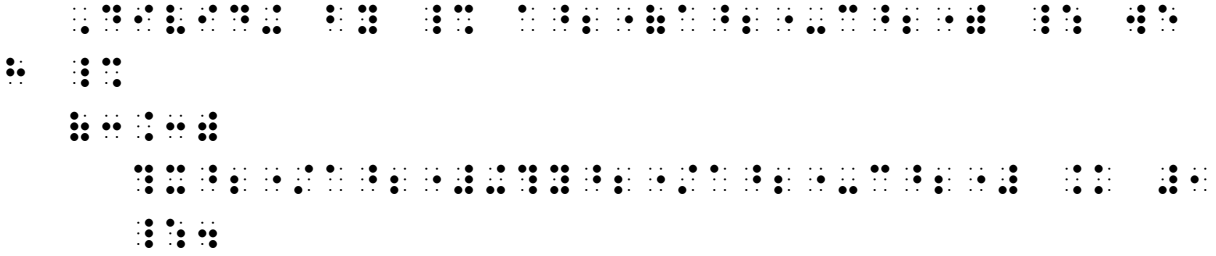


Although "⊈ {x" will fit at the end of the first line, only the comparison sign outside of the grouped expressions is an appropriate division site.

If the expression will fit on one line, do not divide it. Sometimes this means starting the expression on a new line following an identifier. The new line begins in the appropriate runover cell.

Example 14.13-8 Dividing by $a^2(a^2 - c^2)$ we have

$$\frac{x^2}{a^2} + \frac{y^2}{a^2 - c^2} = 1. \qquad (3.3)$$

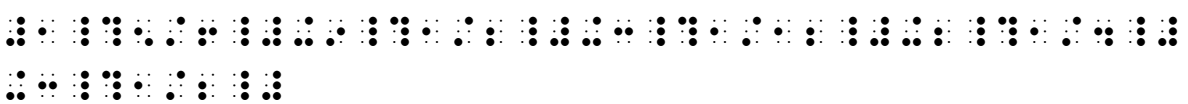


Lines 3-5: Displayed margins to narrative are 3-5.
Line 3: The label may sit alone on a line. The math will not fit on this line.
*(Review **Lesson 11** regarding displayed material with labels.)*
Line 4: The displayed expression will fit on one line in the runover position.
Line 5: The Nemeth Code terminator and punctuation may stand alone on a line.

14.14 PRIORITY #2—Before a Sign of Operation: A new line may begin before a sign of operation on the baseline of writing. Use as much of the current line as possible before dividing. It is not necessary to divide at every operation sign.

Example 14.14-1 $1\frac{5}{6} + 9\frac{1}{2} + 3\frac{1}{12} + 2\frac{1}{4} + 3\frac{1}{2}$

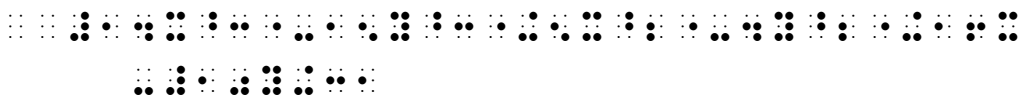
Embedded:



If the transition to a new line is made before a minus sign and a numeral, or before a minus sign and a decimal point and a numeral, a numeric indicator is required.

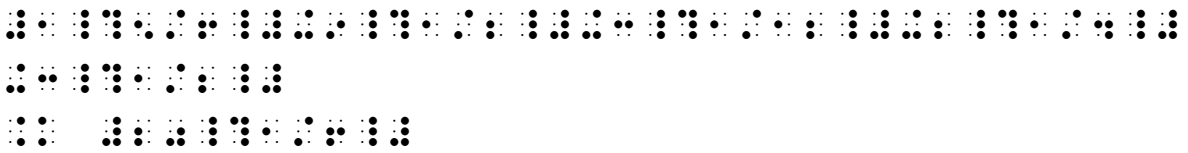
Example 14.14-2 $14x^3 - 15y^3 + 5x^2 - 4y^2 + 16x - 10y + 31$

Displayed:



14.14.1 *Logical Mathematical Units*—If the expression also contains a baseline sign of comparison, the "priority 1" division is applied first in order to maintain an orderly representation of logical mathematical units.

Example 14.14-3 $1\frac{5}{6} + 9\frac{1}{2} + 3\frac{1}{12} + 2\frac{1}{4} + 3\frac{1}{2} = 20\frac{1}{6}$

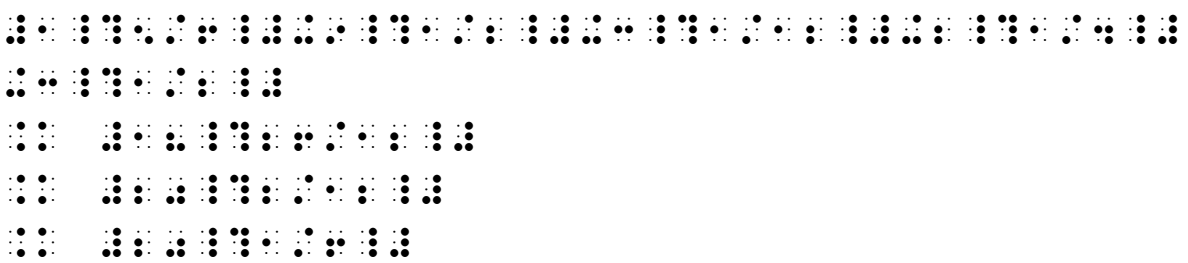


The first division decision is to begin a new line with the equals sign (priority #1). Since the anchor does not fit on one braille line, it is divided before a plus sign (priority #2).

Apply this concept in order to maintain an orderly representation of logical mathematical units: <u>If an expression must be divided at a site lower on the priority list, a new line is required at each sign which occurs higher on the list.</u>

Example 14.14-4 $1\frac{5}{6} + 9\frac{1}{2} + 3\frac{1}{12} + 2\frac{1}{4} + 3\frac{1}{2} = 18\frac{26}{12} = 20\frac{2}{12} = 20\frac{1}{6}$

Embedded:

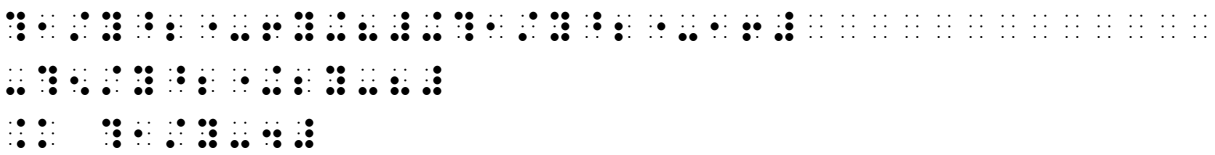


A new line is started at <u>each</u> sign which occurs higher on the list. Because the first link must be divided at a "priority 2" location (before an operation sign), every "priority 1" division site is also applied.

An operation sign within a modifier, superscript, subscript, fraction, radical expression, etc. is not a logical division site.

Example 14.14-5 $\frac{1}{y^2-6y+8}+\frac{1}{y^2-16}-\frac{5}{y^2+2y-8}=\frac{1}{y-4}$

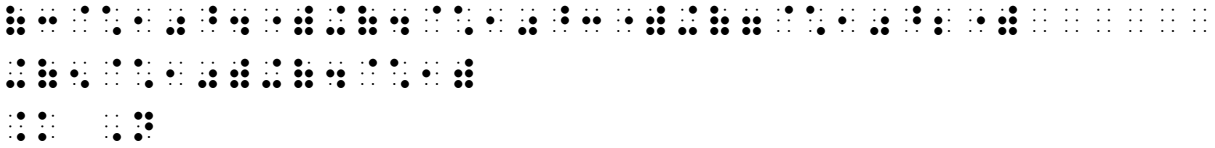
Embedded:



Only the operation signs on the baseline are appropriate division sites, not those in the denominators of the fractions. To maintain an orderly representation of logical mathematical units, the "priority 1" division (before the equals sign) is still applied.

Material within grouping signs is a unit whose division should be avoided (if possible).

Example 14.14-6 $(3\times10^4)+(4\times10^3)+(7\times10^2)+(5\times10)+(4\times1)=N$



Only the operation signs (plus signs) outside of the grouped expressions are appropriate division sites, not those between the parentheses (the multiplication crosses).

Instructions: Keep the following points in mind as you select runover sites. (1) keep logical units intact on one line, if possible; (2) the highest priority available in the expression is utilized first.

PRACTICE 14E

A. $2\times423=(2\times400)+(2\times20)+(2\times3)=800+40+6=846$

B. $\angle1+\angle2+\angle3+\angle4+\angle5+\angle6+\angle7=490°$

C. $2\frac{3}{4}\text{ yd}+1\frac{3}{4}\text{ yd}+\frac{3}{4}\text{ yd}=5\frac{1}{4}\text{ yd}$

D. $\sqrt{(x+a^2)+(y+a^2)}-\sqrt{(x-a^2)+(y-a^2)}=\pm2a$

Sommer's Routine: Sommer's routine can be represented by the following expression.

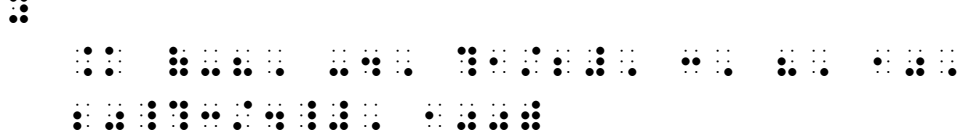
Routine S: $5+3+4(1+-1)+(-3)+(-5)+3\big(5+(-2)\big)+1$

Draw a simple diagram to represent *Routine S*.

CHALLENGE PROBLEM #1

The Nemeth Code lists "after a comma which occurs between items in an enclosed list" as the first item in the priority list. Since material within grouping signs is a unit whose division should be avoided, when an enclosed list is part of a larger math expression, division within the enclosed list should be made only if the entire list will not fit on one braille line. As seen below, dividing before the comparison sign has the desired effect of presenting the equation in logical units.

$$y = \left(-8, -4, \frac{1}{2}, 3, 8, 10, 20\frac{3}{4}, 100\right)$$

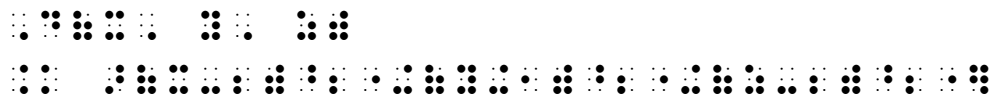


Since the expression will not fit on one line, a division is made before the equals sign. The link will not fit on one line. The link is an enclosed list– it is divided after a comma.

CHALLENGE PROBLEM #2

Similarly, there is no need to separate the items in the enclosed list in the following anchor.

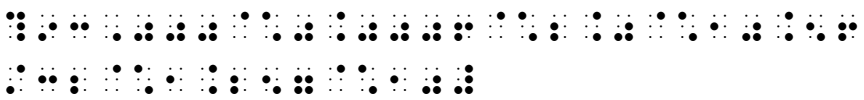
$$D(x, y, z) = \sqrt{(x-2)^2 + (y+1)^2 + (z-2)^2}$$



Priority list so far:
— before a comparison sign on the baseline of writing;
— before an operation sign on the baseline of writing;

14.15 PRIORITY #3—Before a Fraction Line: A division may be made before a fraction line belonging to a fraction on the baseline of writing.

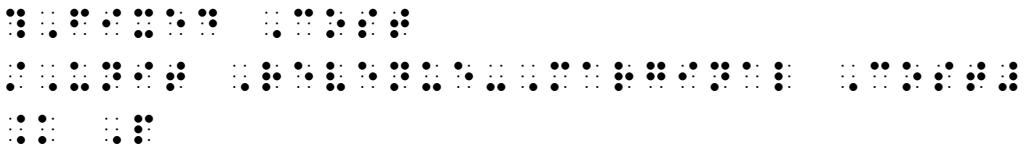
Example 14.15-1 $\dfrac{93{,}000\times0.0006\times2.0\times10.56}{32\times1.257\times10}$



If the expression contains more than one such fraction line, use as much of the line as possible before dividing the expression. It is not necessary to divide at every fraction line.

14.15.1 *Logical Mathematical Units*—An expression that requires division before a fraction line must also be divided before every previous item on the priority list—that is, before every baseline comparison sign and every baseline operation sign. Avoid division within a numerator or within a denominator.

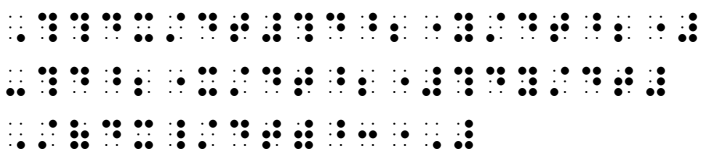
Example 14.15-2 $\dfrac{\text{Fixed Cost}}{\text{Unit Revenue}-\text{Marginal Cost}} = P$



The entire fraction will not fit on one line, so a division is made at the fraction line. Even though "= P" will fit on the second line (following the fraction's denominator), a division before the "priority 1" equals sign is required.

If the numerator or denominator requires a division, division at the fraction line must also be applied.

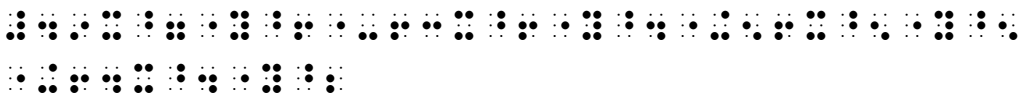
Example 14.15-3 $\dfrac{\dfrac{\mathrm{d}x}{\mathrm{d}t}\dfrac{\mathrm{d}^2y}{dt^2}-\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2}\dfrac{\mathrm{d}y}{\mathrm{d}t}}{(\mathrm{d}x/\mathrm{d}t)^3}$



*The numerator will not fit on one braille line. A division is made before the minus sign. A division is also required before the fraction line. (Review complex fractions in **Lesson 9**.)*

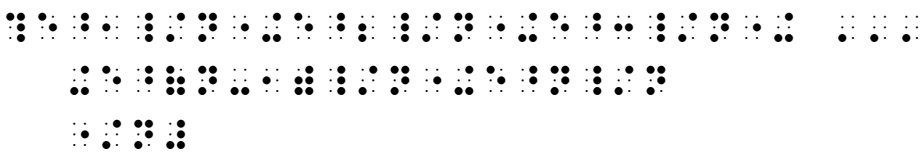
14.16 PRIORITY #4—Before a Baseline Indicator: If a baseline indicator is required to show a return to the baseline of writing after a superscript or subscript, it will be the first symbol on the new braille line of a divided expression. Use as much of the line as possible before dividing the expression.

Example 14.16-1 $49x^7y^6 - 63x^6y^4 + 56x^5y^5 + 64x^4y^2$



The baseline indicator is the first symbol in the runover line.

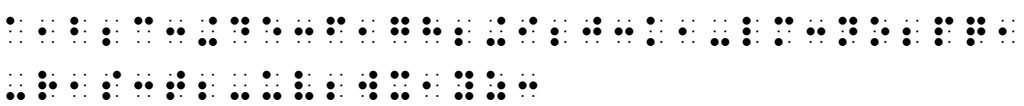
Example 14.16-2 $\dfrac{e^{1/n}+e^{2/n}+e^{3/n}+\ldots+e^{(n-1)/n}+e^{n/n}}{n}$



A baseline indicator is needed before this fraction line, even when the symbol begins in the runover position on a new line of a divided expression.

Note that a baseline indicator is not inserted if otherwise not required.

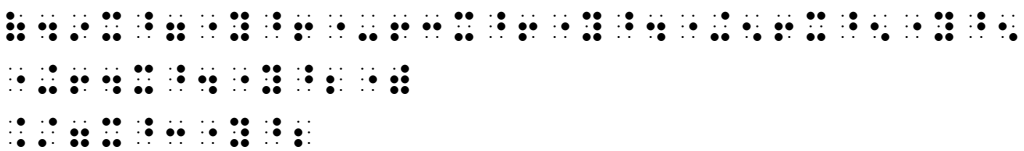
Example 14.16-3 $a_1b_2c_3 + de_3f_1gh_2 + i_2j_3k_1 - lm_3no_2pq_1 - r_1s_3t_2 - uv_2wx_1yz_3$



*Review **8.10** in Lesson 8 regarding nonuse of the subscript indicator.*

14.16.1 *Logical Mathematical Units*—An expression that requires division before a baseline indicator must also be divided before every previous item on the priority list.

Example 14.16-4 $(49x^7y^6 - 63x^6y^4 + 56x^5y^5 + 64x^4y^2) \div 7x^3y^2$



This expression must also begin a new line before the operation sign that is not within the enclosed expression (the division symbol) — a priority #2 site.

Instructions: Keep the following points in mind as you select runover sites. (1) keep logical units intact on one line, if possible; (2) the highest priority available in the expression is utilized first; (3) if necessary, choose additional sites as you descend the list.

PRACTICE 14F

Compute and/or simplify.

1. $\dfrac{(1.21\times10^{4})(6.937\times10^{8})}{3.75(10^{2})}$

2. $\dfrac{\left(\frac{3}{2}\right)\times\left(\frac{1}{2}\right)\times\left(-\frac{1}{2}\right)}{1\times2\times3}$

3. $(\pm)a_{1i_1}a_{2i_2}a_{3i_3}a_{4i_4}\ldots\ a_{ni_n}$

4. $(16x^4+8x^3y+4x^2y^2+2xy^3+y^4)(2x-y)=32x^5-y^5$

5. $144\ \text{ft}^2+107\ \text{ft}^2+112\ \text{ft}^2+145\ \text{ft}^2-131\ \text{ft}^2=N$

6. $\dfrac{\sqrt{0.4523}\times(2.6275)^3}{43690\times0.000982}$

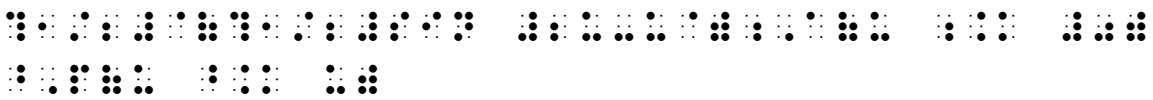
7. $\pm\left(\dfrac{a^{\frac{2}{3}}+a^{\frac{1}{3}}b^{\frac{1}{3}}+b^{\frac{2}{3}}}{3}\right)^{\frac{3}{2}}$

8. $\dfrac{1.793\ \text{g Cu/g Cl}}{0.8964\ \text{g Cu/g Cl}}=$ ___

Priority list so far:
— before a comparison sign on the baseline of writing;
— before an operation sign on the baseline of writing;
— before a fraction line;
— before a baseline indicator.

14.17 PRIORITY #5—Before a Level Indicator: A division may be made before a superscript or subscript indicator. The level indicator will be the first symbol in the runover line.

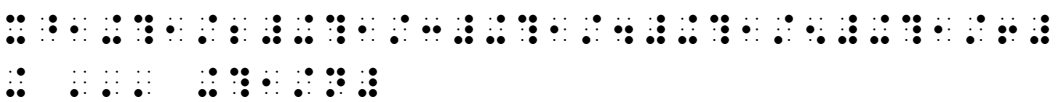
Example 14.17-1 $\frac{1}{2}\left[\frac{1}{2}\sin 2u - u\right]_{A(u=0)}^{P(u=u)}$



Reminders: Keep the expression enclosed between brackets together on one line if possible. The subscript is brailled first.

If transition to a new braille line must be made *within* a superscript or subscript, the level in effect continues when the division is made before a sign of operation or a fraction line, just as it would if the expression were not divided.

Example 14.17-2 $x^{1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\ \ldots\ +\frac{1}{n}}$



The plus sign beginning line 2 is still in the superscript position.

However, if the transition is made before a sign of comparison the level in effect is terminated (just as it would if the expression were not divided) and the level must be restated before the sign of comparison on the new braille line. [No example given.]

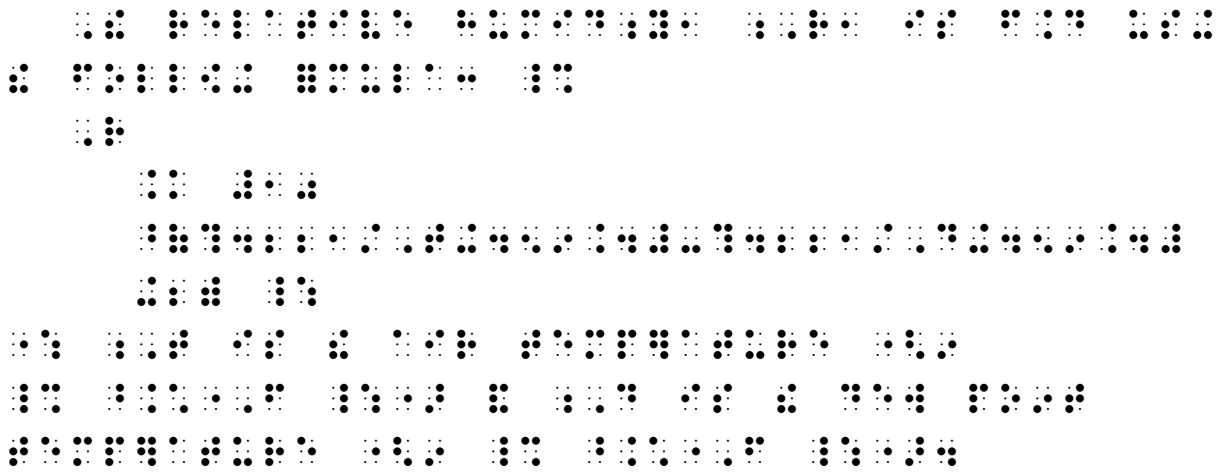
CHALLENGE PROBLEM #3

The relative humidity, *R*, is found using the following formula:

$$R = 10^{\left(\frac{4221}{T+459.4} - \frac{4221}{D+459.4} + 2\right)}$$

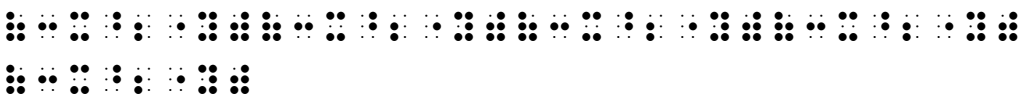
where T is the air temperature (in °F) and D is the dew point temperature (in °F).

The superscript is too long to fit on one line.
Can you explain the division decisions made in the transcription?



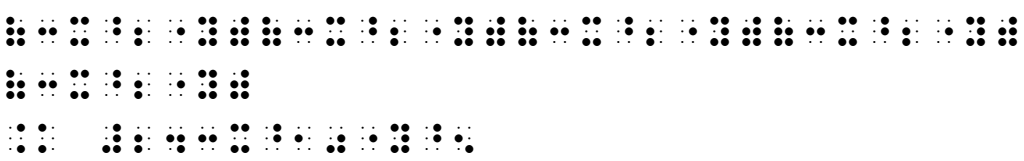
14.18 PRIORITY #6—Between Grouped Factors: A division may be made between factors that are enclosed within grouping signs. Use as much of the line as possible before dividing the expression. Begin a new line with the opening enclosure symbol of the next factor.

Example 14.18-1 $(3x^2y)(3x^2y)(3x^2y)(3x^2y)(3x^2y)$



14.18.1 *Logical Mathematical Units*— Also divide before any previous item on the priority list.

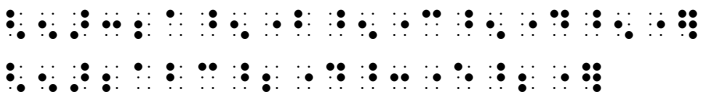
Example 14.18-2 $(3x^2y)(3x^2y)(3x^2y)(3x^2y)(3x^2y) = 243x^{10}y^5$



If division within an enclosed factor is unavoidable, divide the factor according to the priority list principles. The **challenge** problem at the end of this section illustrates.

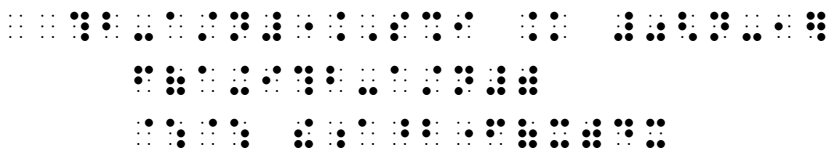
14.19 PRIORITY #7—After a Termination Indicator: A division may be made after a termination indicator of a radical expression or after a termination indicator of a modified expression.

Example 14.19-1 $\sqrt[5]{32a^5b^5c^5d^5}\ \sqrt[5]{2abc^2d^3e^2}$



Example 14.19-2 $\frac{b-a}{n}\sum_{i=0}^{n-1} f\left(a+i\frac{b-a}{n}\right) \approx \int_a^b f(x)\,\mathrm{d}x$

Displayed:



The five components of a modified expression should not be separated from each other. If, however, the modified expression as a whole is so long that it cannot fit inside the current margins, a division is made in the order of preference listed in this section. [No example given.]

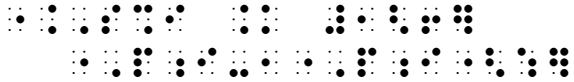
Instructions: Assume this transcription is restricted to a 30-cell line. All of these expressions are longer than 30 cells and require only one division. Draw a vertical line showing where you would divide each expression, keeping in mind the priority sites discussed in this section: before a level indicator, between factors, or after a termination indicator. Then comment on the rule. If you care to transcribe this practice, answers are also given. An example is given first.

Example: $\sum_{i=1}^{6} \overline{P_{i-1}P_i}$

Answer: Divide after the termination indicator of the modified Sigma expression.

$\sum_{i=1}^{6} \overline{P_{i-1}P_i}$

Transcription:



PRACTICE 14G

1. Multiply these four factors: $(4x^9y)(7x^8y^2)(5x^7y^3)(6x^6y^4)$

2. $\sqrt[3]{x^1}\ \sqrt[6]{x^2}\ \sqrt[12]{x^3}\ \sqrt[24]{x^4}$ ...

3. $N_{x_{y_{z_{\ldots}}}}^{a^{b^{c^{\cdots}}}}$

14.20 Application of this Rule to Special Linked Expressions: Braille format regarding a "special linked expression" is discussed in **Lesson 9**. Here is a review of the layout required in the print copy in order to apply the special formatting in braille:

- The expression is displayed.
- The comparison signs are vertically aligned.
- Other than the anchor on the first line, no sign of comparison is preceded by any expression on its left, with the exception of the last line of the expression which may contain more than one link.

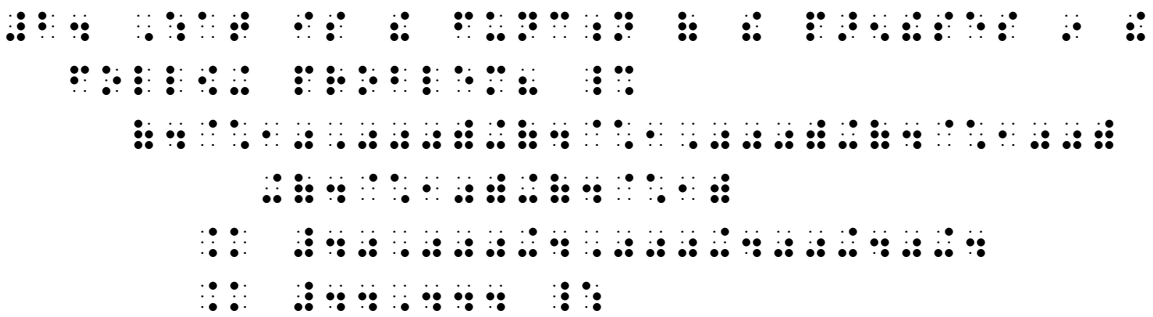
14.20.1 Long Anchor: If the anchor will not fit on one braille line, the guidelines regarding division of mathematical expressions are applied. The runover line to the anchor is indented four cells from the first cell of the anchor.

Example 14.20-1

2. What is the function of the parentheses in the following problem?

$$(4 \times 10{,}000) + (4 \times 1{,}000) + (4 \times 100) + (4 \times 10) + (4 \times 1)$$
$$= 40{,}000 + 4{,}000 + 400 + 40 + 4$$
$$= 44{,}444$$

1
2
3
4
5
6



Lines 3-4: This anchor begins in cell 5. The runover begins in cell 9.
Lines 5-6: When the anchor begins in cell 5, each link begins in cell 7.

14.20.2 Long Link: Similarly, if a link will not fit on one braille line, the guidelines regarding division of mathematical expressions are applied to that link. The runover line is indented two cells from the first cell of the link.

Example 14.20-2

We can find the product of 6 and 44,444 by the distributive principle:

$$\begin{aligned} 6 \times 44{,}444 &= 6 \times (40{,}000 + 4{,}000 + 400 + 40 + 4) \\ &= (6 \times 40{,}000) + (6 \times 4{,}000) + (6 \times 400) + (6 \times 40) + (6 \times 4) \\ &= 240{,}000 + 24{,}000 + 2{,}400 + 240 + 24 \\ &= 266{,}664 \end{aligned}$$

1 ⠀⠀⠀

CHALLENGE PROBLEM #4

By looking at a long expression as a series of logical mathematical units, a clear transcription is possible even if the mathematics is unfamiliar to you. We will analyze the following expression in order to come up with a workable transcription.

$$\frac{1}{2}\left|\int_{\alpha}^{\beta}\left\{x(t)\frac{\mathrm{d}y}{\mathrm{d}t}-y(t)\frac{\mathrm{d}x}{\mathrm{d}t}\right\}\mathrm{d}t\right|$$

There are nested grouping signs: braces within vertical bars.

$$\left|\int_{\alpha}^{\beta}\left\{x(t)\frac{\mathrm{d}y}{\mathrm{d}t}-y(t)\frac{\mathrm{d}x}{\mathrm{d}t}\right\}\mathrm{d}t\right|$$

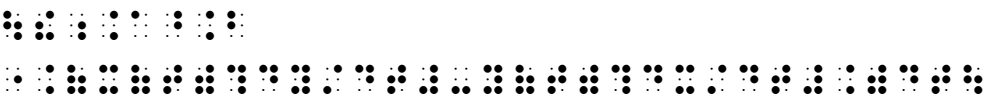
Material between grouping signs should be kept together on one line, if possible. The material between the braces will fit on one line, so do not divide it.

$$\left\{x(t)\frac{\mathrm{d}y}{\mathrm{d}t}-y(t)\frac{\mathrm{d}x}{\mathrm{d}t}\right\}$$



The entire grouped expression between the vertical bars will not fit on one line. By dividing before the opening brace, that nested grouping remains undivided.

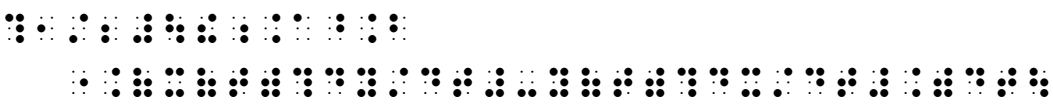
$$\left|\int_{\alpha}^{\beta}\left\{x(t)\frac{\mathrm{d}y}{\mathrm{d}t}-y(t)\frac{\mathrm{d}x}{\mathrm{d}t}\right\}\mathrm{d}t\right|$$



Since a baseline indicator precedes the opening brace, division is made before the baseline indicator according to priority #4.

Here is a clear transcription of the entire expression using "displayed" margins.

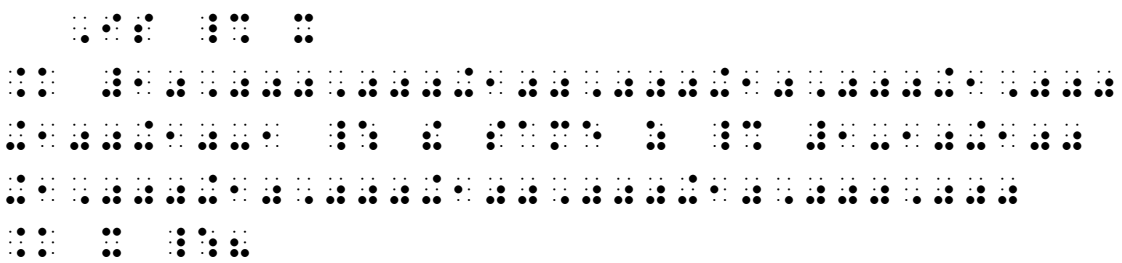
$$\frac{1}{2}\left|\int_{\alpha}^{\beta}\left\{x(t)\frac{\mathrm{d}y}{\mathrm{d}t}-y(t)\frac{\mathrm{d}x}{\mathrm{d}t}\right\}\mathrm{d}t\right|$$



14.21 Margins for Embedded Expressions: When an embedded math expression must be divided, it may begin on the current line provided division is made in accordance with the principles defined above. This is illustrated below using the same expressions in four different layouts.

3-1 narrative format

Example 14.21-1 Is $x = 10{,}000{,}000 + 100{,}000 + 10{,}000 + 1{,}000 + 100 + 10 - 1$ the same as $1 - 10 + 100 + 1{,}000 + 10{,}000 + 100{,}000 + 10{,}000{,}000 = x$?



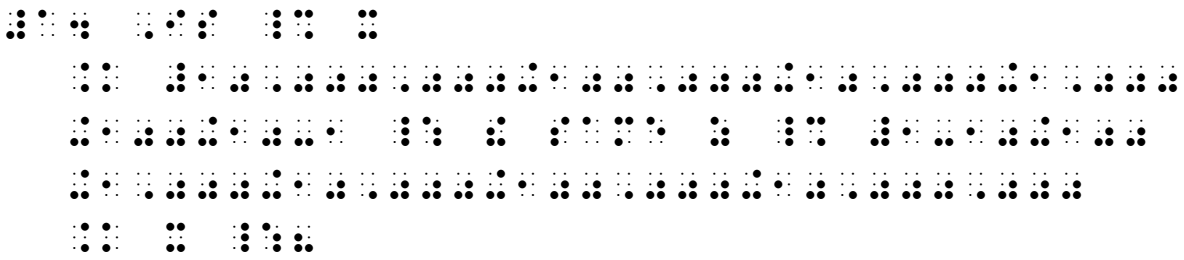
First long expression: Within this narrative format, the anchor "x" fits on line 1. The expression is divided before the equals sign (priority #1) and the link continues on the same line. The long link will not fit on one line so it is divided before an operation sign (priority #2).

Second long expression: Since the long anchor will not fit on one line, it begins on line 3 as it falls within this narrative format. The anchor is divided before an operation sign (priority #2). The last line begins with the equals sign (priority #1) and the short link "x". Note that, although the link will fit on line 4, priority #1 must be observed and so a division is made before the equals sign.

1-3 itemized format

Example 14.21-2

1. Is $x = 10{,}000{,}000 + 100{,}000 + 10{,}000 + 1{,}000 + 100 + 10 - 1$ the same as $1 - 10 + 100 + 1{,}000 + 10{,}000 + 100{,}000 + 10{,}000{,}000 = x$?



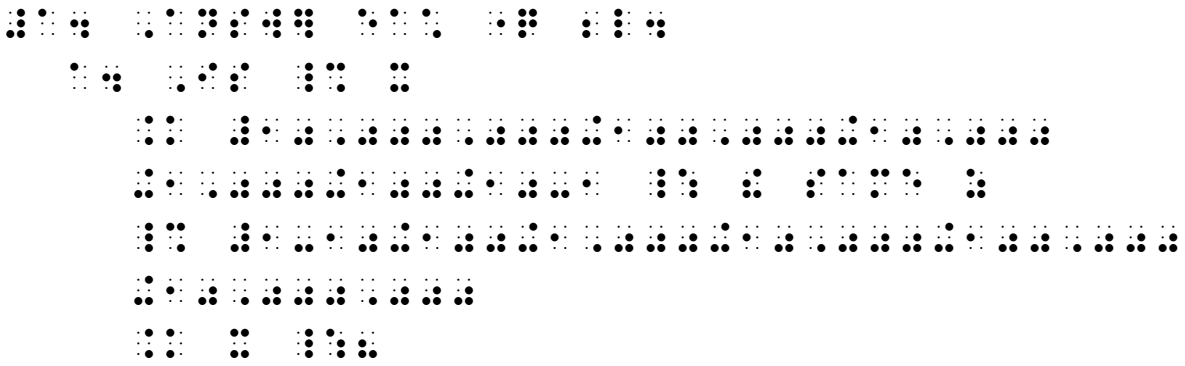
Comments are similar to Example 14.21-1, above. Runovers begin in cell 3.

3-5 subitem format

Example 14.21-3

1. Answer each question below.

a. Is $x = 10{,}000{,}000 + 100{,}000 + 10{,}000 + 1{,}000 + 100 + 10 - 1$ the same as $1 - 10 + 100 + 1{,}000 + 10{,}000 + 100{,}000 + 10{,}000{,}000 = x$?



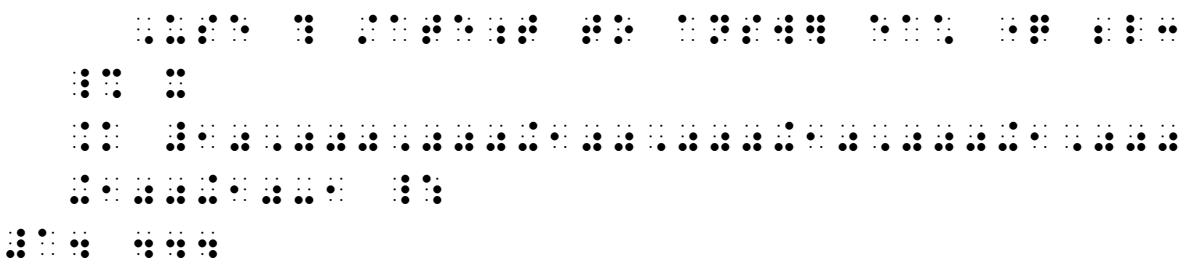
Comments are similar to Example 14.21-1, above. Runovers begin in cell 5, restricting the number of cells available on each runover line. Although the short link "x" will fit on line 6, priority #1 must be observed and so a division is made before the final equals sign.

5-3 instructions format

Example 14.21-4

Use this statement to answer each question below: $x = 10{,}000{,}000 + 100{,}000 + 10{,}000 + 1{,}000 + 100 + 10 - 1$

1. ...



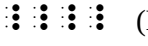
The embedded expression begins in the runover cell (cell 3) since the first logical unit (the anchor) does not fit on the first line of the instructions. Division is then made according to the priority list – first before the equals sign (priority #1) and also within the link, before an operation sign (priority #2).

The BANA Nemeth Code Technical Committee is discussing details regarding division of mathematical expressions between braille pages. This section will be completed after decisions are made.

You may have noticed that the current edition of the Nemeth Code has eight items in the priority list, not seven. The BANA Nemeth Code Technical Committee is discussing the reason behind the Nemeth Code's inclusion of "after a comma which occurs between items in an enclosed list" in the priority list rather than as a separate item, as presented in this lesson manual.

14.22 Items That Must Not Be Divided: When a long expression needs to be divided between lines, keep in mind that many symbols and expressions must not be divided.

14.22.1 Symbols to Keep Together: The components of the following symbols must not be divided between braille lines:

- Signs of operation using plus and minus (Lesson 6)
- Signs of comparison compounded vertically or horizontally (Lesson 6)
- Signs of shape with structural or interior modification (Lesson 11)
- Superposed signs (Lesson 13)
- Tally marks belonging to the same group  (Lesson 13)

14.22.2 Expressions to Keep Together: The following expressions must not be divided between braille lines, even if divided in print:

- A hyphenated expression containing one or more mathematical components (Lesson 3)
- An abbreviation and its related numeral or letter (Lessons 4 and 5)
- A sign of shape and its identifying numeral, letter, or sequence of letters (Lesson 11)
- Characters within a keystroke construction (Lesson 11)
- The components of an expression modified according to the 5-step rule (Lesson 12)
- A function name and the sign following it (Lesson 14)

SQUARE ROOT DIVISION

14.23 Review of Terminology: Radical expressions were presented in **Lesson 9**. When an answer is shown, a spatial arrangement is required. Here are the names of the parts of a radical expression.

$\overline{\quad 12}$	*root*	The line above the radicand is the *vinculum*.
$\sqrt{\ 144}$	*radicand*	$\sqrt{\ }$ is the *radical sign*.

14.24 Spatial Arrangement for Square Root Problems: In the spatially-arranged radical expression, the first cell of the vinculum is brailled directly above the radical symbol. The last cell of the vinculum extends one cell beyond the radicand. A termination indicator is not required in a spatial square root arrangement.

Radical Sign (with Vinculum)	$\sqrt{\ }$	⠩⠩⠩ ⠜

⠩
⠩⠩⠩⠩⠩
⠜⠂⠩⠩
⫸ $\sqrt{144}$

A blank line precedes and follows a spatial arrangement.

Example 14.24-1 $12 \times 12 = 144$ so the square root of 144 is 12, written as follows.

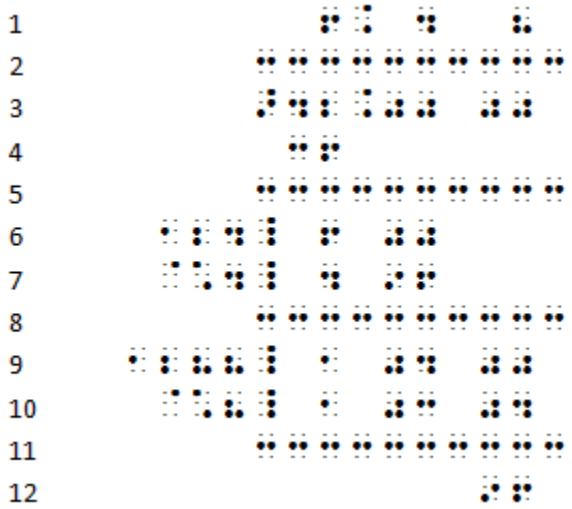
$\sqrt{144}$

⠩
⠩⠩⠩⠩⠩
⠜⠂⠩⠩

The procedures used with long division arrangements are applied to a spatially-arranged square root problem. (Review alignment and spacing rules for long division in **Lesson 13**.) The vertical line that separates the parts of the problem is represented by dots 456. Spacing between digits replicates spacing in the print copy.

Example 14.24-2

```
    6. 4  8
√42.00 00
  36
124 | 6 00
×4  | 4 96
1288| 1 04 00
× 8 | 1 03 04
      96
```

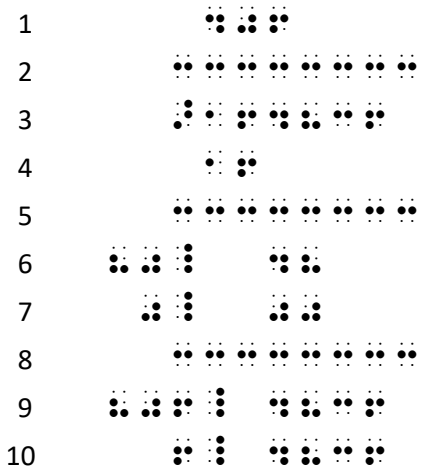


All lines: Spacing between digits matches print in order to attain proper vertical alignment.
Line 2: The vinculum begins in the cell above the radical sign and ends one cell beyond the rightmost character in the entire arrangement.
Lines 5, 8, 11: Separation lines are all the same width.
Lines 6, 7, 9, 10: These vertical lines align below the radical sign.
Lines 7, 10: The multiplication cross is unspaced from the multiplier.

Follow print regarding the alignment of indented vertical lines.

Example 14.24-3

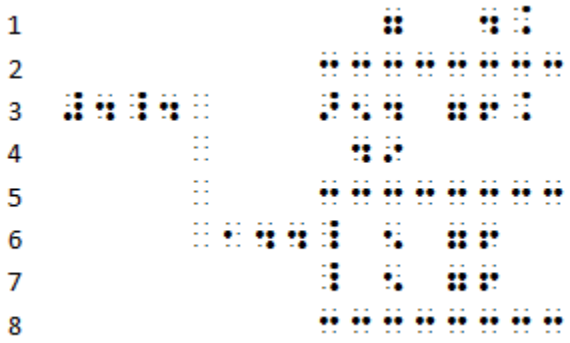
```
  406
√164836
  16
80 | 48
 0 | 00
806 |4836
  6 |4836
```



14.25 Placement of Identifiers with Spatial Radical Expressions: An identifier, if present, is placed on the line with the radicand. One blank space is left between the last symbol in the identifier and the symbol furthest left in the overall arrangement, including separation lines.

Example 14.25-1

```
      7  4.
4.  √54 76.
      49
   144| 5 76
      | 5 76
```



PRACTICE 14I

(A)

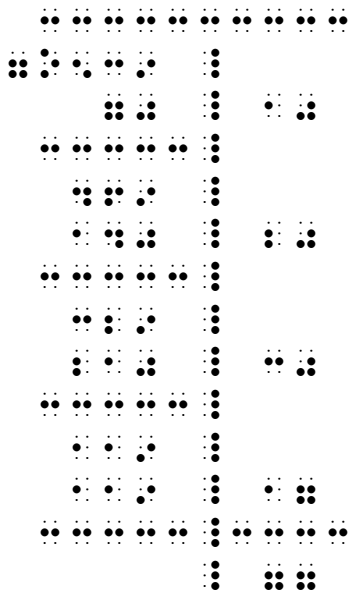
```
      5. 7  4
   √33.00 00
     25
  107| 8 00
  ×7 | 7 49
 1144|51 00
  ×4 |45 76
     | 5 24
```

OTHER PRINT LAYOUTS SHOWING DIVISION

14.26 Partial Quotients: This layout shows partial quotients printed to the right of the division problem. A vertical line separates the partial quotients from the rest of the problem. The vertical line may be either drawn as a tactile graphic or it may be represented by dots 456. One space (one blank cell) is left between the vertical line and any digit preceding or following it.

Example 14.26-1

```
7)539  |
  70   | 10
 ---   |
 469   |
 140   | 20
 ---   |
 329   |
 210   | 30
 ---   |
 119   |
 119   | 17
 ---   |
       | 77
```



Notice the comparative lengths of the separation lines as well as their vertical alignment.

14.27 Synthetic Division: Synthetic division is a method of showing division of polynomials. There is not a standard print layout. The braille transcription replicates the print design, following alignment rules discussed below, and using the standard separation line and vertical line of the Nemeth Code. Here is an example of one possible layout of a synthetic division problem.

```
+2 | 1  −3  +4  +5
   |    +2  −2  +4
   ---------------
     1  −1  +2 | +9
```

The parts to this problem are labeled as follows.

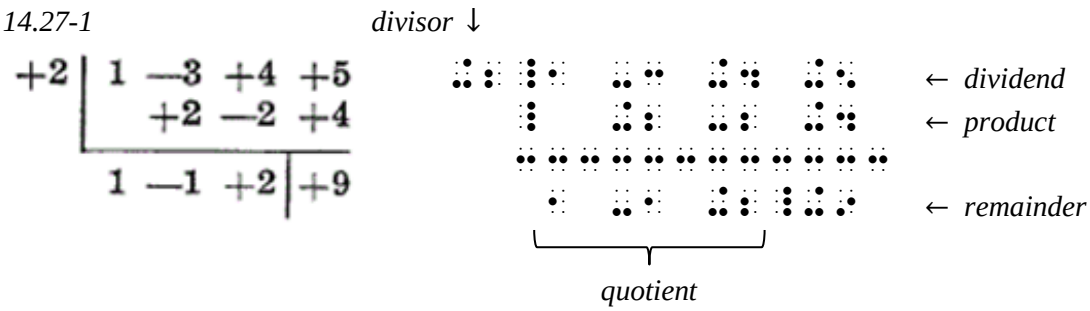
```
divisor    +2 | 1  −3  +4  +5   dividend
              |    +2  −2  +4   product
              ---------------
quotient        1  −1  +2 | +9  remainder
```

14.27.1 Alignment and Spacing: In the examples which follow, look carefully at the vertical alignment. The numerals in the dividend, product, and quotient are aligned in vertical columns as in the print copy. Signs of operation, if any, are also vertically aligned. At least one blank cell is left between adjacent columns.

14.27.2 Vertical Line: Braille dots 456 represents the vertical line in the print copy. The braille symbol is shown between the divisor and the division arrangement, beginning on the line with the dividend and ending on the line with the product. No space is left between the vertical line and the

dividend or divisor. The separation line (dots 25) extends from the vertical line to one cell beyond the entire arrangement. Another unspaced vertical line is brailled between the quotient and the remainder.

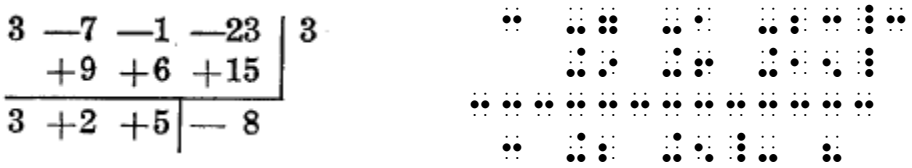
Example 14.27-1



Note the vertical alignment of the numerals and the operation signs.

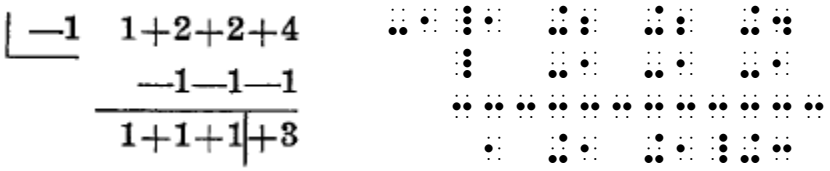
14.27.3 Another Print Style—Divisor on the Right: If the divisor is printed to the right of the overall problem, the same layout is followed in braille. Follow the alignment and spacing rules outlined above, particularly noting that at least one blank cell must be left between adjacent columns. The vertical lines are unspaced from the dividend and the divisor, as well as from the quotient and the remainder.

Example 14.27-2

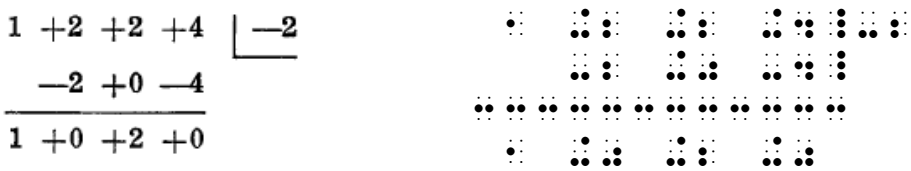


14.27.4 Another Print Style—Boxed Divisor: If the divisor appears boxed in on two sides, the boxing is omitted. A vertical line between the divisor and the dividend is brailled in order to differentiate the divisor from the rest of the arrangement, even though this vertical line does not appear in print. Follow the same alignment and spacing rules outlined above. The first example shows the divisor at the left; the second shows the divisor at the right.

Example 14.27-3



Example 14.27-4

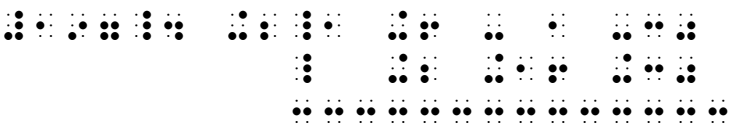


Note that this example has no remainder.

14.27.5 Placement of Identifiers with Synthetic Division: An identifier, if present, is placed on the line with the dividend. One blank space must be left between the last symbol in the identifier and the symbol furthest left in the overall arrangement, including separation lines.

Example 14.27-5

197. | +2 1 +6 — 1 —30
+2 +16 +30



← *dividend*

Notice the vertical alignment of the operation signs. The numerals are aligned by place value, with the "1" directly above the "6" of "16".

Instructions: In the partial quotients problem, align the minus signs with the division symbol.

PRACTICE 14J

```
   1 3 2
6 | 7 9 2
  - 6 0 0 | 1 0 0
    1 9 2
  -   6 0 | 1 0
    1 3 2
  -   6 0 | 1 0
      7 2
  -   6 0 | 1 0
      1 2
  -   1 2 | 2
        0 |
```

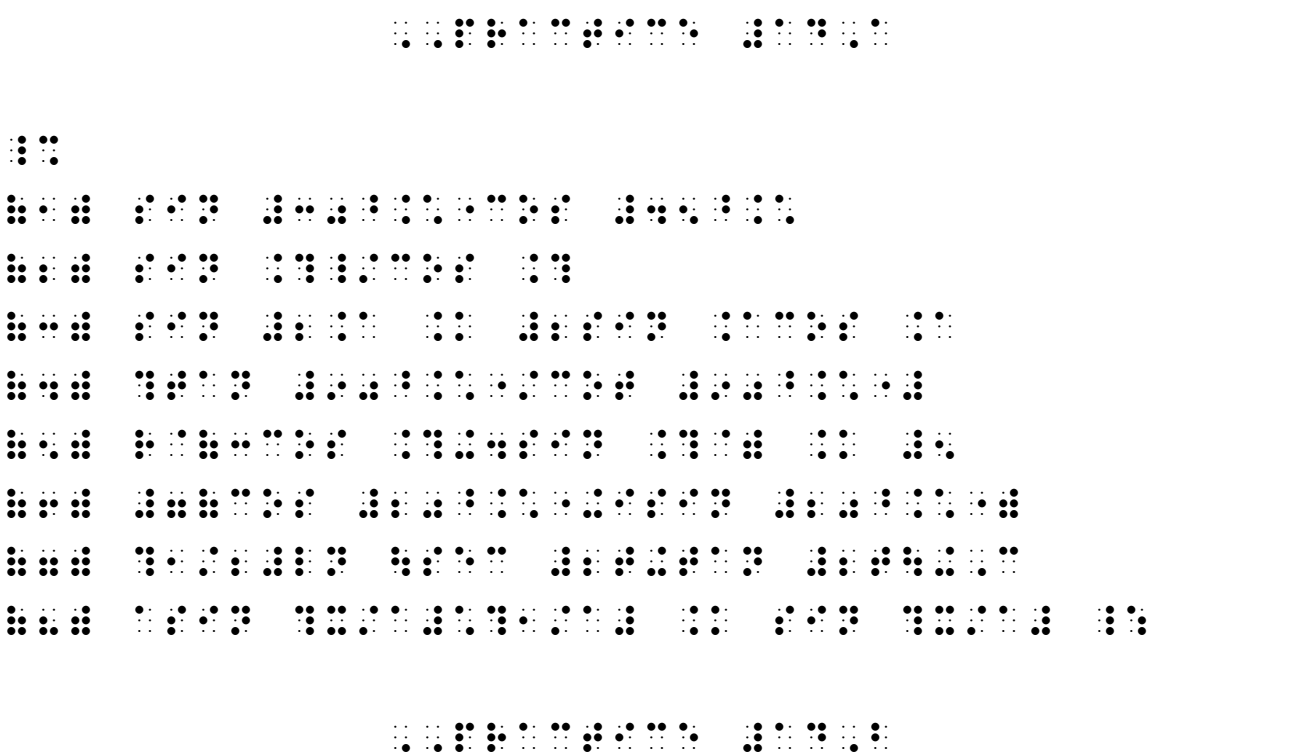
Dividing Polynomials: Divide $(3x^4 + 12x^3 - 5x^2 - 18x + 8) \div (x + 4)$

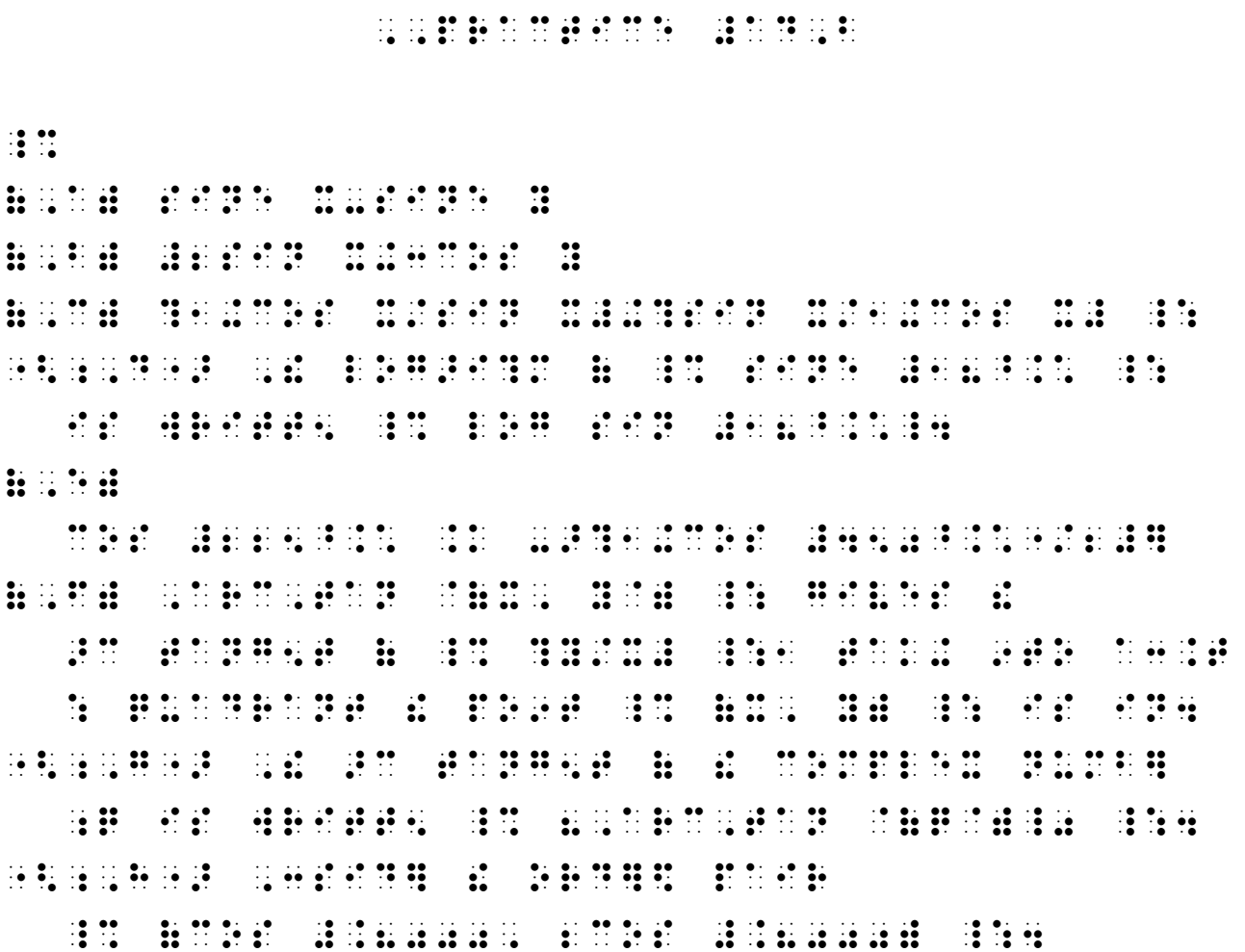
```
–4 | 3   12   –5   –18    8
       –12    0    20   –8
   ---------------------------
     3    0   –5     2 |  0
```

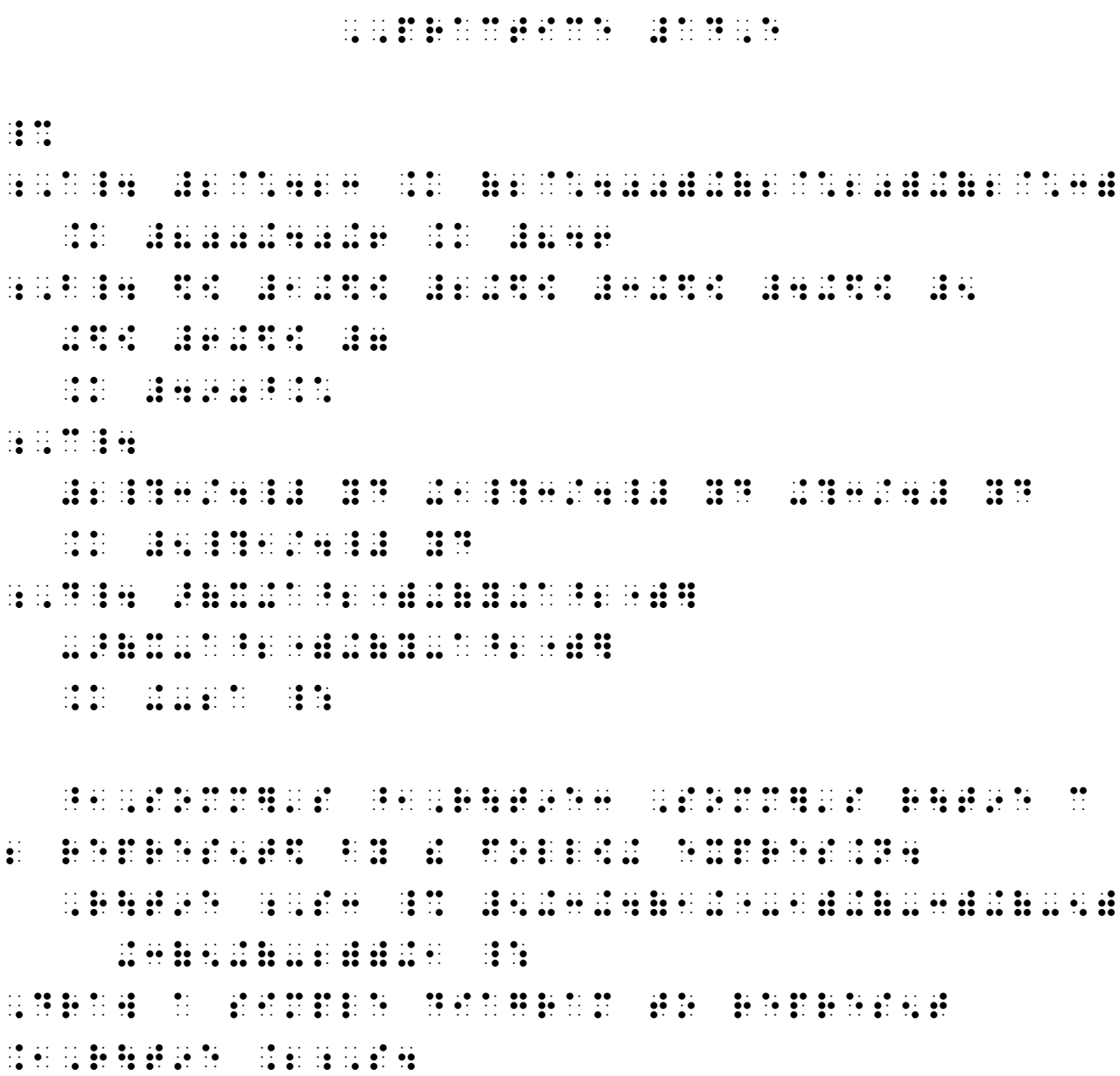
Answer: $3x^2 - 5x - 2$

For further practice, see Appendix A—Reading Practice.

ANSWERS TO PRACTICE MATERIAL

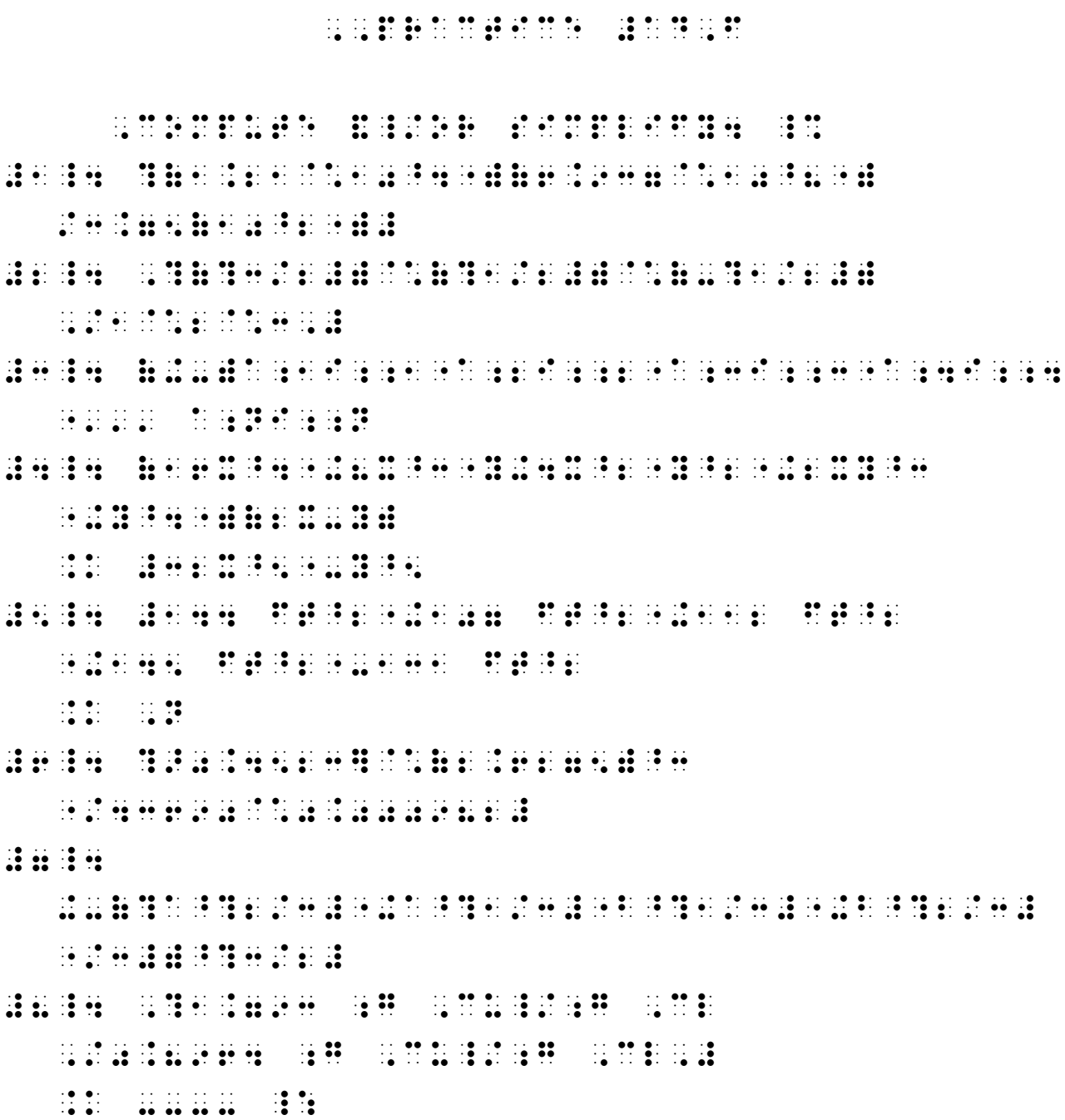






B and D: Since a division must be made before an operation sign (priority #2), a division must also be made before the equals sign (priority #1).

C: The anchor will fit on one line if it starts in the runover position of this itemized problem (cell 3). The only necessary division is made before the equals sign (priority #1).

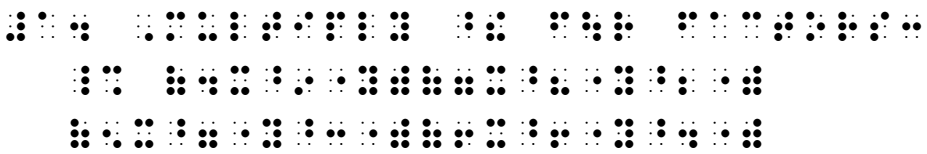


In #8, although the link will fit on the previous line, because a division was made at the fraction line (priority #3), a division must also be made before the comparison sign (priority #1).

PRACTICE 14G

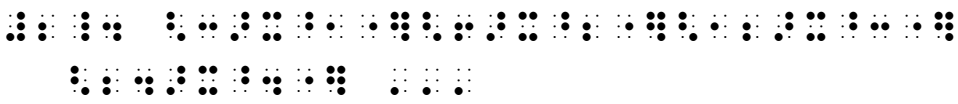
1. Multiply these four factors: $(4x^9y)(7x^8y^2)|(5x^7y^3)(6x^6y^4)$

Divide between factors.



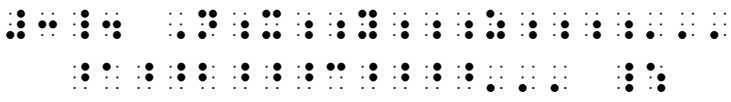
2. $\sqrt[3]{x^1}\ \sqrt[6]{x^2}\ \sqrt[12]{x^3}\ |\ \sqrt[24]{x^4}$ ...

Divide after the termination indicator of a radical expression.

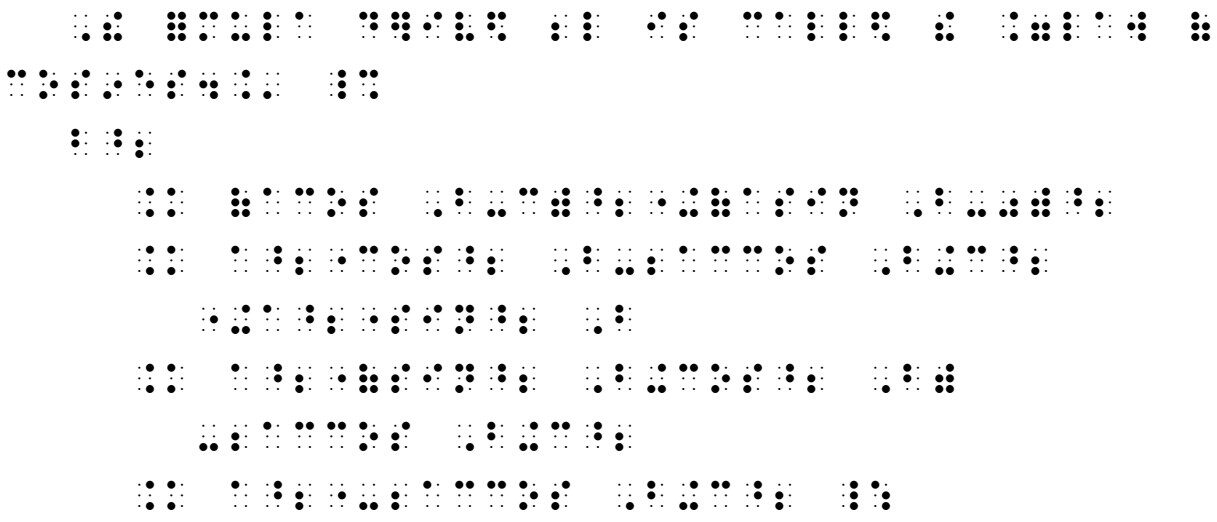


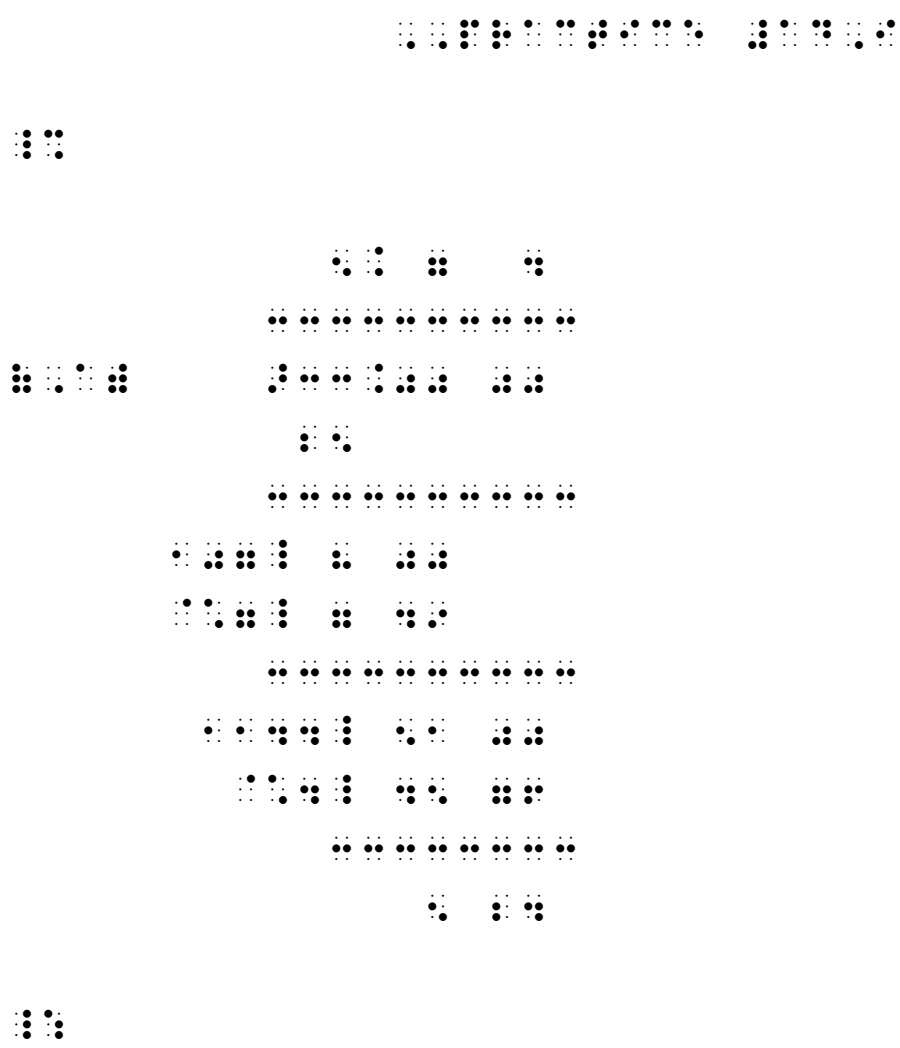
3. $N_{x_{yz}}^{a^{b^{c\cdots}}}$

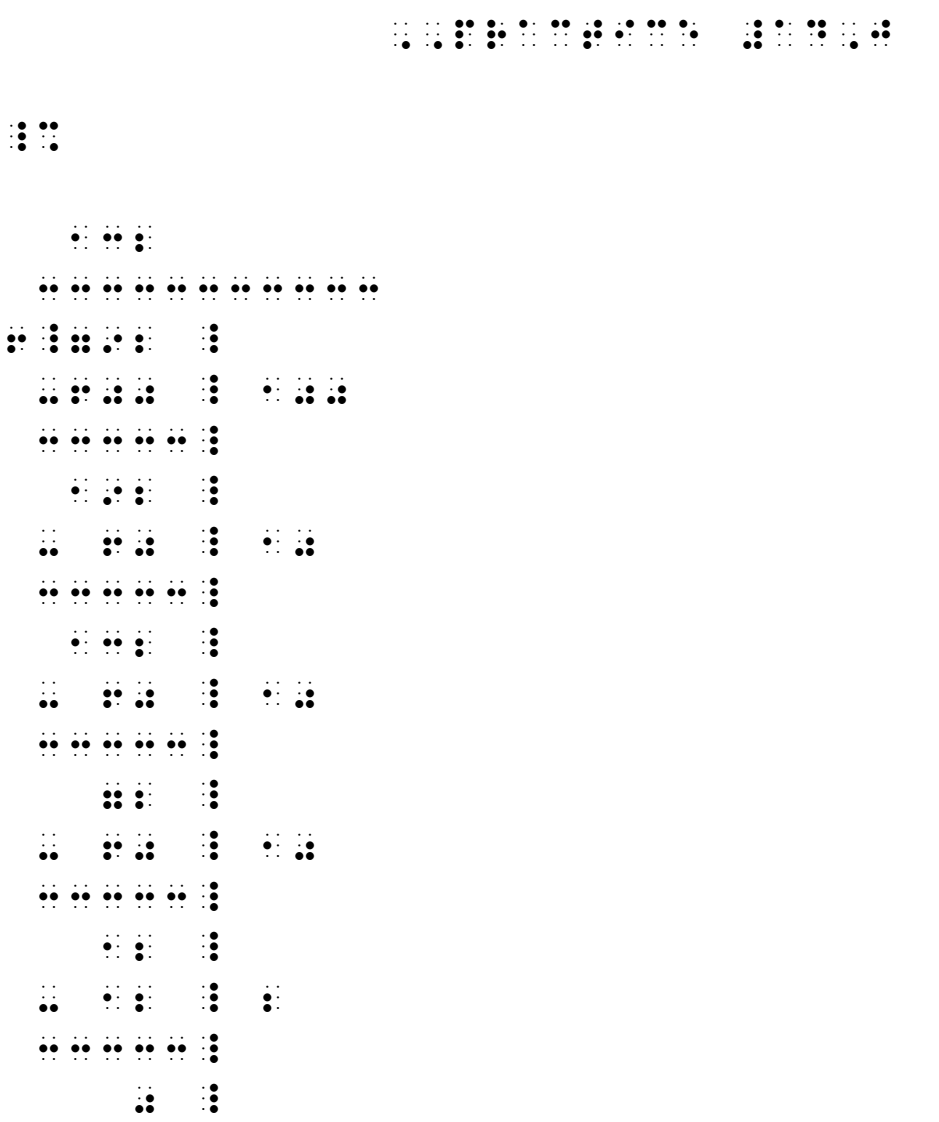
Divide before the superscript level indicator.











EXERCISE 14

Exercise 14 will be available when this course is finished being written and is no longer "Provisional".

Proceed to Lesson 15.

LESSON 15

Read about this PROVISIONAL EDITION in the front matter to this book.
Check the NFB website periodically for updates to this lesson.

- *SPATIAL ARRANGEMENTS OF FRACTIONS*
- *HYPERCOMPLEX FRACTIONS*
- *CONTINUED FRACTIONS*

FORMAT
Remarks and Comments
Stem-and-Leaf Plots

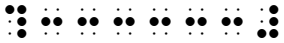
SPATIAL ARRANGEMENTS OF FRACTIONS

Fractions are usually transcribed linearly, as discussed in **Lesson 9**. However, certain situations require a spatial arrangement. Before addressing those conditions, the construction of a spatially-arranged fraction will be illustrated. Recall the terminology used for the parts of a fraction.

$$\frac{3}{4}$$

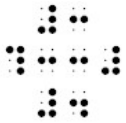
numerator
fraction line
denominator

15.1 Spatial Fraction Line: The spatial fraction line is comprised of a series of dots 25. An opening fraction indicator (1456) marks the beginning of the fraction line; a closing fraction indicator (3456) signals the end.

Spatial Fraction Line (varying length) **with opening and closing fraction indicators**	

In a spatially-arranged fraction the numerator is placed above the spatial fraction line and the denominator is placed below it. The fraction line is comprised of the same number of cells as the longest line of the braille above (numerator) or below (denominator), with the fraction indicators placed one cell beyond the width on either end, as illustrated in the next section.

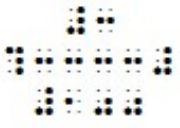
15.2 Numerator and Denominator: Since fractions do not contain material aligned for computation, the numeric indicator must be used in the numerator and/or denominator according to the rules of the Nemeth Code.

Example 15.2-1 $\frac{3}{4}$ 

Likewise, the English letter indicator must be used where necessary in the numerator and/or denominator.

Example 15.2-2 $\frac{x}{y}$ 

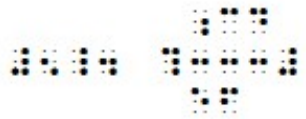
Terms of the fraction are centered above and below their fraction lines.

Example 15.2-3 $\frac{3}{100}$ 

If exact centering is not possible, the term is moved to the left one cell.

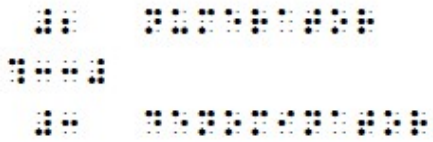
Example 15.2-4 $\frac{25}{100}$ 

15.3 Placement of Identifiers with Spatially Arranged Fractions: An identifier, if present, is placed on the same braille line as the fraction line. One blank space is left between the last symbol in the identifier and the symbol furthest left in the overall arrangement.

Example 15.3-1 **5.** $\frac{cd}{ef}$ 

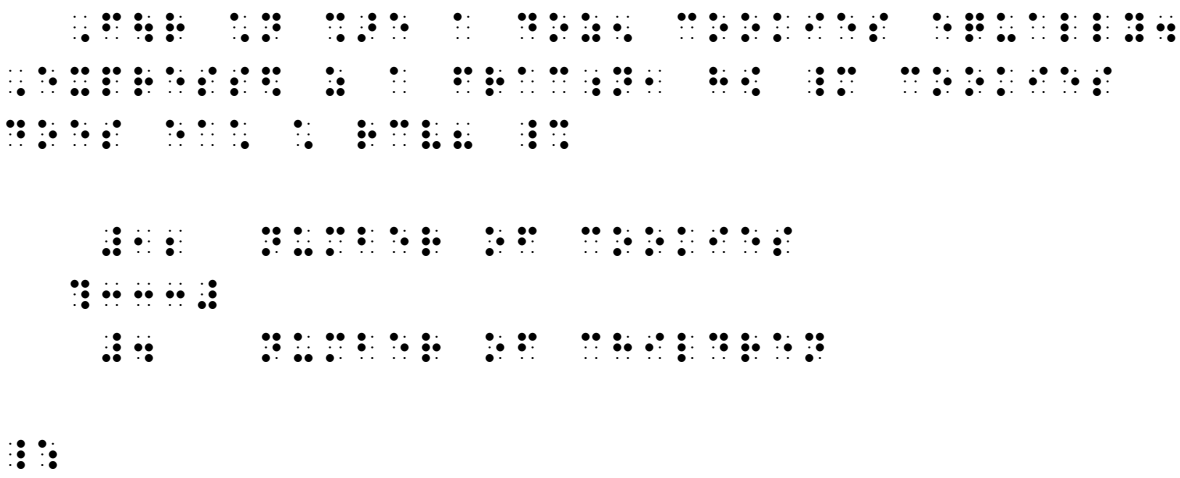
Situations Requiring Spatial Presentation

15.4 Simple Fractions Arranged Spatially for Illustration: When the parts of a simple fraction are labeled, it may be helpful to use a spatial arrangement. Labels are included within the code switch and are uncontracted.

⩥ $\frac{2}{3}$ numerator denominator 

Example 15.4-1 Four children share a dozen cookies equally. Expressed as a fraction, how many cookies does each child receive?

$\frac{12}{4}$ number of cookies
number of children



PRACTICE 15A

1. Shandra invited seven friends to a pizza party. Two pizzas were ordered. Each pizza had eight slices. Which fraction shows how many slices of pizza each child can have if they share equally?

   a. $\frac{2}{8}$ number of pizzas / number of children

   b. $\frac{8}{8}$ number of slices / number of children

   c. $\frac{16}{8}$ number of slices / number of children

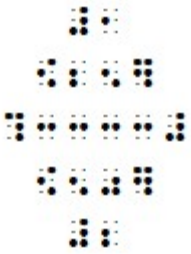
15.5 Cancellation Within Fractions: Recall that a spatial arrangement is required whenever numbers, letters, or abbreviations are canceled in print by any type of stroke through them.

Opening Cancellation Indicator	⠨
Closing Cancellation Indicator	⠬

When cancellation occurs within a fraction, replacement items (the result of cancellation) are placed above canceled items in the numerator and below canceled items in the denominator. Each replacement item is centered with respect to the canceled term.

Example 15.5-1

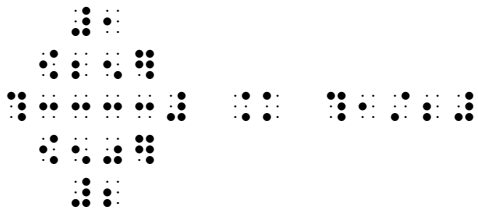
1
~~25~~
~~50~~
2



Line 1:Replacement "1" is centered above canceled "25". A numeric indicator is required.
Line 2: A numeric indicator is not required because the numeral is not preceded by a space.
Line 3: The fraction line is as long as the longest line in the fraction – in this case, both numerator and denominator have a 4-cell item.
Line 4: A numeric indicator is not required because the numeral is not preceded by a space.
Line 5: Replacement "2" is centered below canceled "50". A numeric indicator is required.

Fractions which do not require a spatial arrangement are brailled using the linear method.

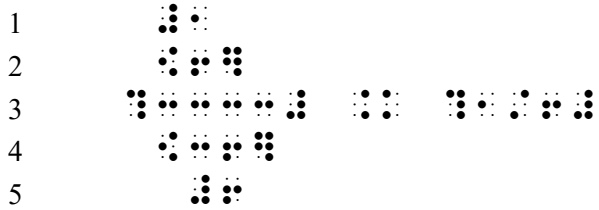
Example 15.5-2 $\frac{\cancel{25}^{1}}{\cancel{50}_{2}} = \frac{1}{2}$



Line 3: The second fraction ("one-half") is brailled as a linear fraction because it doesn't contain cancellation. The equals sign and the linear fraction are brailled on the same line as the spatial fraction line.

If exact centering is not possible, the term is moved to the left one cell.

Example 15.5-3 $\frac{\cancel{6}^{1}}{\cancel{36}_{6}} = \frac{1}{6}$

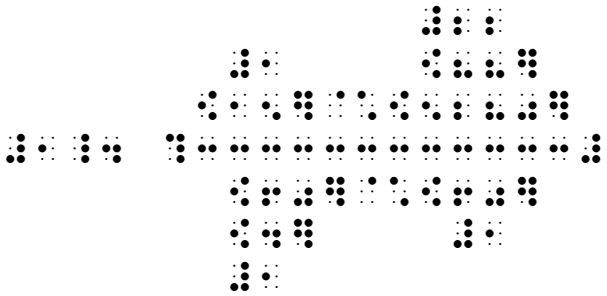


Line 1:Replacement "1" is centered above canceled "6". Exact centering is not possible so it is moved left one cell.
Line 2:Canceled "6" is centered above the fraction line. Exact centering is not possible so it is moved left one cell.
Line 3: The fraction line is as long as the longest line in the fraction (the denominator).
Line 4: Canceled "36" is centered below the fraction line.
Line 5: Replacement "6" is centered below canceled "36".

More than one level of cancellation may be encountered. *Reminder: The print copy may show any type of stroke to indicate cancellation.*

Example 15.5-4

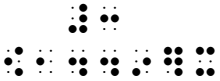
1. $\frac{\overset{1}{\cancel{15}}\times\overset{22}{\cancel{5280}}}{\underset{4}{\cancel{60}}\times\underset{1}{\cancel{60}}}$



Reminder: Vertical alignment of factors is not a consideration in spatial fractions because the material is not being aligned for computation. Instead, terms of the fraction are centered above or below their fraction lines, and replacement items are centered above or below their canceled item.

Clarification: Placement of the replacement item is relative to the width of the canceled item, without respect to place value.

⋙ $\overset{3}{\cancel{1449}}x$



15.5.1 Extent of Cancellation: Items which are individually canceled in print are individually canceled in the transcription. Compare the next two examples.

Example 15.5-5 $\frac{\cancel{rst}}{\cancel{rst}v}$



Each letter "r" "s" and "t" is printed with a slash through it. Letter indicators are not required because each letter is touching an indicator.

Example 15.5-6 $\frac{\cancel{(rst)}}{\cancel{(rst)}v}$

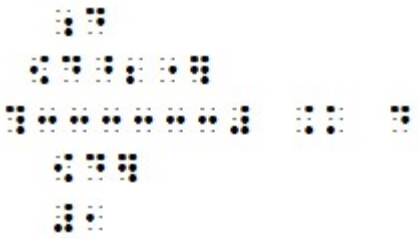


Each letter combination "rst" is printed with a slash through it.

15.5.2 Cancellation and Level Indicators: Paired cancellation indicators must be placed on the same level of writing.

⋙ $\cancel{d^2}$ 

"d^2" is canceled. The cancellation indicators are on the baseline of writing.

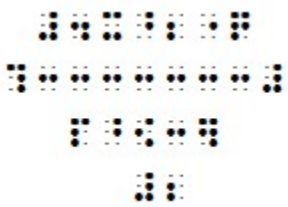
Example 15.5-7 $\frac{\cancel{d^2}}{\cancel{d}} = d$ 

The replacement items are centered above and below the canceled items.

In the next example, only the superscript is canceled.

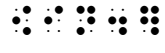
⋙ $p^{\cancel{3}}$ 

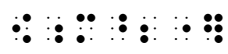
The cancellation indicators are in the superscript position.

Example 15.5-8 $\frac{4x^2q}{p^{\cancel{3}\,2}}$ 

The replacement numeral "2" is placed below the canceled superscript in the denominator, centered below the term it replaces.

15.5.3 Canceled Abbreviations: A canceled abbreviation is unspaced from the cancellation indicators.

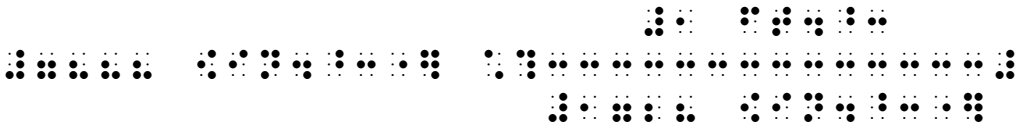
⋙ $\cancel{in.}$ 

⋙ $\cancel{m^2}$ 

The spaces required before and after the abbreviation are inserted before the opening cancellation indicator and after the closing cancellation indicator.

Example 15.5-9 $7888\ \text{in.}^3 \cdot \frac{1\ \text{ft.}^3}{1728\ \text{in.}^3}$

1
2
3



All lines: A space is required between each abbreviation and its related numeral (line 1) even when the abbreviation is canceled (lines 2 and 3).
Line 2: The abbreviation "in.^{3}" is spaced away from the operation symbol (multiplication dot) even when enclosed between cancellation indicators.

15.6 Chemistry Exception: An exception is made in the *Chemistry Code* regarding cancellation in chemical equations. Refer to *Braille Code for Chemical Notation 1997* for details.

PRACTICE 15B

1. How many meters are in 100 yards? By multiplying 100 yd by several expressions of "1" and canceling units of measure, an answer is found: There are 191.44 m in 100 yd.

$$?\ \text{m} = 100\ \cancel{\text{yd}} \times \frac{3\ \cancel{\text{ft}}}{1\ \cancel{\text{yd}}} \times \frac{12\ \cancel{\text{in}}}{1\ \cancel{\text{ft}}} \times \frac{2.54\ \cancel{\text{cm}}}{1\ \cancel{\text{in}}} \times \frac{1\ \text{m}}{100\ \cancel{\text{cm}}}$$

$$= \frac{100 \times 3 \times 12 \times 2.54}{100}\ \text{m}$$

$$= 91.44\ \text{m}$$

2. $2 \times \frac{8}{\cancel{2}} = \frac{1 \times 8}{1} = 8$ (1 over the 2, 1 under the 2)

3. $\frac{2b+a}{b} \div \frac{4b^2-a^2}{b^2} = \frac{\cancel{(2b+a)}}{\cancel{b}} \cdot \frac{\cancel{b^2}}{\cancel{(2b+a)}(2b-a)} = \frac{b}{2b-a}$ (1 over $(2b+a)$, b over b^2, 1 under b, 1 under $(2b+a)$)

4. Simplify using cross-canceling. Multiply the numerators and multiply the denominators.

$$\frac{\cancel{3}^{1}}{\cancel{15}_{3}} \times \frac{\cancel{5}^{1}}{\cancel{6}_{2}} = \frac{1}{6}$$

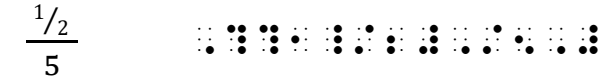
HYPERCOMPLEX FRACTIONS

15.7 Definition and Recognition: A hypercomplex fraction is one whose numerator or denominator, or both, contain at least one complex fraction. (The term "hypercomplex" fraction is used only in the context of the Nemeth Code—it is unlikely that you will encounter this term in a math book.)

Recall from **Lesson 9** that a complex fraction is one whose numerator and/or denominator are, or contain, one or more simple fractions or mixed numbers. In other words, a complex fraction is a fraction within a fraction. Here is an example.

Complex Fraction

$$\frac{1/2}{5}$$



Here is a print example of a hypercomplex fraction where the denominator is "5" and the numerator contains the complex fraction "one-half over three-fourths."

$$\frac{\frac{1/2}{3/4}}{5}$$

This is a hypercomplex fraction.

A fraction is not a hypercomplex fraction if the only complex fractions it contains are at the superscript or subscript level. The following is <u>not</u> a hypercomplex fraction. The numerator is "1" and the denominator is "2 raised to the (one-half over three-fourths) power."

$$\frac{1}{2^{\frac{1/2}{3/4}}}$$

This is not a hypercomplex fraction.

This fraction is brailled linearly, as a simple fraction whose denominator contains a complex fraction in the superscript position. Review the transcription of complex fractions in **Lesson 9**.



15.8 Hypercomplex Fraction Indicators: Opening and closing hypercomplex fraction indicators mark the beginning and end of a hypercomplex fraction or the beginning and end of a hypercomplex fraction line in a spatial arrangement.

Hypercomplex Fraction Indicators

Opening 

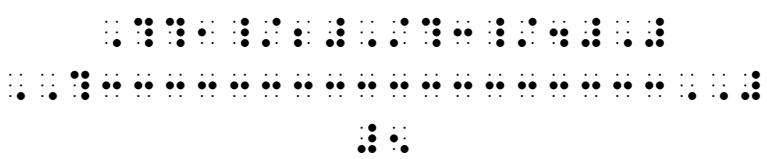
Closing 

Spatial Hypercomplex Fraction Line with opening and closing fraction indicators

 (varying length)

Here is a transcription of the example from the previous page where the complex fraction in the numerator is brailled as a linear arrangement within the overall spatial arrangement.

Example 15.8-1 $\dfrac{\frac{1}{2}/\frac{3}{4}}{5}$

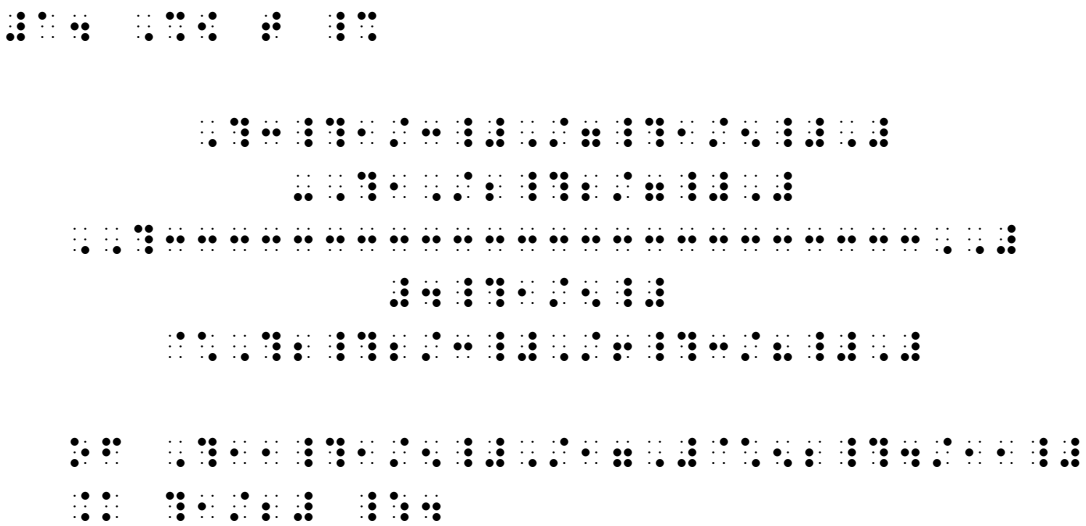


Terms of the fraction are centered above and below their fraction lines.

If the numerator and denominator are too long to fit on one line, follow the rules regarding division of mathematical expressions between braille lines. (See **Lesson 14**.)

Example 15.8-2

1. Show that $\dfrac{\frac{3\frac{1}{3}}{7\frac{1}{5}}-\frac{1}{2\frac{2}{7}}}{4\frac{1}{5}\times\frac{2\frac{2}{3}}{6\frac{3}{8}}}$ of $\dfrac{11\frac{1}{5}}{17}\times 52\,\frac{4}{11}=\frac{1}{2}$.

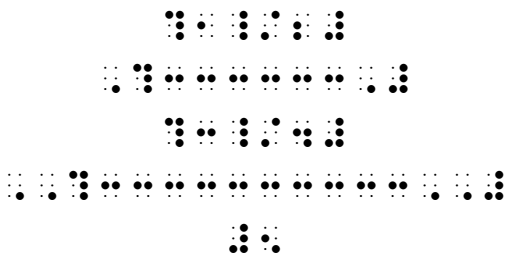


Numerator and denominator are divided before an operation sign (priority #2). The numerator is divided before the minus sign; the denominator is divided before the multiplication cross. Terms of the fraction are centered above and below the spatial fraction line.

The combined transcription method illustrated above is the preferred way of brailling a hypercomplex fraction. However, it is permissible to use an entirely spatial arrangement or an entirely linear arrangement. Example 15.8-1 is illustrated below using these two alternate methods.

$$\cfrac{\cfrac{1/2}{3/4}}{5}$$

Entirely spatial



Entirely linear A linear arrangement uses the horizontal hypercomplex fraction line.

Horizontal Hypercomplex
Fraction Line (used in linear arrangements)



Linear hypercomplex fractions are difficult to assemble mentally.

15.9 Higher Orders of Complexity: Hypercomplex fractions of higher order may be transcribed in the manner described above, with dot 6 added the appropriate number of times before the fraction indicators and the matching fraction lines. No examples are shown.

Instructions: Transcribe this hypercomplex fraction using the combined method—that is, braille each complex fraction as a linear arrangement within the overall spatial arrangement. Numerators and denominators can be determined by noting the length of each fraction line in print.

PRACTICE 15C

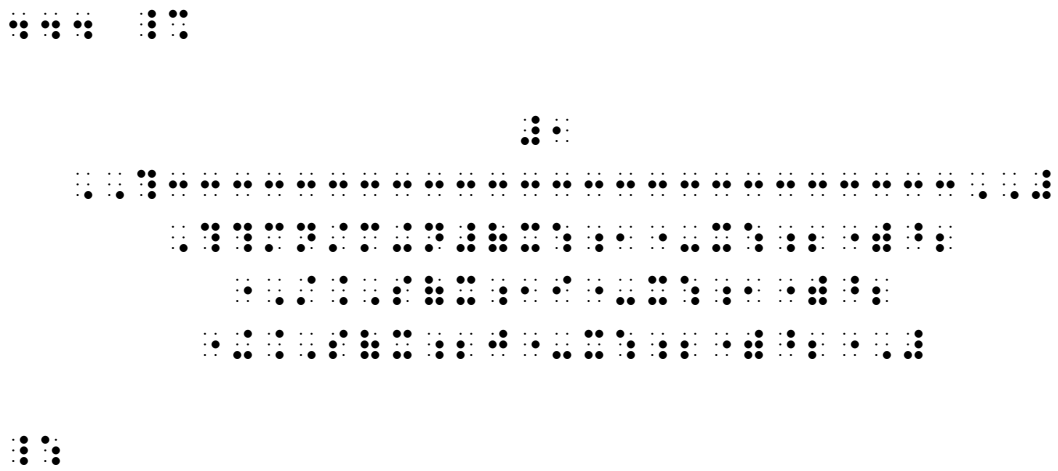
$$\cfrac{\cfrac{\cfrac{r^2-4s^2}{s^2}}{\cfrac{r+2s}{s}}}{\cfrac{\cfrac{4r-2s^2}{3s}}{\cfrac{2s^2-3r}{4r}}}$$

CHALLENGE PROBLEM

This transcription uses the combined method of brailling a hypercomplex fraction. The numerator is "1". The complex fraction in the denominator is brailled as a linear arrangement within the overall spatial arrangement. There are runovers in the denominator. The runover sites are selected according to the priority list (see **Lesson 14**). Runovers are centered to the fraction line.

Assuming this complex fraction is displayed to a narrative paragraph, the leftmost braille cell of the fraction will be placed in cell 3.

$$\frac{1}{\dfrac{\dfrac{mn}{m+n}(\overline{x}_1-\overline{x}_2)^2}{\Sigma(x_{1i}-\overline{x}_1)^2+\Sigma\left(x_{2j}-\overline{x}_2\right)^2}}$$



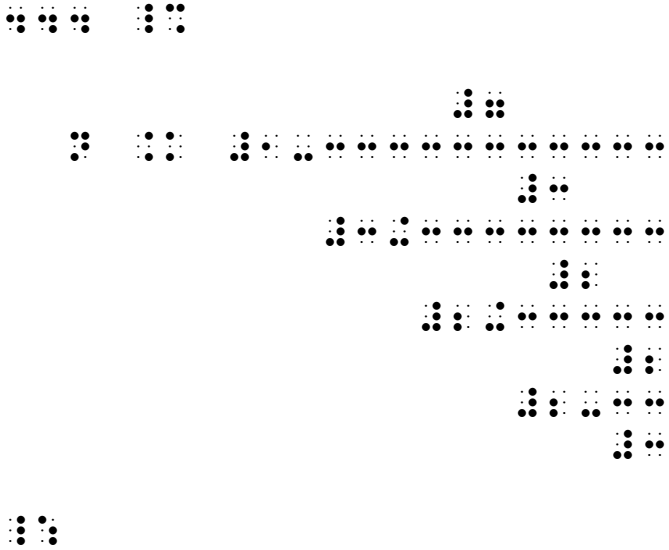
CONTINUED FRACTIONS

15.10 Definition and Recognition: A continued fraction is one in which each denominator is the sum of a whole number and a fraction. Such a fraction is transcribed entirely as a spatial arrangement.

Example 15.10-1 $$n = 1 - \cfrac{7}{3 + \cfrac{3}{2 + \cfrac{2}{2 - \cfrac{2}{3}}}}$$

Description: The fraction begins with $1 - \frac{7}{3}$ *... the first denominator is* $3 + \frac{3}{2}$ *... the next denominator is* $2 + \frac{2}{2}$ *... the final denominator is* $2 - \frac{2}{3}$ *.*

Opening and closing fraction indicators are not used within a continued fraction. Each fraction line (dots 25) is proportionately the same length shown in print. Each numerator is centered with respect to the fraction line below it. If the numerator cannot be exactly centered, place it one cell to the left of center.



Lines 1-2: The opening Nemeth Code indicator is brailled on the line preceding the required blank line.

Line 3: The numerator is centered over its spatial fraction line.

Line 4: The length of this fraction line extends to the rightmost character in the arrangement.

Line 5: This numerator "3" is centered over the spatial fraction line below it. Likewise for the numerators on lines 7 and 9.

Line 6: The first character (numeric indicator) is directly aligned with the first cell of the fraction line two lines above. Likewise for lines 8 and 10.

Line 11: This final denominator consists of two cells. The fraction line above it is the same length.

Lines 12-13: The Nemeth Code terminator is in cell 1 on the line following the required blank line.

PRACTICE 15D

$$\sqrt{3} = 1 + \cfrac{1}{3 + \cfrac{1}{3 + \cfrac{1}{3 + \ldots}}}$$

FORMAT

REMARKS AND COMMENTS

15.11 Guidelines: When short narrative comments alternate with math problems, switch out of Nemeth Code to transcribe the comments in contracted braille. Two ways to handle such remarks are illustrated in this section. Regardless of which method you use, the same format should be used throughout the transcription and an explanation should be included on the Transcriber's Notes page according to the guidelines given in *Braille Formats*.

Author's comments are often printed in a different color or typeface. The distinction is disregarded in the braille transcription according to guidelines given in *The Rules of Unified English Braille*. In the examples in this section, author's remarks are printed in italics.

15.11.1 OPTION #1—Continue the Commentary on the Same Line: This option is the preferred treatment of authors' remarks. The comments continue on the line on which the related math ends, with runovers in the appropriate location for the math expression. By placing the opening Nemeth Code indicator after the last word of each comment rather than at the beginning of the next line of math, the switch indicator does not interfere with the math under study. If there is not room on the line, the indicator is treated as a runover to the comment line.

Example 15.11-1

If y varies inversely as x, and $y = 3$ when $x = 4$, find y when $x = 18$.

$$\frac{x_1}{y_2} = \frac{x_2}{y_1}$$

$\frac{4}{y_2} = \frac{18}{3}$ *Substitute the known values.*

$18y_2 = 12$ *Now cross multiply.*

$y_2 = \frac{12}{18}$ or $\frac{2}{3}$ *Divide each side by 18 and simplify.*

The value of y when $x = 18$ is $\frac{2}{3}$.

1 ⠀⠀⠀

The single-word switch indicator is used for a one-word comment.

Example 15.11-2

What percent of 48 is 54?

$n \cdot 48 = 54$ *Write an equation.*

$\frac{48n}{48} = \frac{54}{48}$ *Divide each side by 48.*

$n = 1.125$ *Simplify.*

$= 112.5\%$ *Change the decimal to a percent.*

54 is 112.5% of 48.

1 ⠀⠀⠀

15.11.2 OPTION #2—Indent the Commentary on the Next Line: This layout provides clarity when a comment is also a math expression or when a comment refers to a spatial arrangement. Begin the remark on the line following the related math expression, indented four cells to the right of the current runover cell. Runovers to the commentary are blocked in the same cell. Whether switching into or out of Nemeth Code, place the switch indicators after the last item of the line rather than at the beginning of the next line. This maintains clarity in the indented margin pattern. Remember, if this method is used in one instance, this format must be followed throughout the transcription.

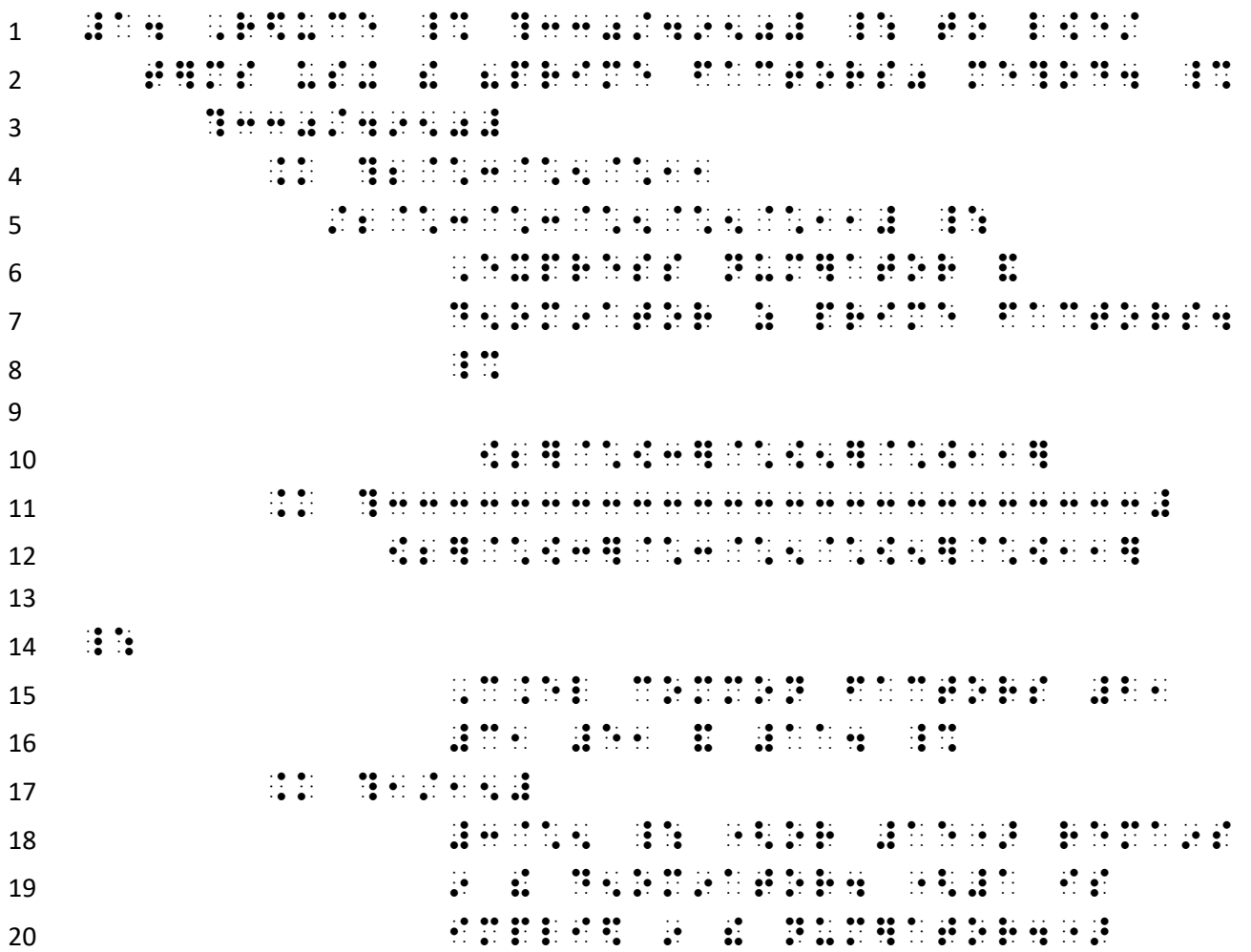
Example 15.11-3

1. Reduce $\frac{330}{4950}$ to lowest terms using the "prime factors" method.

$$\frac{330}{4950} = \frac{2\times3\times5\times11}{2\times3\times3\times5\times5\times11}$$ *Express numerator and denominator as prime factors.*

$$= \frac{\cancel{2}\times\cancel{3}\times\cancel{5}\times\cancel{11}}{\cancel{2}\times3\times\cancel{3}\times\cancel{5}\times5\times\cancel{11}}$$ *Cancel common factors 2, 3, 5, and 11.*

$$= \frac{1}{15}$$ *3 × 5 (or 15) remains in the denominator. (1 is implied in the numerator.)*



Line 3: Math displayed to itemized (1-3) begins in cell 5.
Lines 3-17: This linked expression meets the criteria for "special margins". The anchor starts in cell 5 (line 3). Each link begins in cell 7 (lines 4, 11, 17). Runovers begin in cell 9 (line 5).
Lines 6,15, 18: Each comment begins in cell 13, which is four cells to the right of the runover position of the previous line, even if no runover is present.
Lines 7, 16, 19-20: Runovers to comments are blocked in cell 13.
Line 8: The opening Nemeth Code indicator is placed in the runover cell of the preceding text.
Line 16: The switch indicator is brailled at the end of the comment in order not to interfere with the alignment of the equals sign that begins the next link on line 17.

Instructions: Demonstrate your understanding of the rules by transcribing this practice twice – first using option #1, then using option #2. *NOTE: In print, the equals signs are aligned. This alignment is disregarded in the braille transcription.*

PRACTICE 15E

We write the equation in the slope-intercept form, $y = mx + b$, by solving for y.

$-5x - 2y = 6$

$-2y = 5x + 6$ *Add 5x to both sides.*

$\frac{-2y}{-2} = \frac{5x+6}{-2}$ *Divide both sides by –2.*

$y = \frac{5x}{-2} + \frac{6}{-2}$ *Divide each term by –2 and simplify.*

$y = -\frac{5}{2}x - 3$ *Slope-intercept form*

STEM-AND-LEAF PLOTS

15.12 Definition and Recognition: A stem-and-leaf plot is a method of showing data distribution in columns and rows. A vertical line partitions the information into data on the left, called the *stem*, and data on the right, called the *leaf*. Stem and leaf data may consist of numbers, letters, and/or blank entries and may be spaced or unspaced in the print copy. *Stem data* are right justified to the vertical line; *leaf data* are left justified to the vertical line. The columns may include a heading. Here is an example.

Stem	Leaf
10	0 4
9	3 4 5 7 8 9
8	0 2 2 9
7	5 8 9
6	3

A distinctive feature of a stem-and-leaf plot is the vertical line separating the stems from the leaves.

15.13 The Table: A stem-and-leaf plot is brailled in Nemeth notation using the guidelines for tables outlined in *Braille Formats*. (An introduction to table format in technical materials was given in **Lesson 6**.) The entire table is transcribed in Nemeth Code, including the column headings (if present). Column headings begin in cell 1 and no contractions are used. Column separation lines follow on the next line below the headings, covering the same number of cells as the widest data entry in each column. Column headings are not brailled if they do not appear in print.

The vertical line between the stem column and the leaf column is brailled as dots 456, preceded and followed by one blank cell. This line is brailled in every row, including between column headings and between column separation lines. The presence of this Nemeth Code symbol necessitates the use of Nemeth Code throughout the table.

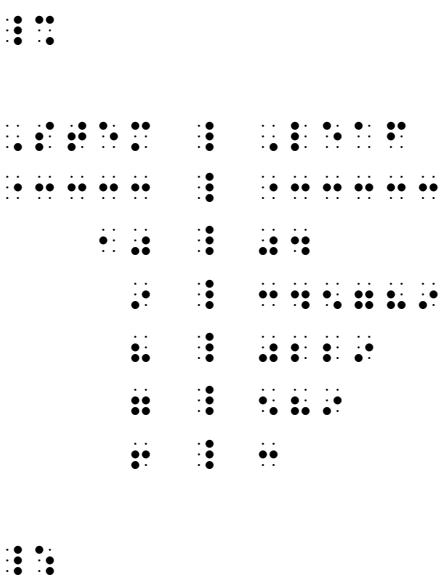
Vertical Line	⠳	*(preceded and followed by a space)*

The following Nemeth Code rules apply to this example.

- The numeric indicator is omitted in the body of the plot.
- Spaces are not brailled between single-digit entries.
- The data are justified to the vertical line as shown in print.

Example 15.13-1

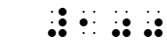
Stem	Leaf
10	0 4
9	3 4 5 7 8 9
8	0 2 2 9
7	5 8 9
6	3



The table is preceded and followed by a blank line. Code switch indicators do not replace required blank lines.

15.14 The Key: A key is often provided in the print copy and may be located above, below, next to, or within the table. The key is brailled first, regardless of its placement in the print copy. Numeric indicators are not used for the portion of the key that replicates a numeric entry in the plot. (In this illustration, 10|0 is the portion of the key replicating the stem "10" and the leaf "0".) The value assigned to the key is brailled using the numeric indicator as required by the Nemeth Code. (In this illustration, a numeric indicator is used for the value "100".)

⋙ *10 | 0 = 100*



The numbers in this key are printed in italics. Typeform is not retained in braille. A key is not constructed by the transcriber if one does not appear in print.

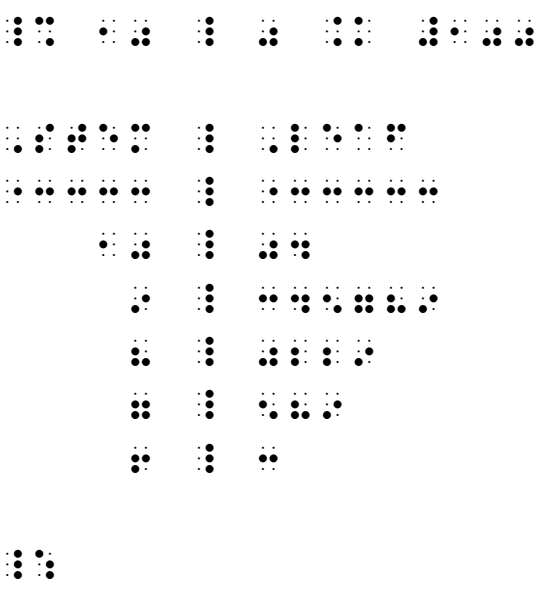
The following Nemeth Code rules apply to this example.

- The key is brailled first, beginning in cell 1.
- The numeric indicator is omitted in the body of the plot.
- Spaces are not brailled between single-digit entries.

Example 15.14-1

Stem	Leaf
10	0 4
9	3 4 5 7 8 9
8	0 2 2 9
7	5 8 9
6	3

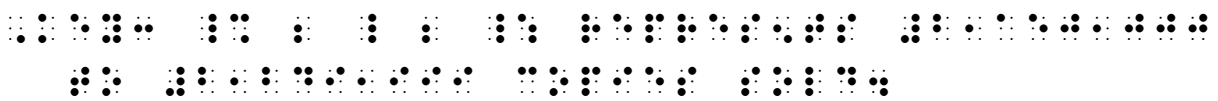
10 | 0 = 100



If the key requires a runover line, the runover begins in cell 3. When there are words in the key it may be brailled in UEB, switching to Nemeth Code for the portion that reflects the stem and leaf. This key also includes a descriptive label ("Key:"). A label is brailled only if it appears in print. If the key is set off by shading or in a box, box lines are not brailled.

Example 15.14-2

Key: 2 | 2 represents
2,150,000 to
2,249,999 copies sold.



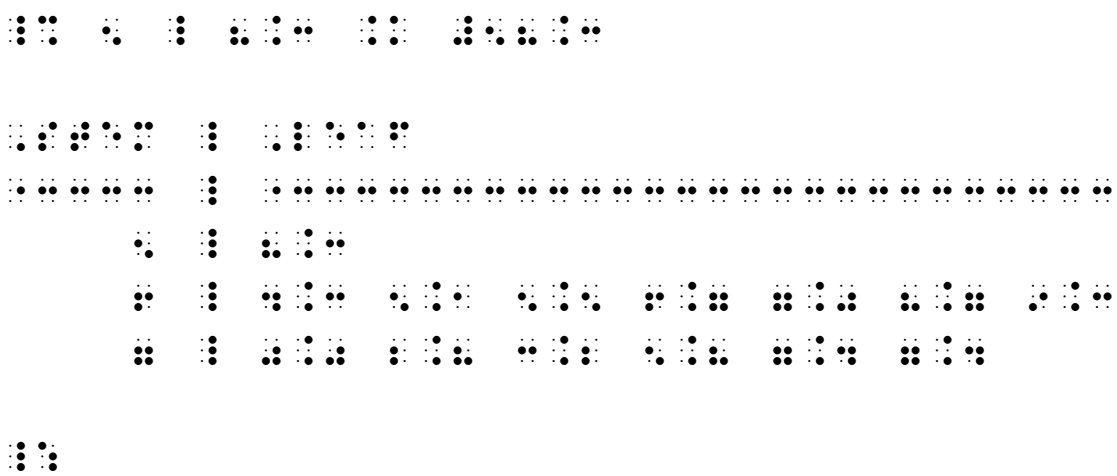
15.15 Data Consisting of More Than One Character; Punctuation Between Entries: The next example shows leaf entries with a decimal point and more than one digit. It also shows commas between leaf entries. The key is printed within the body of the table. The following rules are applied:

- The key is brailled first.
- The numeric indicator is omitted in the body of the plot.
- Data consisting of groups of two or more digits require one blank cell between entries.
- Punctuation printed between units of data is omitted in the transcription.

Example 15.15-1

Stem	Leaf
5	8.3
6	4.3, 5.1, 5.5, 6.7, 7.0, 8.7, 9.3
7	0.0, 2.8, 3.2, 5.8, 7.4, 7.4

5 | 8.3 = 58.3



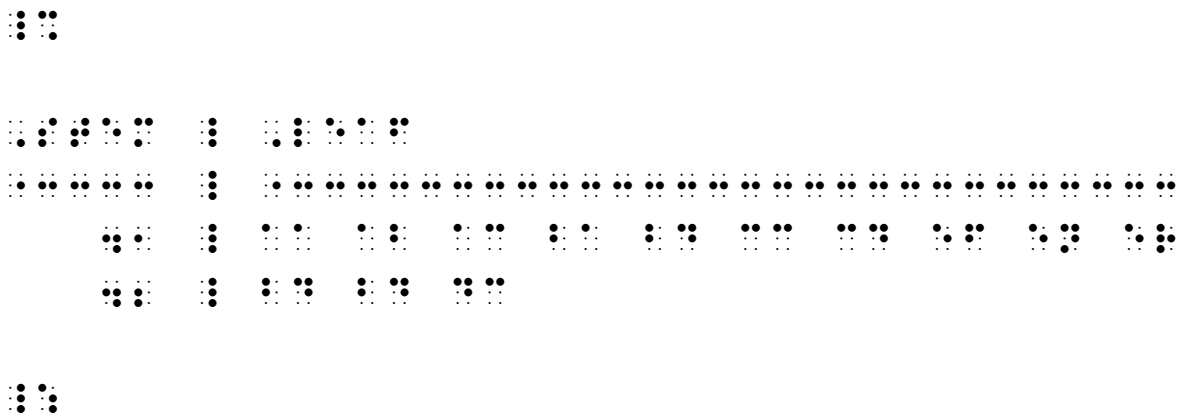
15.16 Alphabetic Data: For alphabetic data, the following rules apply.

- The English letter indicator is omitted in the body of the plot.
- Spaces are not brailled between single-letter data entries.
- Data consisting of groups of two or more letters require one blank cell between entries.

The leaves in the next example consist of pairs of lowercase letters. There is no key.

Example 15.16-1

Stem	Leaf
41	aa ab ac ba bd cc cd ef en er
42	bd bd dc



Reminder: The numeric indicator is omitted in the body of the plot (Stem column).

15.16.1 Alphabetic Key: In an alphabetic key, the English letter indicator is not used for the portion of the key that replicates an entry in the plot. The value assigned to the key is brailled using the English letter indicator as required by the Nemeth Code. If the key is printed in a box, box lines are not brailled.

⋙ $c \mid d = cd$ 

*Reminder: "cd" does not require an English letter indicator because it immediately follows a sign of comparison. See **4.16.2** in Lesson 4.*

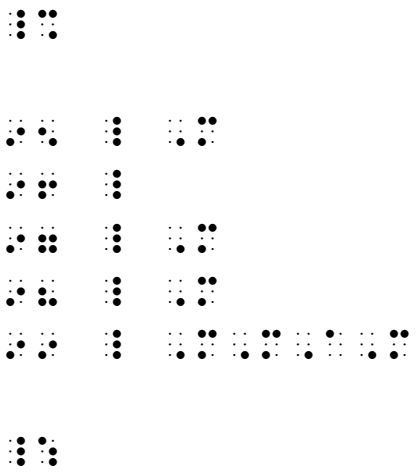
15.17 Blank Entries: A blank entry in a stem-and-leaf plot is shown as blank space in the braille transcription. Do not braille a general omission symbol. Do not fill the width of the column with dot 5s.

The following rules also apply to the next example.

- The numeric indicator and the English letter indicator are omitted in the body of the plot.
- Spaces are not brailled between single-letter data entries.
- Uppercase letters are capitalized individually.

Example 15.17-1

95	M
96	
97	M
98	M
99	M M A M



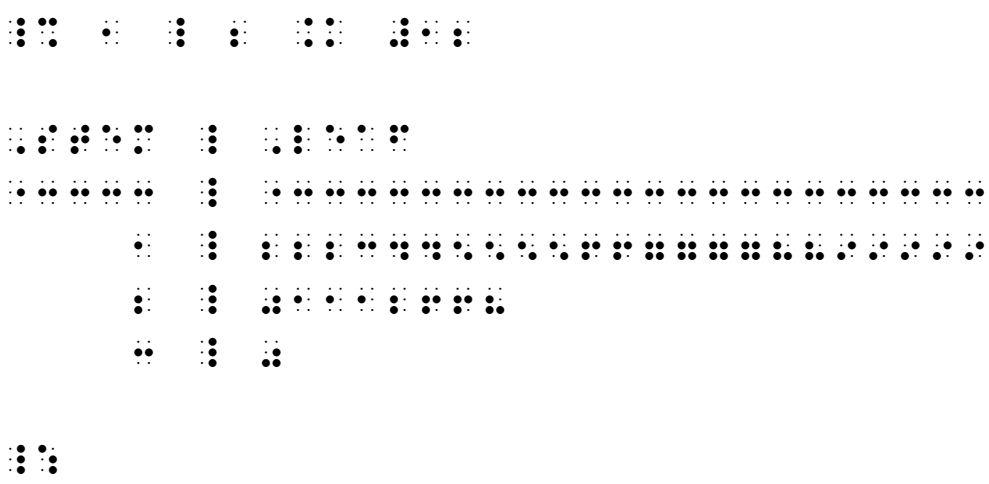
This plot has no key and no column headings—none are added in braille. The blank space in the second row indicates a blank entry in the leaf column.. Single letters are unspaced and are capitalized individually.

15.18 Runovers Within the Table: A runover of leaves shown in print should be ignored if all leaves will fit on one row in braille. Use the full width of the available cells in the row.

Example 15.18-1

Stem	Leaf
1	2 2 2 3 4 4 5 5 5 5 6 6 7
	7 7 7 8 8 9 9 9 9 9
2	0 1 1 1 2 6 6 8
3	0

1|2=12

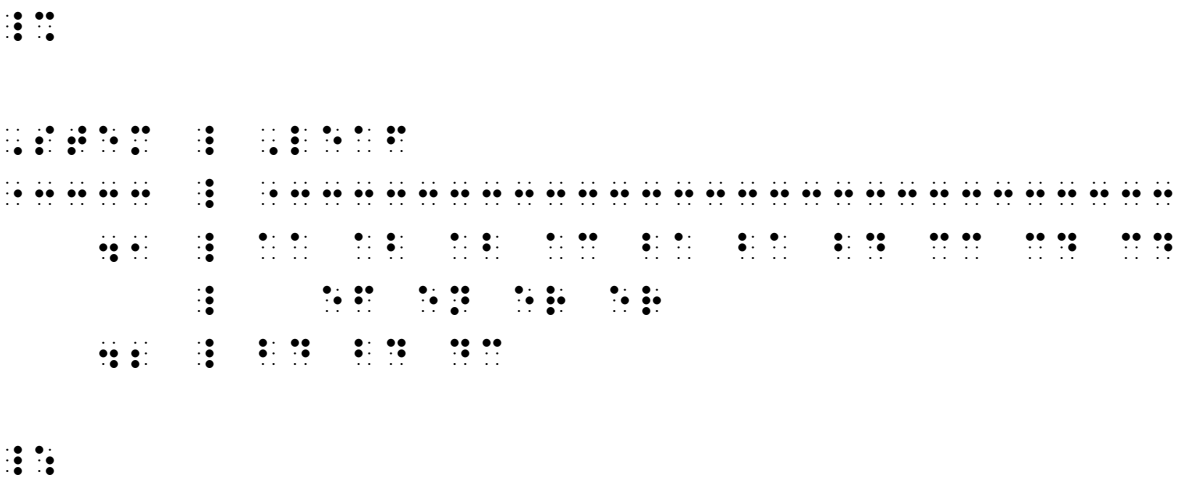


The first row of leaf data requires two lines in print but not in braille.

If leaf data will not fit on one line in the braille transcription, indent the runover line two cells to the right after using as much of the braille line as possible. Do not repeat the stem row heading. The next example illustrates, expanding the two-letter data example shown previously.

Example 15.18-2

Stem	Leaf
41	aa ab ab ac ba ba bd cc cd cd ef en er er
42	bd bd dc



The vertical line is brailled on every line in the table.

15.19 Back-To-Back Plot: A back-to-back stem-and-leaf plot is used when two sets of data are to be compared. There are three columns. The stem is the middle column. Data is read from the stem outward, which means that data in the left leaf is read from right to left. There will be two keys. In this example, the keys are printed within the body of the table.

Example 15.19-1

Ms. Abel's Test Scores

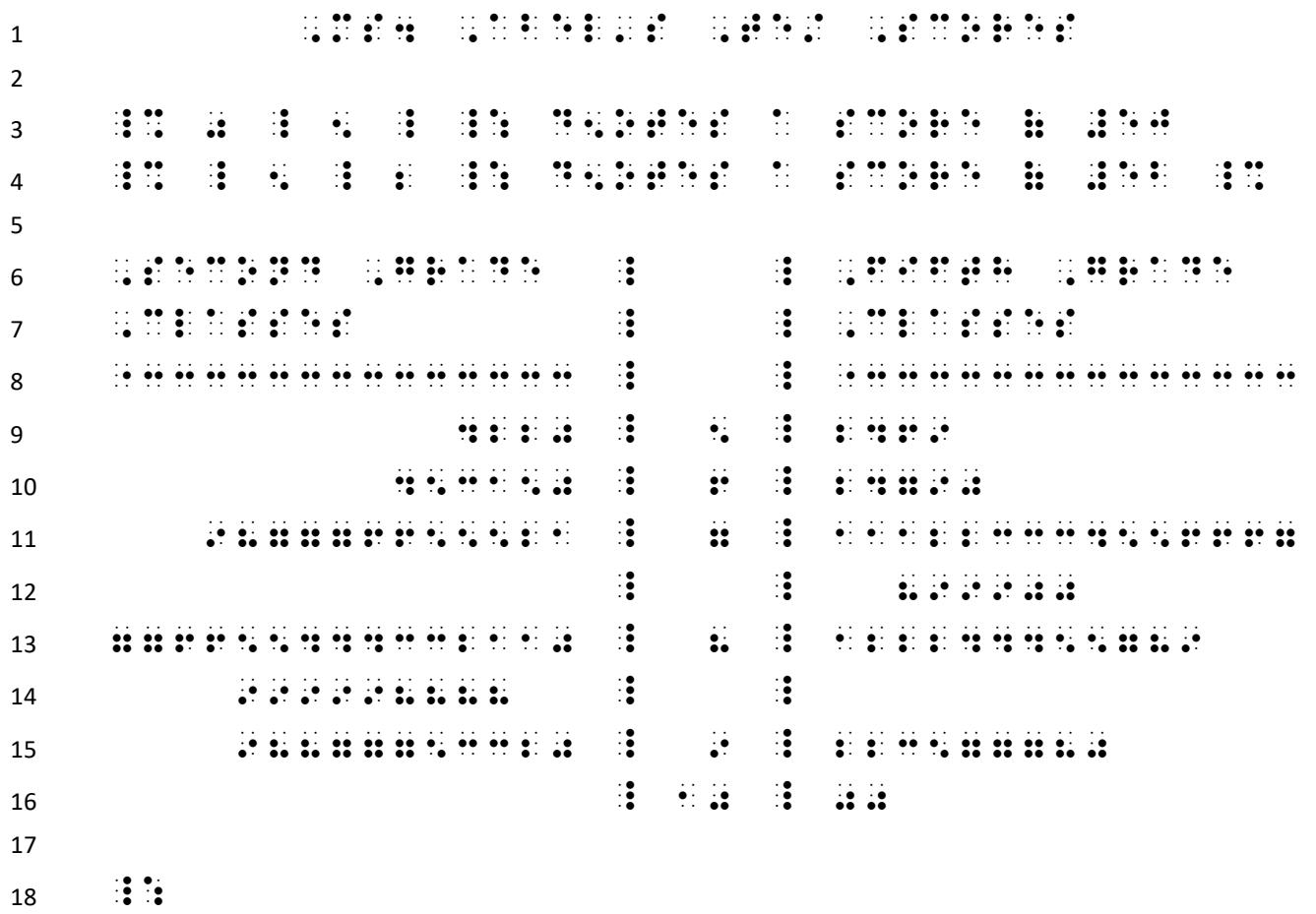
Second Grade Classes		**Fifth Grade Classes**
0 | 5 | *denotes a* 4220	5	2469 | 5 | 2 *denotes a*
score of 50 453150	6	24790 *score of 52*
987776655521	7	111223334556667899900
99999888877665544332110	8	122244455789
98877753320	9	223577780
	10	00

Keys: The left column key is brailled first, followed on the next line by the right column key.

Columns: Column separation lines are brailled below only the lines that contain column headings (line 8). In this table, the stem column (middle column) has no heading.

Runovers:

- Right leaf data are read from left to right; runovers to the right leaf data column are two cells to the right (line 12).
- Left leaf data are read from right to left; runovers to the left leaf data column are two cells to the left (line 14).



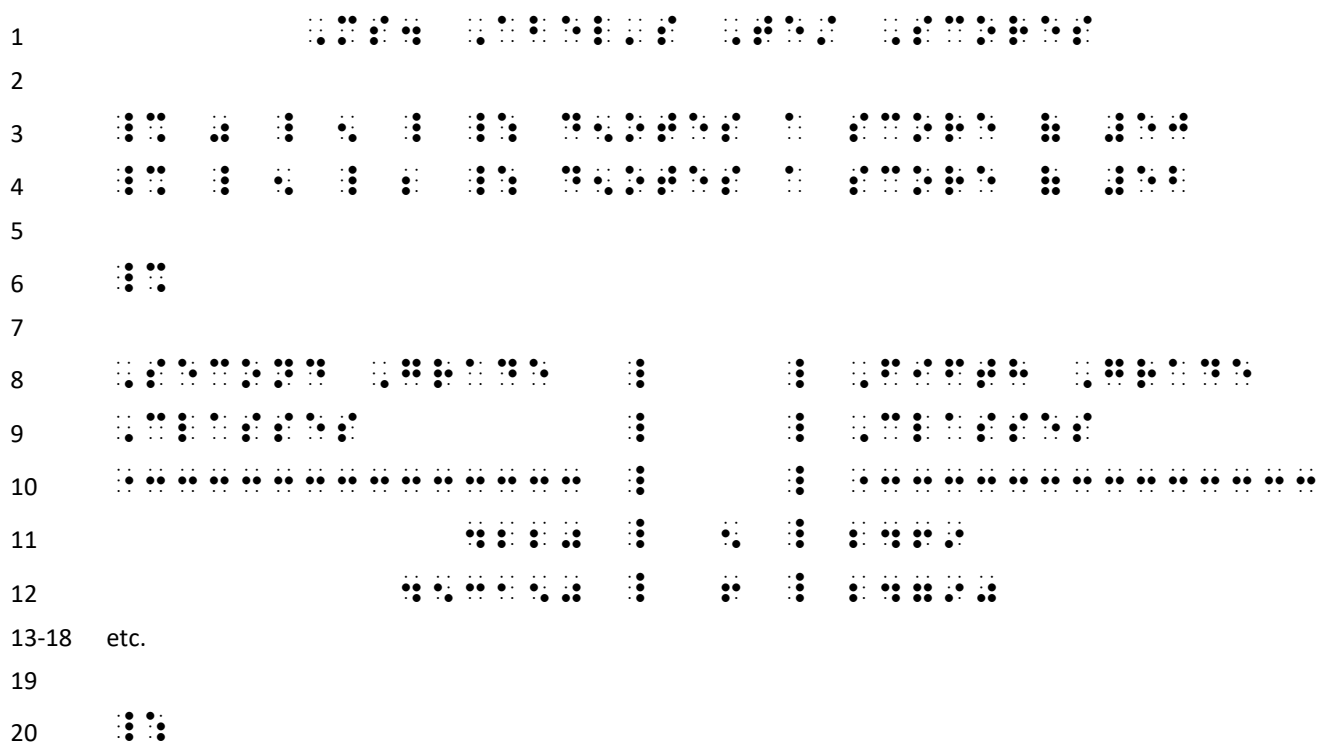
Line 3: Key to left column.

Line 4: Key to right column.

Line 16: The blank left-leaf entry in the last row is a blank space in braille.

Code Switch Decision: The opening Nemeth Code indicator needed for the table falls at the end of line 4. An alternate method would be to place it on a new line in cell 1. A blank line would then be necessary both after the key list and before the table. This alternate method is shown on the next page.

ALTERNATE METHOD



PRACTICE 15F

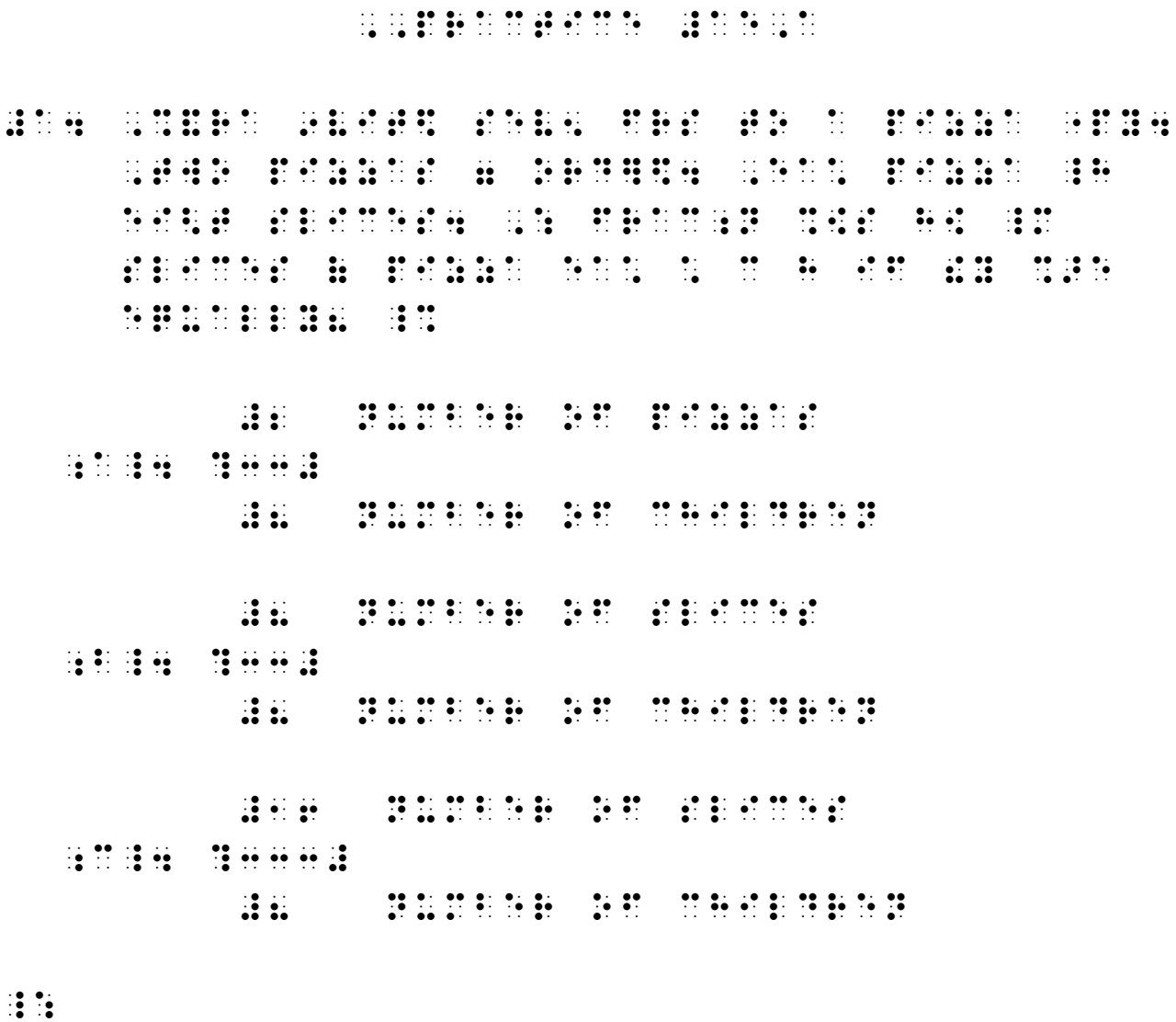
A pet store owner constructed the following stem-and-leaf plot showing the number of guinea pigs at each of her seventeen stores:

Stem	Leaf
0	7 8
1	
2	0 6 8 8 8
3	0 2 6 6 7 8
4	1 2 6 6
5	
	Key: 2 | 0 represents 20 guinea pigs

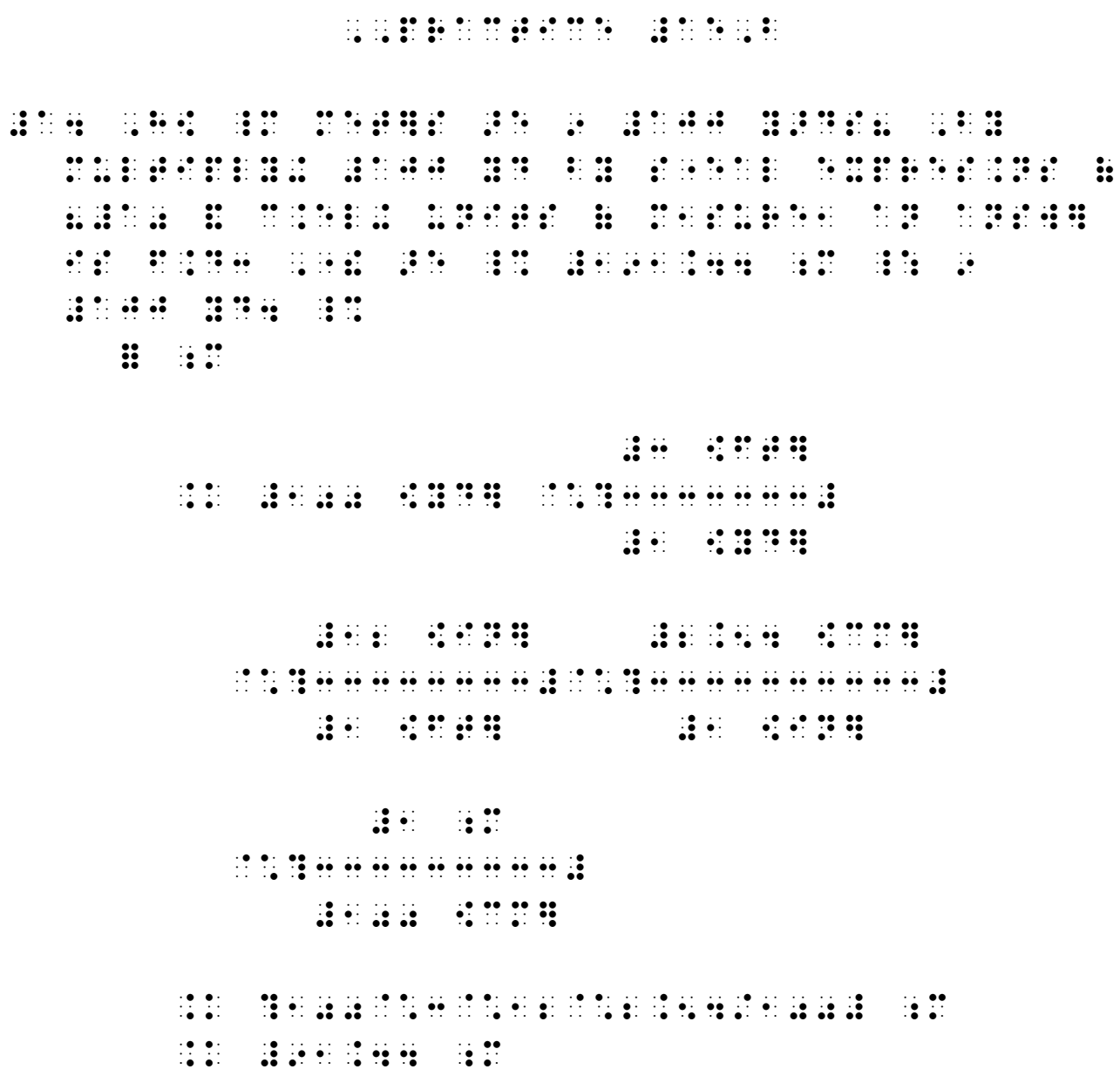
How many stores have fewer than 36 guinea pigs?

For further practice, see Appendix A—Reading Practice.

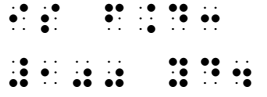
ANSWERS TO PRACTICE MATERIAL

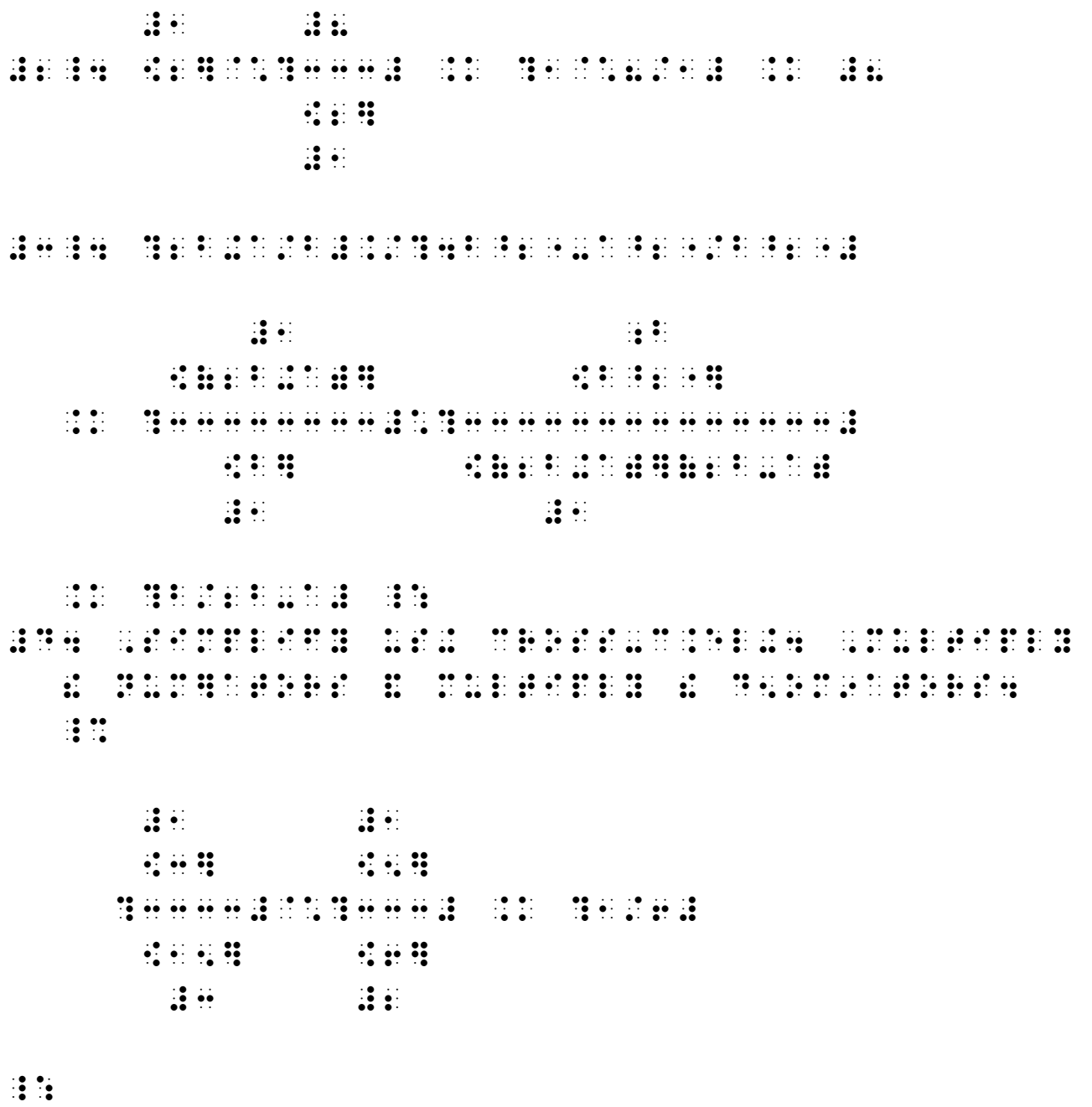


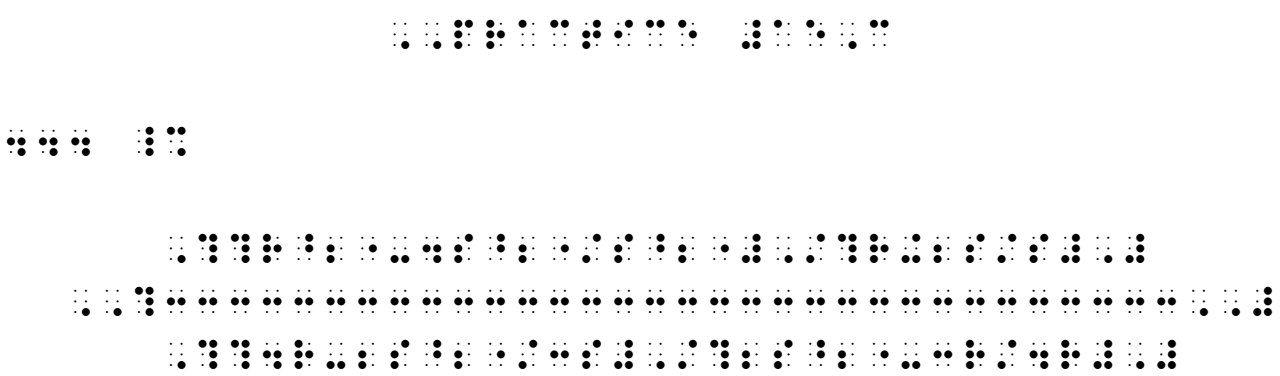
Regarding the last line, recall that, following a spatial arrangement, the Nemeth Code terminator is placed in cell 1 by itself on the line following the required blank line.

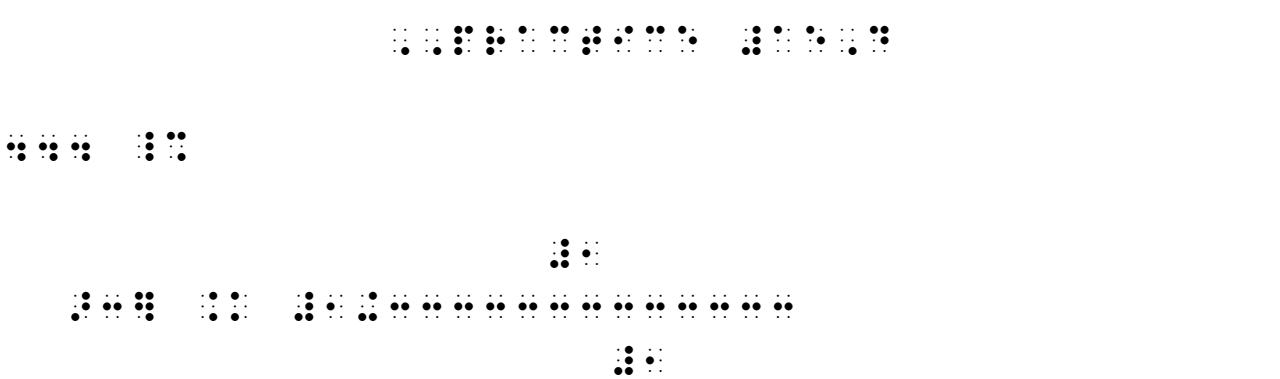


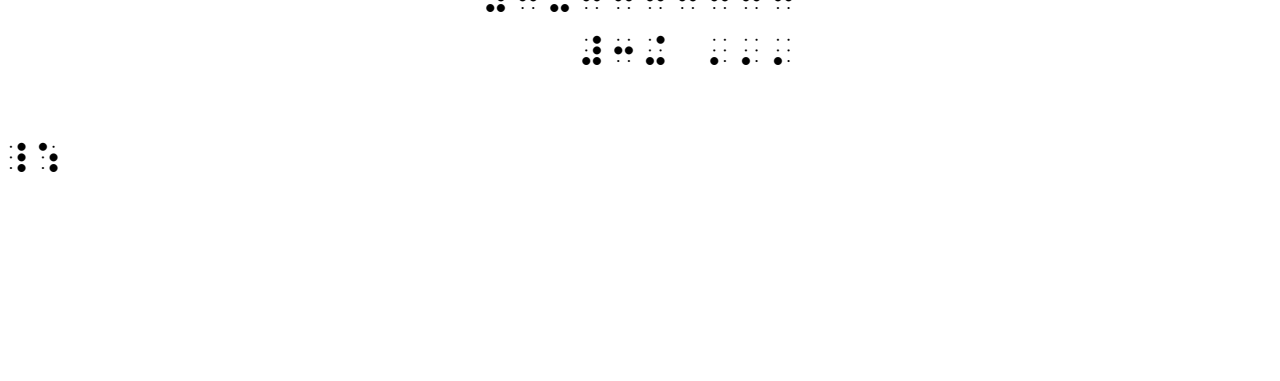
In #1, the fourth and fifth line may also be transcribed as follows. In either case, the number "100" must be brailled on the same line as its abbreviation "yd".





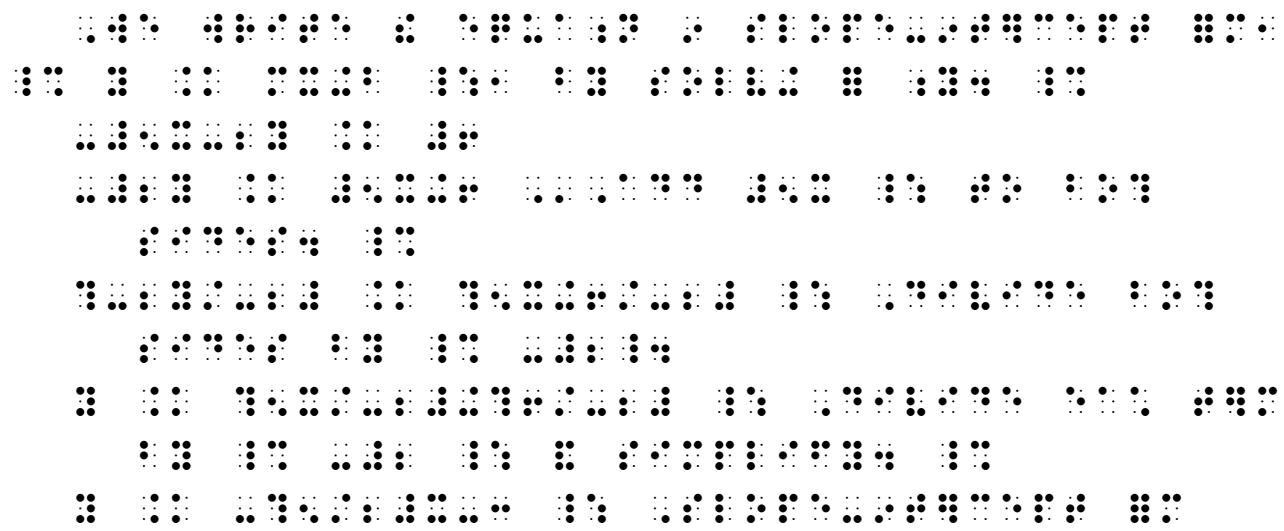




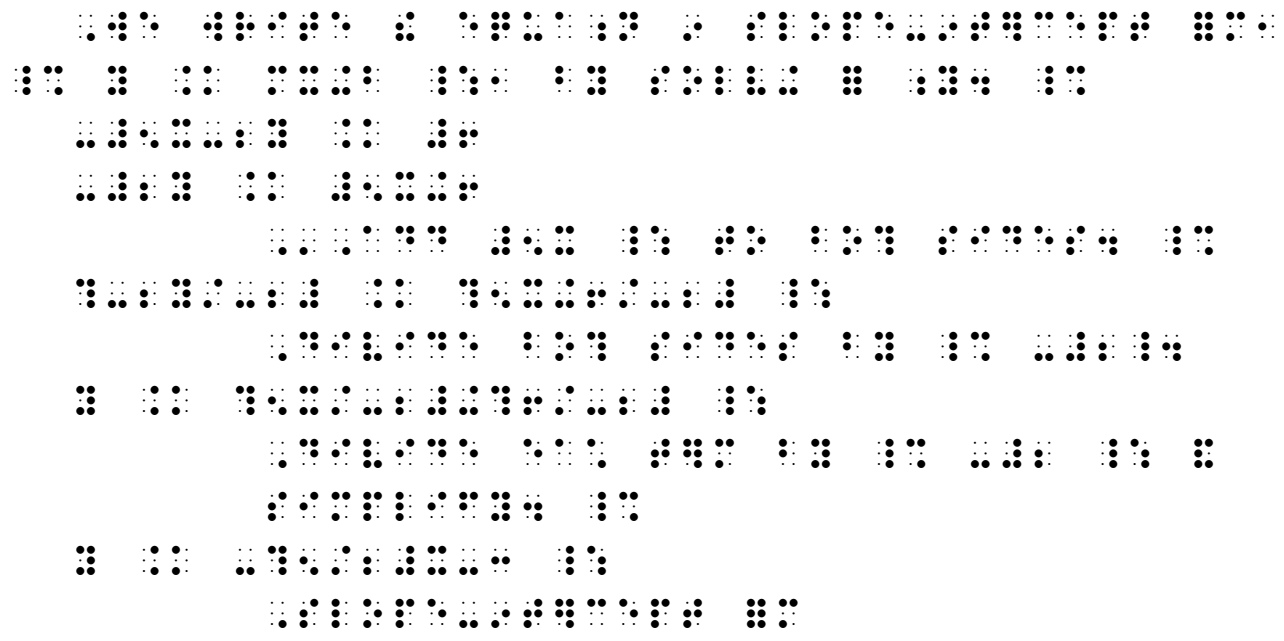


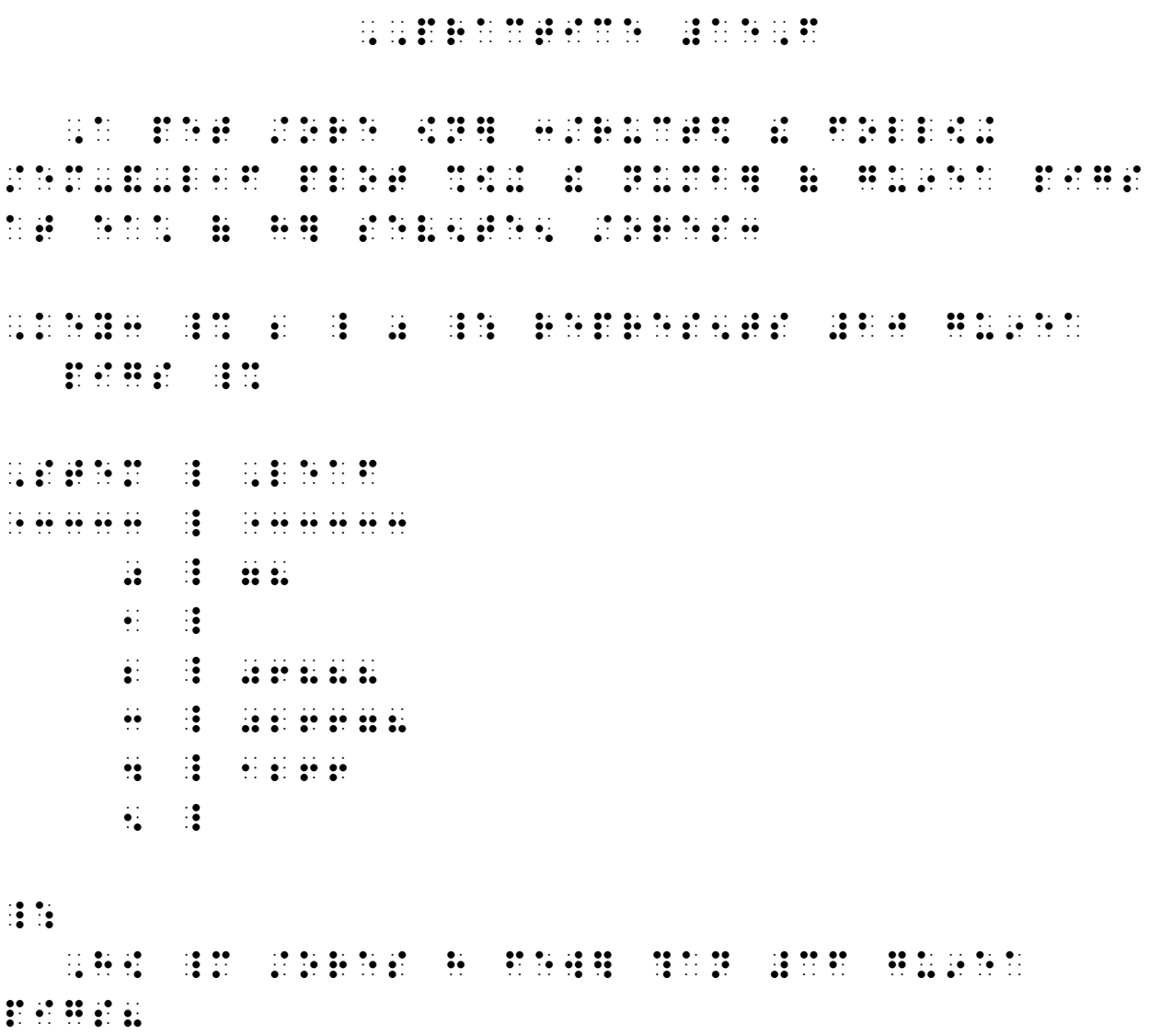


OPTION #1



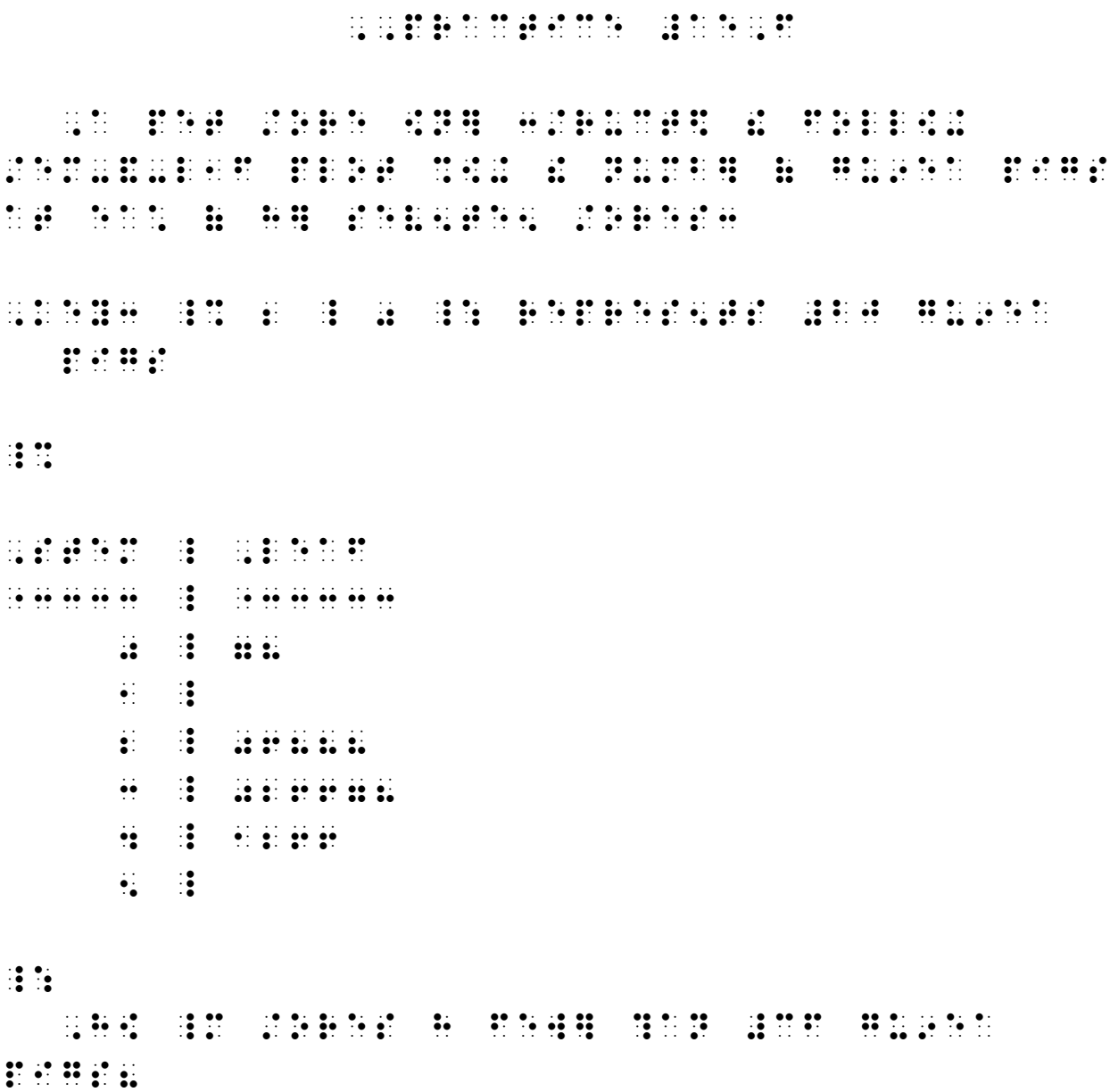
OPTION #2





An alternate method is shown on the next page.

ALTERNATE METHOD



EXERCISE 15

Exercise 15 will be available when this course is finished being written and is no longer "Provisional".

Proceed to Lesson 16.

LESSON 16

Read about this PROVISIONAL EDITION in the front matter to this book.
Check the NFB website periodically for updates to this lesson.

- *SYSTEMS OF EQUATIONS*
  - *Enlarged Signs of Grouping*
- *DETERMINANTS AND MATRICES*

SYSTEMS OF EQUATIONS

16.1 Definition and Recognition: A *system of equations* is a collection of two or more equations which share variables. The student identifies the value of each variable by "solving the system." You can recognize a system by noticing the arrangement of equations on two or more lines. If the system consists of two equations, there will be two variables to solve (typically *x* and *y*); if the system consists of three equations, there will be three variables to solve (typically *x*, *y*, and *z*). The equations may or may not be joined by an enlarged grouping sign. Here is a print example of two equations, each with variables *x* and *y*.

$$4x - y = 10$$
$$2x = 12 - 3y$$

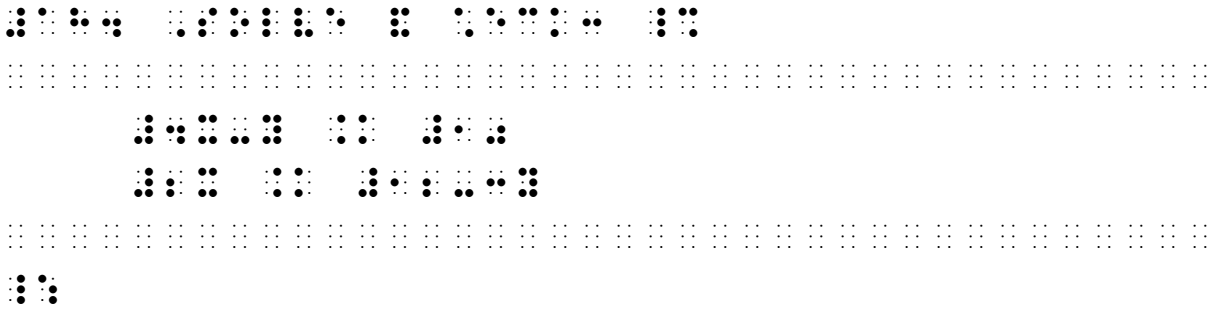
16.2 Transcription Rules for Systems of Equations: Systems of equations are brailled following rules of both spatial and nonspatial material, as follows.

- One blank line is left above and below the system, according to rules of the Nemeth Code for *spatial* material.
- The numeric indicator is (or is not) used according to rules of the Nemeth Code for *nonspatial* material.
- Alignment is maintained only if all terms and symbols are aligned in print.

Example 16.2-1 18. Solve and check:

$$4x - y = 10$$
$$2x = 12 - 3y$$

Print observations: This system has two variables (x, y) – two equations. The terms are not vertically aligned.



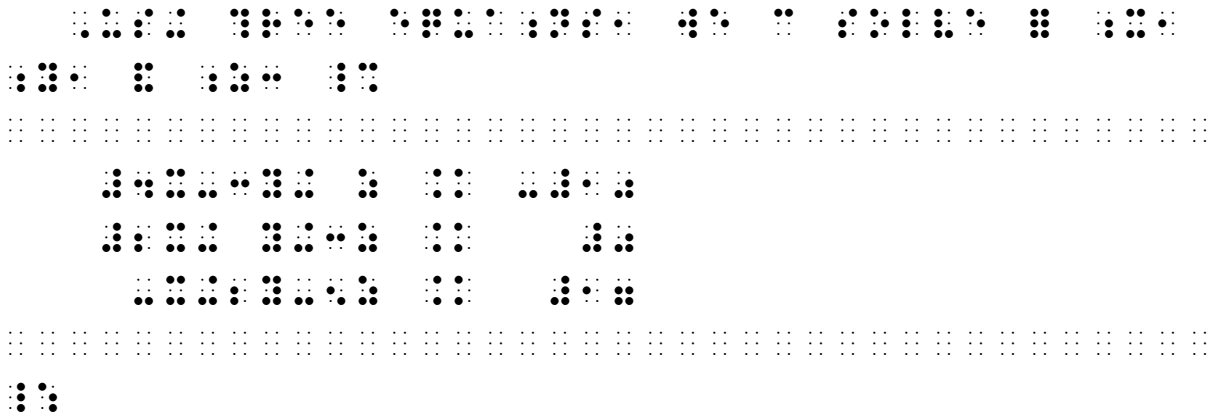
Braille observations: Each equation begins in the same cell because the terms are not vertically aligned in print. A blank line is inserted before and after the system. Numeric indicators are used as needed according to rules of the Nemeth Code for nonspatial material. Code switch indicators do not interfere with the arrangement. The Nemeth Code terminator is placed in cell 1 after a blank line following the spatial arrangement.

Note: The Nemeth Code calls such an arrangement an "ununified system of equations" because the equations are not joined by a grouping symbol.

Example 16.2-2 Using three equations, we can solve for x, y, and z:

$$\begin{aligned} 4x - 3y + z &= -10 \\ 2x + y + 3z &= 0 \\ -x + 2y - 5z &= 17 \end{aligned}$$

Print observation: The terms on both sides of the equals signs are vertically aligned.



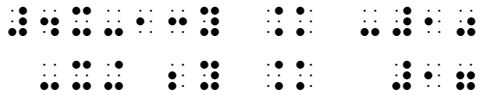
Braille observations: A blank line is inserted before and after the system. Spaces are inserted in order to maintain alignment as printed. Numeric indicators used as needed according to rules of the Nemeth Code for nonspatial material.

The BANA Nemeth Code Technical Committee is discussing details regarding the situation encountered below. A provisional solution is provided.

When numerical coefficients with a different number of digits need to line up by place value in order to follow print alignment, a numeric indicator is not inserted in the space.

Example 16.2-3

$$\begin{aligned} 4x - 13y &= -10 \\ -x + 2y &= 17 \end{aligned}$$



A space is inserted before the single digit "2" to achieve alignment with the "13" above it.

PRACTICE 16A

1. Solve and check:

$$2x + 3y = 2$$
$$8x - 4z = 3$$
$$3y - 8z = -1$$

2. Solve:

$$\begin{aligned} 2x - 5y + 6z &= 11 \\ 3x - 2y + 3z &= 9 \\ 2x + 4y - 9z &= -3 \end{aligned}$$

16.3 Unified System of Equations and Enlarged Grouping Signs: The Nemeth Code calls an arrangement a "unified system of equations" when the equations are joined by an enlarged grouping symbol. Here is a print example, using an enlarged left brace.

$$\begin{cases} x = y \\ 5x - y = 4 \end{cases}$$

Enlarged Signs of Grouping

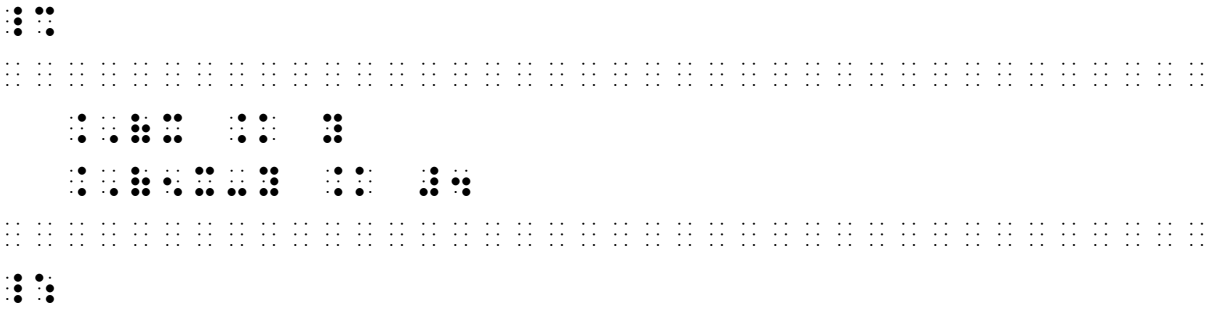
16.4 Transcription Rules for Enlarged Signs of Grouping: In braille, enlarged signs of grouping are used on each line of the unified system of equations and are vertically aligned. The opening and the closing enlarged grouping symbol are placed in the cells next to the items which extend furthest left and furthest right, with no intervening space. If only the opening or closing sign of grouping is shown in print, only that sign is shown in braille.

16.4.1 Enlarged Left Brace: The enlarged left brace curves and points to the left in print. Notice that the enlarged braille symbol is formed by inserting a dot 6 before the ⠸⠪ symbol of the normal brace symbol.

Enlarged Left Brace	{	⠐⠸⠪	(on two or more lines)

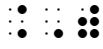
Example 16.4-1 $\begin{cases} x = y \\ 5x - y = 4 \end{cases}$

Print observations: An enlarged left brace groups the unified system of equations. Terms are not vertically aligned.



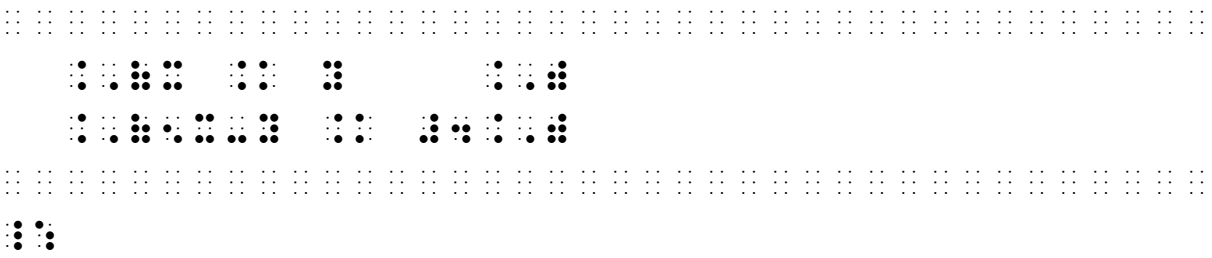
Braille observations: Each enlarged grouping symbol begins in the same cell. When terms are not aligned in print, each equation begins in the cell following the left enlarged brace symbol. Numeric indicators are used as needed according to rules of the Nemeth Code for nonspatial material.

16.4.2 Enlarged Right Brace: The enlarged right brace curves and points to the right in print. Notice that the enlarged braille symbol is formed by inserting a dot 6 before the ⠸ symbol of the normal brace symbol.

Enlarged Right Brace	}		(on two or more lines)

The same system is shown below, with an enlarged right brace added.

Example 16.4-2 $\left\{ \begin{array}{l} x = y \\ 5x - y = 4 \end{array} \right\}$



Closing grouping symbols are vertically aligned, starting in the cell next to the longest equation.

16.5 Embedded Vertical Groupings: Enlarged grouping signs are used in other structures as well, not only with systems of equations. If the grouping is embedded within narrative, the required blank lines before and after the arrangement become part of the paragraph. The surrounding text is placed on the top line of the arrangement only – before and/or after the math as it fits in context of the narrative. Text continues in the runover cell of the paragraph after the second required blank line. The opening Nemeth Code indicator and the Nemeth Code terminator are placed only on the top line of the embedded arrangement. The code switch indicators apply to the whole arrangement.

Example 16.5-1 Given $c: \{1, \dots, n\} \to \{1, \dots, n\}$ such that

$$\left.\begin{matrix} c(a_i) = a_{i+1} \\ c(a_l) = a_1 \end{matrix}\right\} \text{for } 1 \leq i < l \text{, solve the problem.}$$

1
2
3
4
5
6
7

Line 1: Paragraph begins in cell 3. (Lines 1-7 comprise the entire 3-1 paragraph.)
Lines 3 and 6: Blank line required before and after vertically-arranged grouping within the paragraph.
Line 4: Text is placed on top line of grouping. Opening switch placed on this line only.
Line 7:Paragraph continues following required blank line.

PRACTICE 16B

After solving for x and y, we determine that the system $\left\{\begin{matrix} 2x - 3y = 17 \\ 3x + 2y = 6 \end{matrix}\right\}$ has the solution set $\{(4, -3)\}$. Name the solution set for this system:

$$\left\{\begin{matrix} x + 2y = 6 \\ 2x - y = 7 \end{matrix}\right\}$$

16.6 Enlarged Parentheses: Notice that the enlarged braille parentheses are formed by inserting a dot 6 before the ⠷ or the ⠾ symbol of the normal parenthesis symbol.

Enlarged Parentheses			
Left	⎛	⠐⠷	(on two or more lines)
Right	⎞	⠐⠾	(on two or more lines)

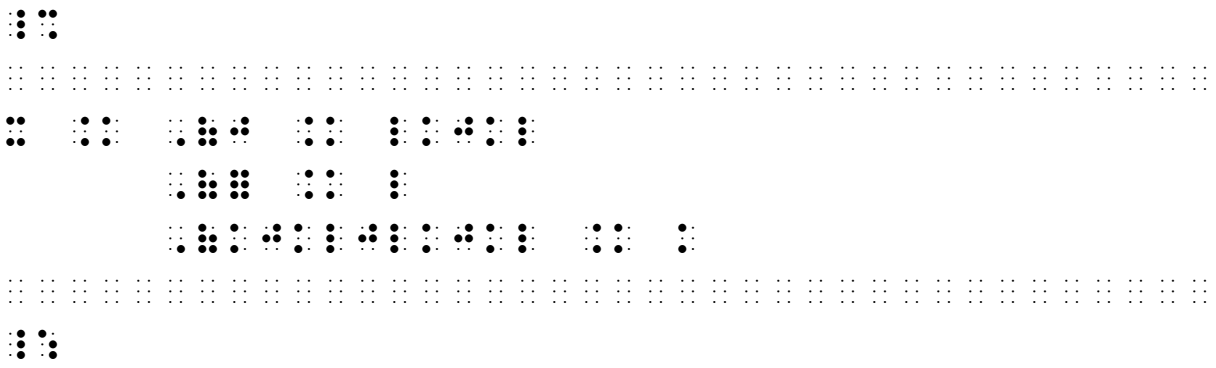
16.7 Placement of Symbols: Symbols which appear outside of the enlarged grouping symbol and which apply to the arrangement are placed on the top line of the arrangement, even if the items are centered in print. Also note the following details:

- The English letter indicator is (or is not) used according to rules of the Nemeth Code for nonspatial material.
- Alignment is maintained only if all terms and symbols are aligned in print.

Example 16.7-1

$$x = \begin{cases} j = lkjkl \\ = l \\ kjkljlkjkl = k \end{cases}$$

Print observations: The anchor is centered with respect to the enlarged grouping symbol to its right. The three-line link is grouped with an enlarged left parenthesis. Equals signs in the grouped arrangement are aligned but terms are not.



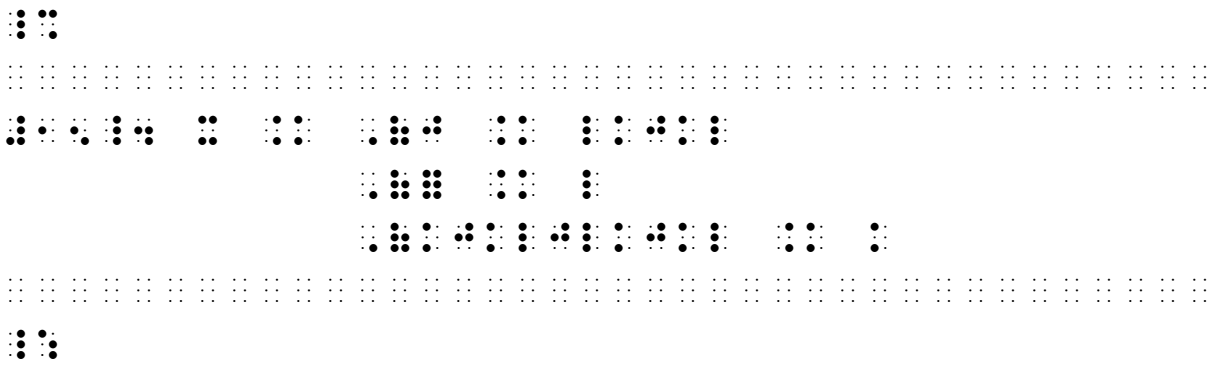
Braille observations: The anchor is aligned with the top line of the arrangement. Each enlarged grouping symbol begins in the same cell. The grouped items are left-adjusted because the terms are not aligned. A general omission symbol indicates the blank space before the middle equals sign. Code switch indicators do not interfere with the spatial arrangement.

16.8 Placement of Identifiers and Punctuation: Identifiers and punctuation which appear outside of the enlarged grouping symbol and which apply to the arrangement are placed on the top line of the arrangement, even if the items are centered in print. Here is the same example, now identified with an item number "15."

Example 16.8-1

15. $x = \begin{cases} j = lkjkl \\ = l \\ kjkljlkjkl = k \end{cases}$

Print observation: The item number and the anchor are centered with respect to the associated enlarged grouping symbol to the right.

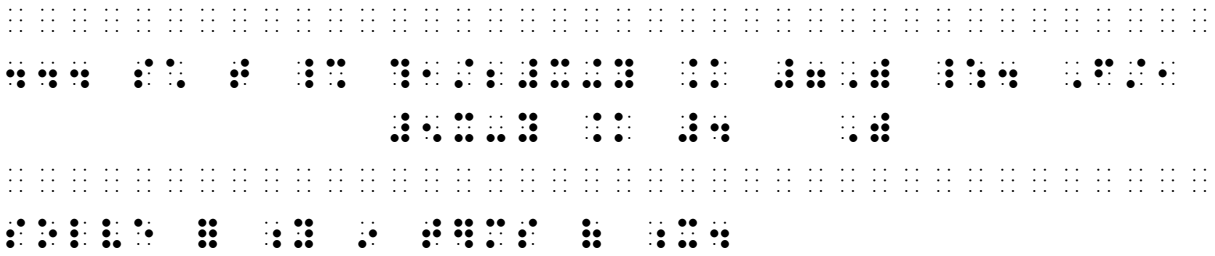


Braille observation: Item number and anchor are aligned with the top (first) line of the arrangement.

Example 16.8-2

... such that $\begin{cases} \frac{1}{2}x + y = 7 \\ 5x - y = 4 \end{cases}$. First, solve for *y* in terms of *x*.

Print observations: A vertical arrangement is embedded within the narrative. It is grouped on the right with an enlarged right parenthesis. A period follows enlarged parenthesis, centered to the arrangement.



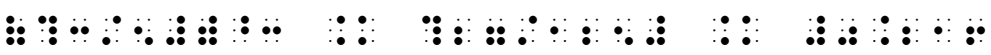
Braille observations: The narrative sentence ends with a displayed expression. The preceding text and the following period are brailled on the top line of the arrangement even though they are centered to the expression in print. A blank line is inserted before and after the lines with the embedded array.

16.9 Grouping Symbols Shown Taller in Print: Recall from **Lesson 3** that nested grouping symbols may be printed in different sizes in order to visually distinguish the nested symbols, but they are not brailled as "enlarged" symbols if they apply to only one line of characters.

⋙ $p\left(q\left(f\left(g\left(h\left(x\right)\right)\right)\right)\right)$ 

Similarly, grouping signs which enclose taller print constructions such as fractions, modified expressions, integrals, or binomial coefficients (**12.7.1**) are brailled as normal grouping symbols because they apply to only one line of braille characters.

⋙ $\left(\frac{3}{5}\right)^3 = \frac{27}{125} = 0.216$



⋙ $\left(\sum_k \binom{n}{k}\right)$ 

PRACTICE 16C

A) Verify (YES or NO):

$$2\left(-\tfrac{4}{3}\right) - 1 \stackrel{?}{=} 4\left(-\tfrac{4}{3}\right) + 9$$

$$\left(-\tfrac{8}{3}\right) - 1 \stackrel{?}{=} \left(-\tfrac{16}{3}\right) + 9$$

$$-8 - 3 \stackrel{?}{=} -16 + 27$$

$$-11 = 11 \text{ . NO}$$

B) $\left(\begin{array}{l} \frac{x}{3} - \frac{y}{2} = 2 \\ 5x + 3y = 51 \end{array}\right) = (?, ?)$

The BANA Nemeth Code Technical Committee is discussing details regarding the treatment of remarks printed beside spatial arrangements. Correct transcriptions of the examples in this section will be confirmed after approval is obtained. For now, provisional solutions are provided.

16.10 Remarks Printed Next to Spatial Arrangements: Remarks which appear beside a spatial arrangement are brailled either beside the arrangement or after the blank line following the arrangement. Code switching is necessary if the remark contains narrative.

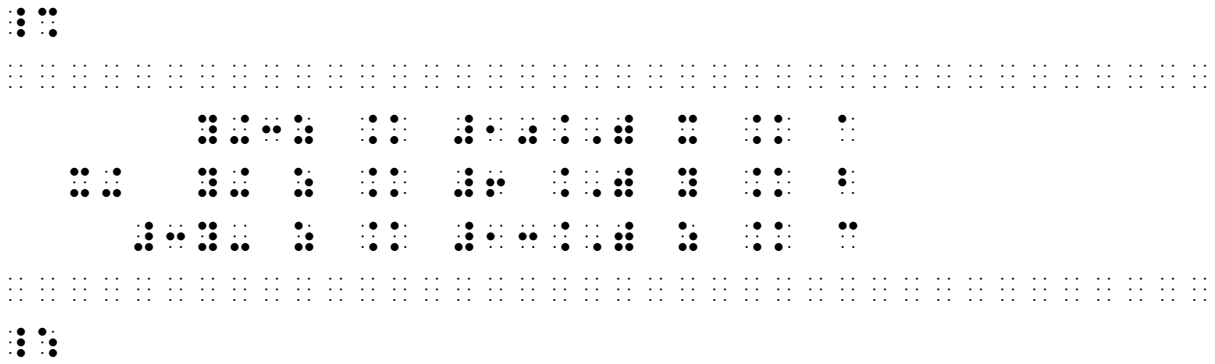
16.10.1 Remarks Brailled Beside Unified Expressions: Remarks printed to the right of an arrangement which is grouped with a right (closing) enlarged grouping sign can be brailled as printed as long as there is enough space remaining on the line for the remark. One blank cell is placed between the grouping symbol and the remark.

Example 16.10-1

$$\left.\begin{aligned} y+3z&=10\\ x+\ y+\ z&=6\\ 3y-\ z&=13 \end{aligned}\right\}\quad \begin{aligned} x&=a\\ y&=b\\ z&=c \end{aligned}$$

Print observations: Terms y and z on the left side of the equals signs are vertically aligned, although place values of the numbers to the right of the equals signs are not. Three individual remarks are printed to the right of the enlarged brace. The remarks are mathematical expressions.

PROVISIONAL TRANSCRIPTION



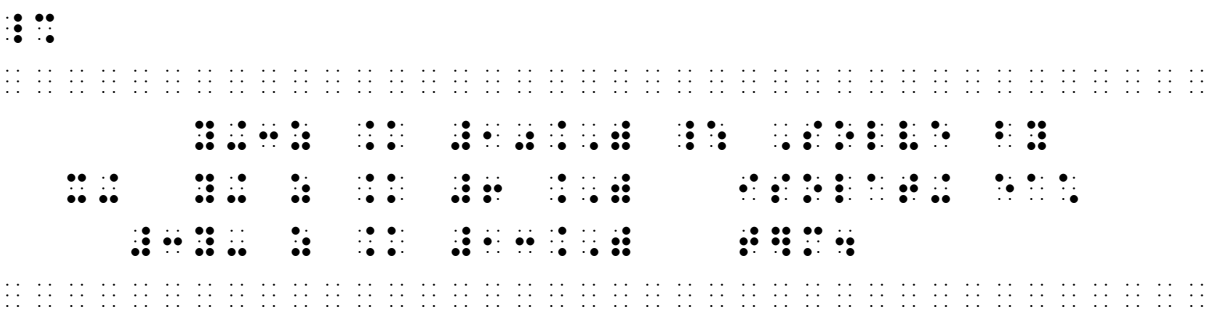
Braille observations: Spaces are inserted in the second equation in order to maintain alignment of factors and terms. Enlarged right braces are vertically aligned and begin in the cell next to the longest equation. One blank cell separates the closing grouping signs from each remark.

The next example shows a three-line unified system. A remark is printed to the right of the middle line of the system. In braille, the remark is moved to begin on the top line of the arrangement. If runovers are needed, they are indented two cells from the starting cell of the comment. Because this remark is not mathematical notation, a Nemeth Code terminator precedes the first word and the comments are transcribed in contracted braille.

Example 16.10-2

$$\left.\begin{aligned} y+3z&=10\\ x+\ y+\ z&=6\\ 3y-\ z&=13 \end{aligned}\right\}$$ *Solve by isolating each term.*

PROVISIONAL TRANSCRIPTION

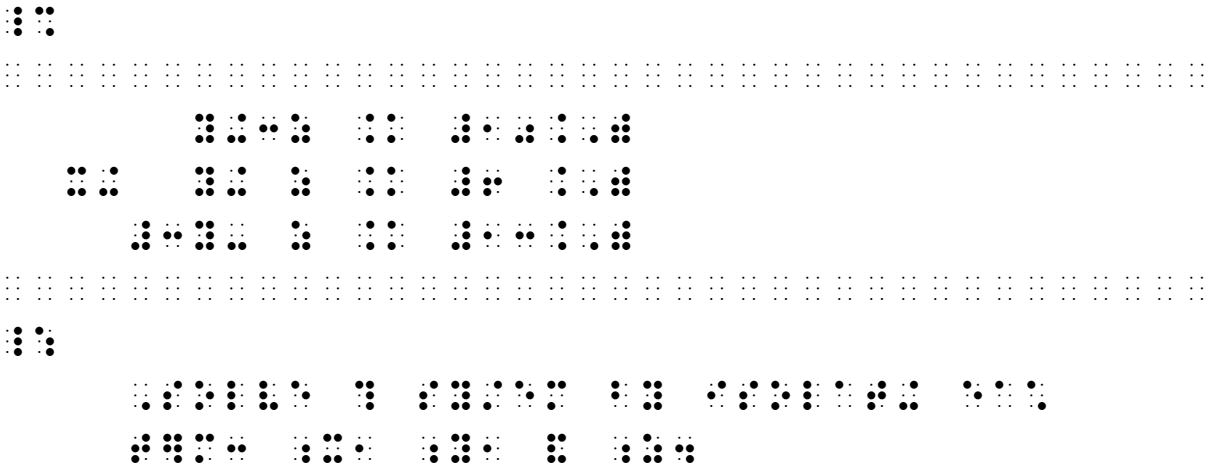


16.10.2 Remarks Brailled Below the Arrangement: The next example shows the same three-line unified system but the remark is longer. Because this remark takes more than the three available lines beside the system, it is placed after the arrangement in the runover cell for displayed material. If a Nemeth Code terminator is needed before the remark begins, the terminator is placed in cell 1 on the line following the required blank line.

Example 16.10-3

$$\left.\begin{aligned} y+3z&=10\\ x+\ y+\ z&=6\\ 3y-\ z&=13 \end{aligned}\right\}$$ *Solve this system by isolating each term: x, y, and z.*

PROVISIONAL TRANSCRIPTION

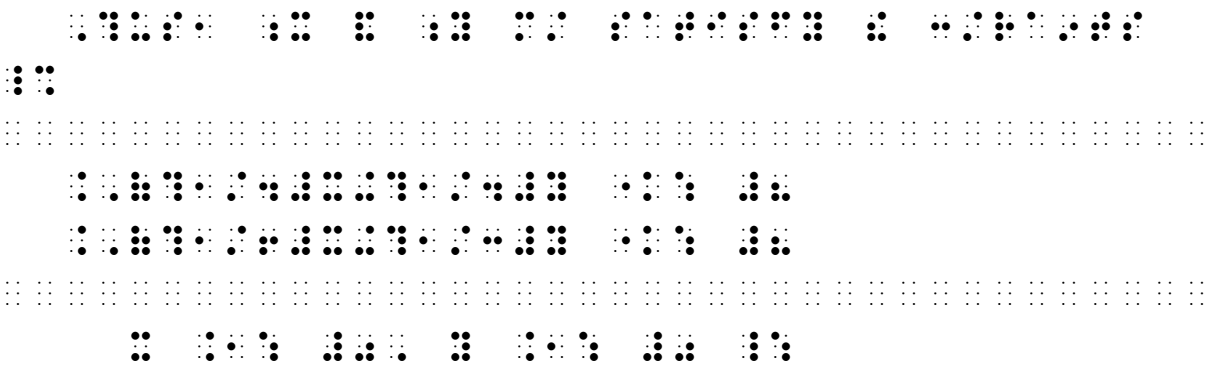


If there is no enlarged grouping sign to separate the math material from the remarks, place the remarks after the required blank line. The remark begins in the runover position.

Example 16.10-4 Thus, x and y must satisfy the constraints

$$\begin{cases} \frac{1}{4}x + \frac{1}{4}y \leq 8 \\ \frac{1}{6}x + \frac{1}{3}y \leq 8 \end{cases} \qquad x \geq 0, y \geq 0$$

PROVISIONAL TRANSCRIPTION

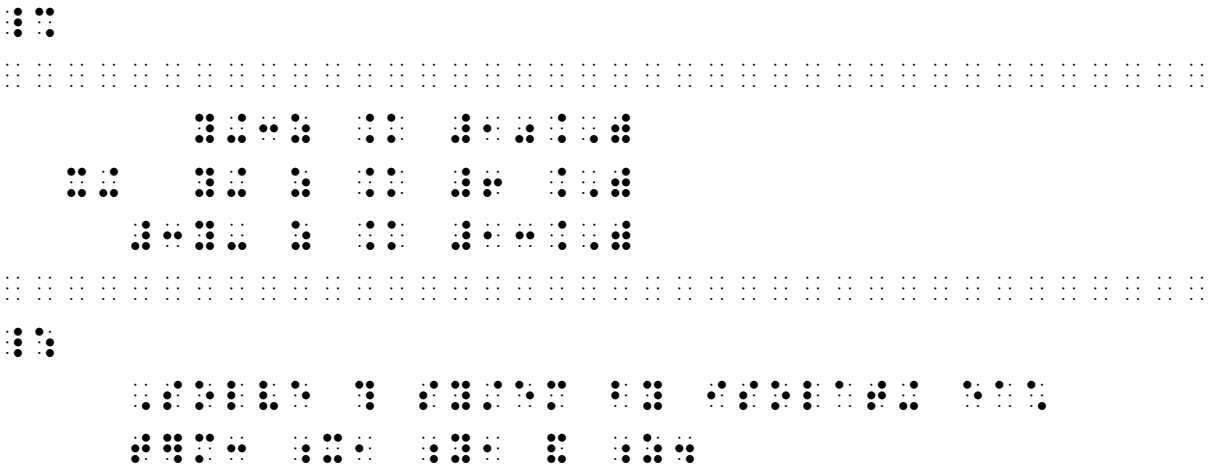


Example 16.10-5

$$\begin{aligned} y + 3z &= 10 \\ x + y + z &= 6 \\ 3y - z &= 13 \end{aligned}$$

Solve this system by isolating each term: x, y, and z.

PROVISIONAL TRANSCRIPTION



PRACTICE 16D

Find x and y in terms of a and b.

1. $\begin{cases} x + y = 0 \\ x + ay = 1 \end{cases}$ $(a \neq 1)$

2. $\begin{cases} ax + by = 0 \\ x + y = 1 \end{cases}$ $(a \neq b)$

3. $\begin{cases} ax + by = 0 \\ a^2x + b^2y = 1 \end{cases}$ $(a \neq 0,\ b \neq 0,\ a \neq b)$

4. $\left.\begin{array}{l} a = \frac{x+y}{x-y} \\ \\ b = \frac{x-y}{x+y} \end{array}\right\}$ $-1 < x < 1, -1 < y < 1$

16.11 More Enlarged Signs of Grouping: In addition to the enlarged braces and parentheses, the Nemeth Code provides symbols for six other enlarged signs of grouping. Notice that each enlarged braille symbol is formed by inserting a dot 6 before the ⠸, ⠼, or ⠨ symbol of the normal symbol.

Enlarged Vertical Bar

Single |

Double ||

Enlarged Brackets

Left [**Right**]

Enlarged Angle Brackets

Left ⟨ **Right** ⟩

Enlarged Barred Brackets

Left **Right**

Enlarged Angle Braces

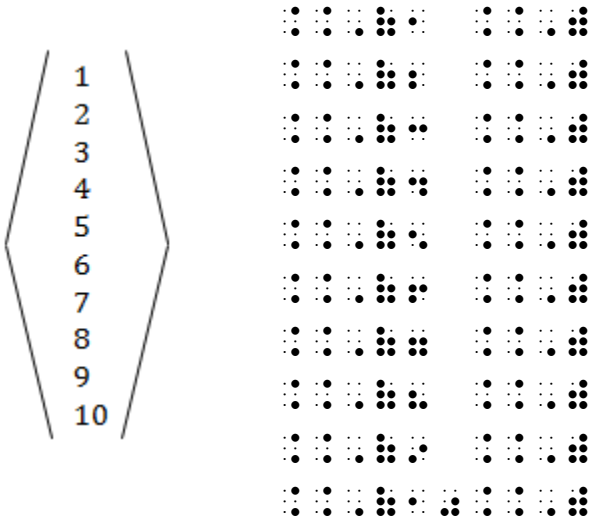
Left **Right**

Enlarged Half Brackets

Upper Left **Upper Right**

Lower Left **Lower Right**

Example 16.11-1



PRACTICE 16E

$$\langle x \rangle \left\langle \begin{matrix} x \\ y \\ z \end{matrix} \right\rangle \left\langle \begin{matrix} u \\ v \\ w \\ x \\ y \\ z \end{matrix} \right\rangle$$

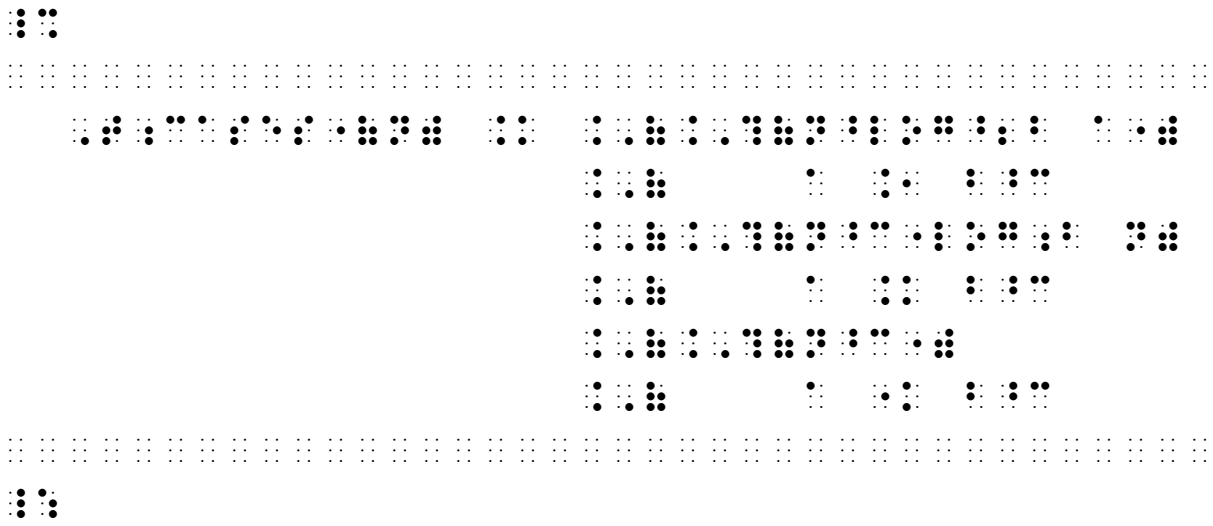
CHALLENGE PROBLEM #1

This problem shows an anchor followed by three lines of expressions grouped with an enlarged left brace. There is no right brace. Conditions are printed to the right of each of the three lines. It can be determined from the text (not shown) that each condition applies specifically to the expression to its left.

$$T_{cases}(n) = \begin{cases} \Theta\left(n^{\log_b a}\right) & a > b^c \\ \Theta(n^c \log_b n) & a = b^c \\ \Theta(n^c) & a < b^c \end{cases}$$

Following the provisional guidelines given in 16.10.2 (brailling the conditions below the arrangement) would obscure the relationship between each expression and its condition. In order to maintain the side-by-side relationship, a layout similar to the alternate layout given in Lesson 15 is proposed. By placing the conditions on the following line, indented four cells, the relationship is preserved.

PROVISIONAL TRANSCRIPTION



DETERMINANTS AND MATRICES

16.12 Definition and Recognition: *Determinants* and *matrices* (singular: *matrix*) are arrangements of items in rows and columns which are enclosed between left and right grouping symbols. Items can be numbers, symbols, or mathematical expressions.

Here is a 2×2 ("two by two") determinant enclosed between enlarged vertical bars.

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

Here is a 2×3 ("two by three") matrix enclosed between enlarged brackets.

$$\begin{bmatrix} 1 & 9 & -13 \\ 20 & 5 & -6 \end{bmatrix}$$

Determinants and matrices may also be referred to as "arrays." An array can be composed of only one column or only one row. These will be studied at the end of this section.

16.13 Transcription Rules for Determinants and Matrices: Observe the following rules regarding the transcription of the two examples shown above.

16.13.1 Blank Lines: Determinants and matrices are spatial arrangements. Thus, a blank line is to be left above and below each array. If the arrangement begins at the top of a braille page, it may begin on line 1 provided no running head is in use; if the arrangement ends at the bottom of a braille page, it may end on line 25. In either case, the rightmost symbol of the arrangement must not fall within three cells of the page number. (See **10.31** in Lesson 10 for details regarding layout at the top or bottom of a page.)

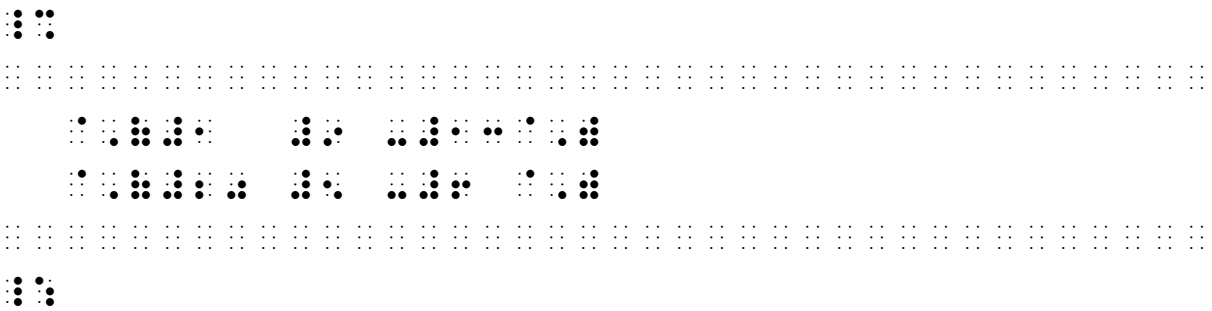
16.13.2 Grouping Symbols: For an array consisting of two or more rows, enlarged grouping symbols are used on each braille line. Grouping symbols are vertically aligned.

16.13.3 Placement of Items: Each entry is moved as far left as possible in its column. Consequently, each opening sign of grouping will be in direct contact with the first entry of each row in the array. Centering and other forms of alignment are not permitted. One column of blank cells is left between the columns of the arrangement. That is, one blank cell separates the widest entry in a column from the beginning of the next column. At least one closing sign of grouping must be in direct contact with an entry in the array. The widest entry in the last column determines the placement of the right (closing) enlarged grouping signs.

16.13.4 Numeric and Letter Indicators: The numeric indicator is used with numeric entries in an array, even when such entries are in direct contact with an opening grouping sign.

Example 16.13-1 $\begin{bmatrix} 1 & 9 & -13 \\ 20 & 5 & -6 \end{bmatrix}$

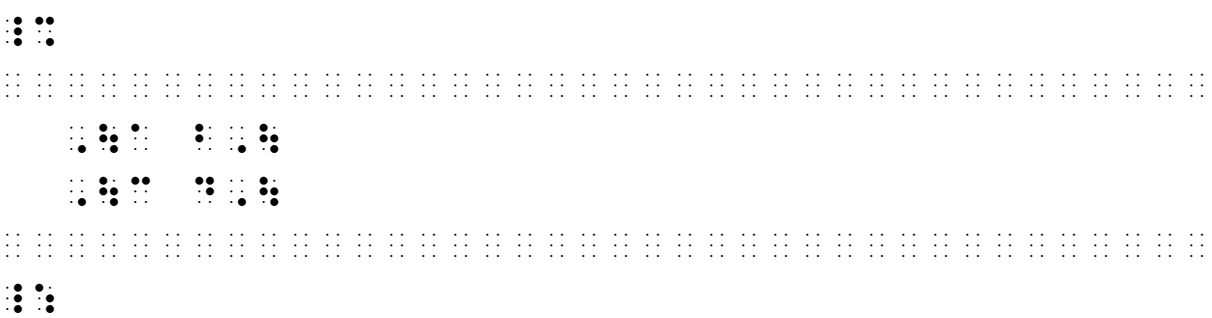
Print observation: The numerical entries are centered in their columns.



Braille observations: The numeric indicator is used for each number. Items begin in the same cell in each column (entries are left-aligned within each column). One blank cell separates the widest entry in a column from the beginning of the next column. The widest entry in the last column determines the placement of the right (closing) enlarged grouping signs. Enlarged brackets enclose this two-row array.

The English letter indicator is not used with any letter or combination of letters in an array.

Example 16.13-2 $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$



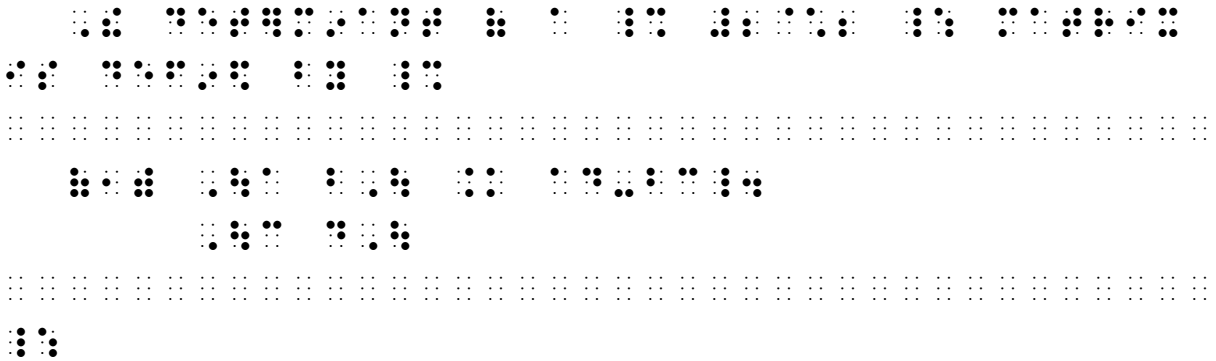
Each English letter is brailled without a letter indicator. Enlarged vertical bars enclose this two-row array.

16.13.5 Placement of Identifiers, Symbols, and Punctuation: Material outside of an array (such as identifiers, punctuation, signs of operation, or signs of comparison) are placed on the top line of the arrangement even though the material may be centered in print.

Example 16.13-3 The determinant of a 2×2 matrix is defined by

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc. \qquad (1)$$

Print observations: An equals sign and expression are centered to the array. A period ends the statement. The expression is identified to the right with a numeral in parentheses.

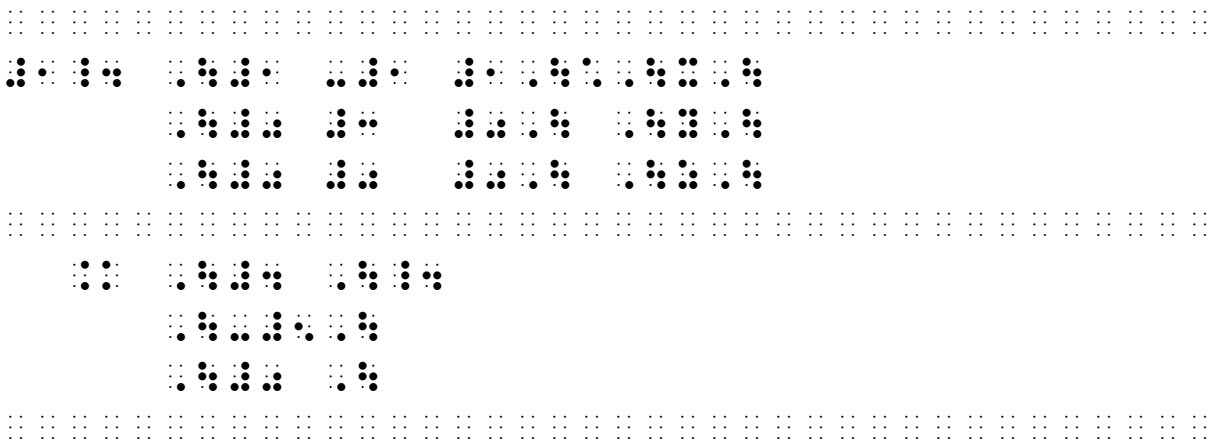


*Braille observations: The material outside of the determinant is placed on the top line of the arrangement. The identifier is moved to the left of the displayed expression according to the rules of the Nemeth Code. (See **Lesson 11,** "Displayed Material with Labels".)*

Example 16.13-4

$$1. \quad \begin{vmatrix} 1 & -1 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{vmatrix} \cdot \begin{vmatrix} x \\ y \\ z \end{vmatrix} = \begin{vmatrix} 4 \\ -5 \\ 0 \end{vmatrix} .$$

Print observations: An item number (1.) is centered to the 3×3 *array. Also centered within the arrangement are a multiplication dot, an equals sign, and a period.*



Braille observations: The material outside of the array is placed on the top line of the arrangement. The multiplication dot is unspaced from the items being multiplied, following spacing rules for operation signs. The entire expression will not fit across the braille page so it is divided before the comparison sign as dictated by the rules of the Nemeth Code.

PRACTICE 16F

1. $\begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix}$

2. $\begin{pmatrix} 1 & -\frac{4}{3} & \frac{5}{3} \\ 2 & 5 & 12 \end{pmatrix}$

3. $\begin{bmatrix} a & b & c \\ 0 & 0 & 0 \end{bmatrix}$

4. $\begin{vmatrix} ab & cd \\ ac & ce \end{vmatrix}$

5. Explain why points $(a_1,\ b_1), (a_2,\ b_2),$ and $(a_3,\ b_3)$ are collinear if and only if

$$\begin{vmatrix} a_1 & b_1 & 1 \\ a_2 & b_2 & 1 \\ a_3 & b_3 & 1 \end{vmatrix} = 0$$

6. The unit vectors in the direction of the *x*, *y*, and *z* axes of a three dimensional Cartesian coordinate system are

$$\hat{\mathbf{i}} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},\ \hat{\mathbf{j}} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},\ \text{and}\ \hat{\mathbf{k}} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Further Considerations with Determinants and Matrices

In order to save space, code switch indicators are omitted from further examples unless both UEB and Nemeth Code are in use. Assume there is a blank line before and after each arrangement.

16.14 Multiplying Arrays: Recall that a multiplication problem can be printed without a multiplication symbol. When each factor is enclosed in grouping signs, it is understood that the side-by-side factors are to be multiplied. For example, $3 \cdot 2$ and $(3)(2)$ both mean "three times two". Similarly, when arrays are being multiplied, the multiplication symbol is often not printed. This array

$$\begin{vmatrix} 1 & -1 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{vmatrix} \cdot \begin{vmatrix} x \\ y \\ z \end{vmatrix}$$

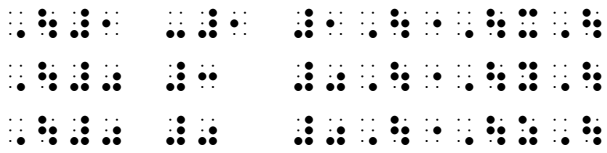
can also be printed without the multiplication dot, like this:

$$\begin{vmatrix} 1 & -1 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{vmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix}$$

The spacing between the arrays shown (in print) is there only to distinguish between the two factors. The space is not brailled. When vertical bars are used, a multipurpose indicator is brailled between the two symbols, otherwise it is misread as a double vertical bar.

Example 16.14-1

$$\begin{vmatrix} 1 & -1 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{vmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix}$$



The multipurpose indicator is brailled between each side-by-side enlarged vertical bar, on every line of the arrangement.

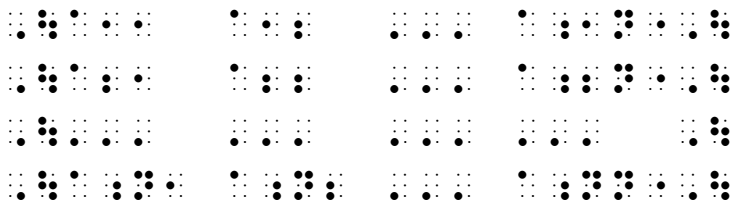
16.15 Omission Dots: When dots are printed in an array to indicate omission of one or more rows or columns, a series of dot 3s is used in braille according to the following observations.

16.15.1 No Dots Are Printed Between Columns: When at least one dot is printed in each column and no dots are printed between columns, each omission is brailled as a 3-cell Nemeth Code ellipsis beginning in the leftmost cell of its column.

Example 16.15-1

$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdot & \cdot & \cdots & \cdot \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

Print observations: Omissions reading down some columns are printed as one dot; omissions reading across some rows are printed as three dots.



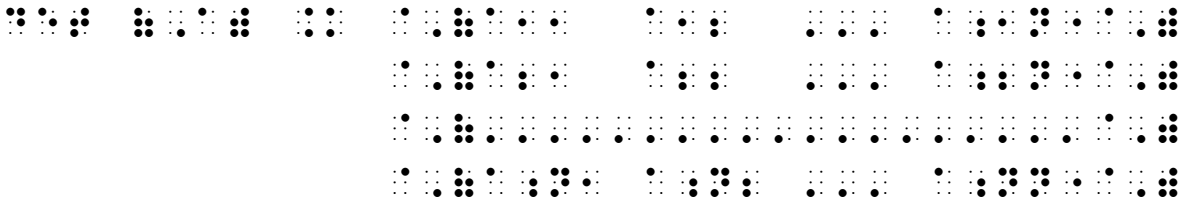
Braille observation: A baseline indicator is needed to assure the closing grouping signs on lines 1, 2, and 4 are on the same level as the opening grouping signs.

16.15.2 Dots Are Printed Between Columns or Some Entries are Blank: Here is the same problem, printed in another style where dots are strung completely across the omitted row (row 3). Notice that the dots occupy space between the columns. A row of omission dots is brailled across the full width of the array.

Example 16.15-2

$$\det(A) = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

Print observation: The third row is printed as a series of dots across the width of the array.



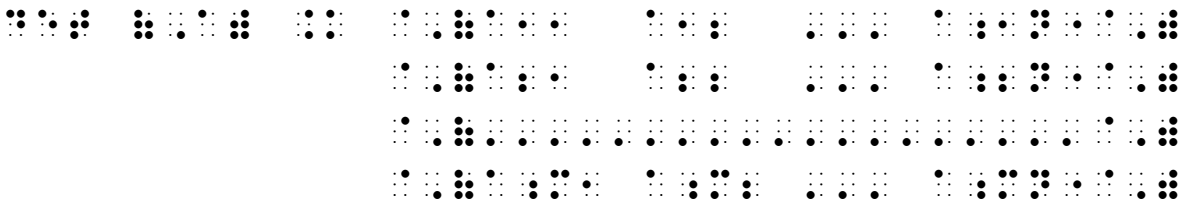
Braille observations: The row of dots is brailled as a series of unspaced dot 3s, beginning in the first cell of the first column and extending to the end of the longest entry in the last column. On lines 1, 2, and 4 a baseline indicator is needed to assure each closing grouping sign is on the same level as the opening grouping sign.

Another print style shows the missing items in the outer columns as single dots, and the missing items in columns 2 and 3 as blank space.

Example 16.15-3

$$\det(A) = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{vmatrix}$$

In braille, the same method shown in the previous example is used for this print style. Only one omission row is brailled across the full width of the array.



PRACTICE 16G

In the next equation, matrix Y is expressed as the product of matrices B and X.

$$\begin{vmatrix} y_1 \\ y_2 \\ y_3 \\ . \\ y_r \end{vmatrix} = \begin{vmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ b_{31} & b_{32} & \dots & b_{3n} \\ . & . & . & . \\ b_{r1} & b_{r2} & \dots & b_{rn} \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \\ . \\ x_n \end{vmatrix}$$

CHALLENGE PROBLEM #2

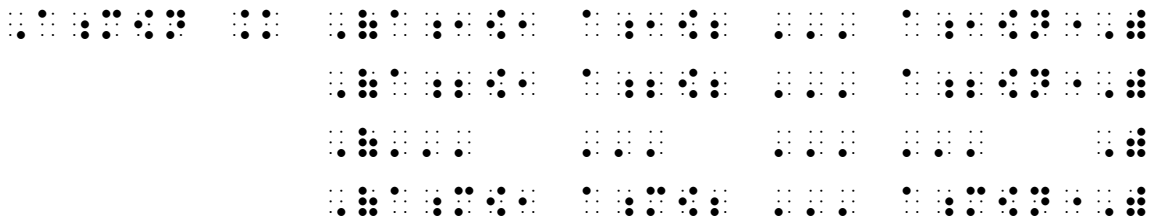
Omissions in an array may also be printed as depicted below.

$$A_{m,n} = \begin{pmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m,1} & a_{m,2} & \cdots & a_{m,n} \end{pmatrix}$$

Print observations: Omissions reading down some columns are printed as three stacked dots (a "vertical" ellipsis); omissions reading across some rows are printed as three dots; one blank entry is printed as an oblique ellipsis.

Which braille rule applies?

As long as at least one dot is printed in each column and no dots are printed between columns, each omission is brailled as a 3-cell Nemeth Code ellipsis beginning in the leftmost cell of its column.



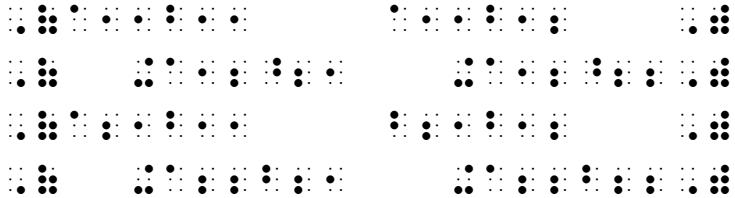
*Review rules regarding the contracted comma in **Lesson 8**.*

16.16 Space-Saving Techniques: Sometimes space-saving techniques are required in order to confine the arrangement to one braille page.

16.16.1 Runovers With Indentation: Entries may be run over to a new line. Each runover is indented two cells from the first cell of the first line of the entry. Attempt to apply the priority list regarding division of long expressions, but those rules can be disregarded if there is no other way around the problem. There is no need to explain the runover format to the reader.

Example 16.16-1

$$\begin{pmatrix} a_{11}b_{11}+a_{12}b_{21} & a_{11}b_{12}+a_{12}b_{22} \\ a_{21}b_{11}+a_{22}b_{21} & b_{21}b_{12}+a_{22}b_{22} \end{pmatrix}$$

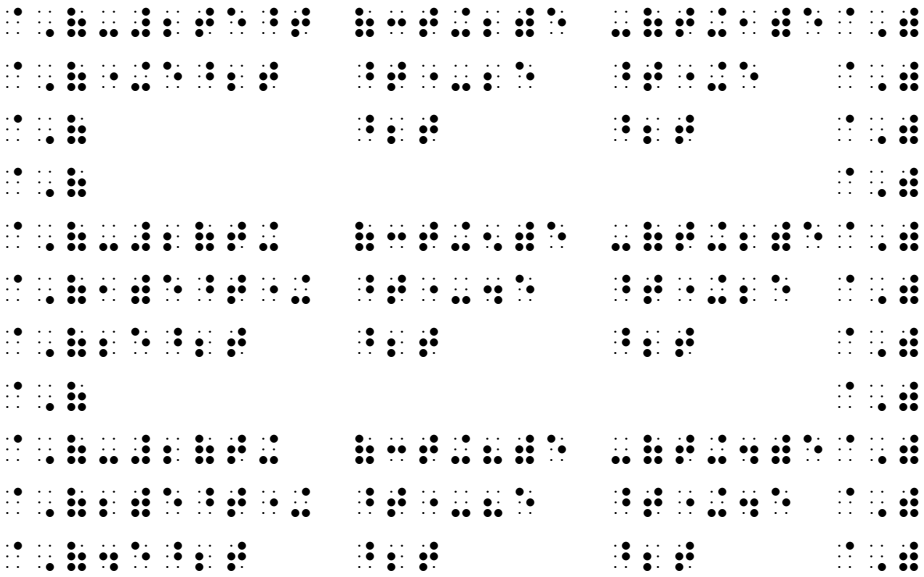


This "runovers with indentation" method is the preferred space-saving technique.

16.16.2 Runovers Without Indentation: If the technique above is not feasible, entries may be run over to new lines without indentation. Preference rules for runovers of mathematical expressions need not be observed if space would be saved. In order to distinguish each row, a blank line is inserted between them. Enlarged grouping symbols are brailled on those blank lines within the arrangement.

Example 16.16-2

$$\begin{bmatrix} -2te^{t}+e^{2t} & (3t+2)e^{t}-2e^{2t} & -(t+1)e^{t}+e^{2t} \\ -2(t+1)e^{t}+2e^{2t} & (3t+5)e^{t}-4e^{2t} & -(t+2)e^{t}+2e^{2t} \\ -2(t+2)e^{t}+4e^{2t} & (3t+8)e^{t}-8e^{2t} & -(t+4)e^{t}+4e^{2t} \end{bmatrix}$$

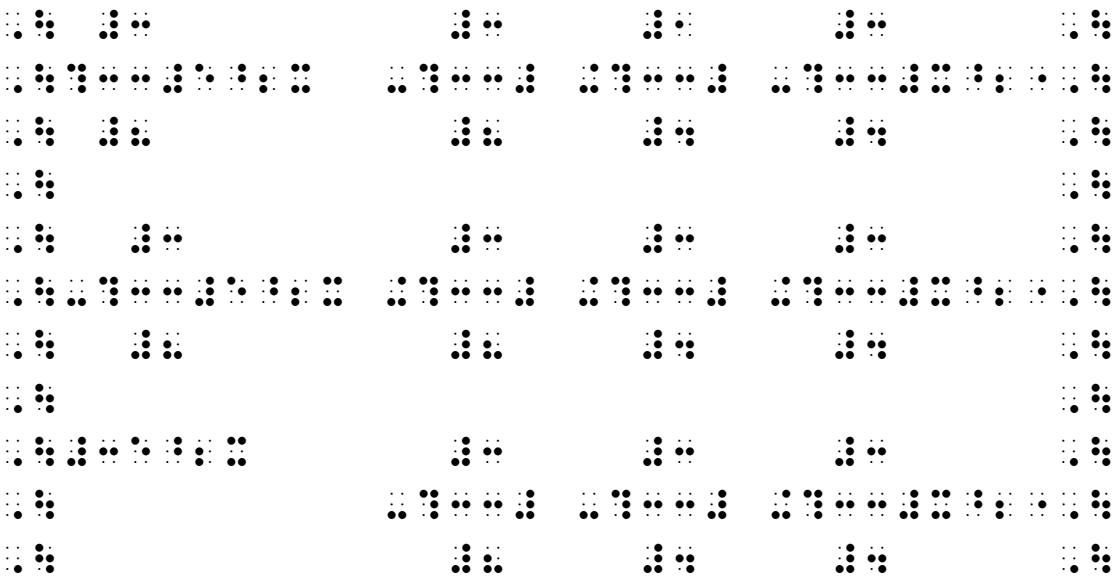


16.16.3 Fractions in Arrays: Fractions may be shown spatially if linear fractions take up too much space. Observe the following details.

- A line is skipped between the rows containing the spatial fraction. Enlarged grouping symbols are brailled on those blank lines within the arrangement.
- Operation signs and variables are placed on the same line as the spatial fraction line.
- Each entry is moved as far up as possible in its row. This includes entries that are not fractions.
- If the last item in a row is a superscript, or a subscript that uses a subscript indicator, a return to the baseline must occur before the closing grouping symbol is brailled. If the row extends fully to the closing grouping symbol, a baseline indicator is required to return to the baseline. If the row does not extend to the closing grouping symbol, the space returns the reader to the baseline.

Example 16.16-3

$$\begin{vmatrix} \frac{3}{8}e^{2x} & -\frac{3}{8} & +\frac{1}{4} & -\frac{3}{4}x^2 \\ -\frac{3}{8}e^{2x} & +\frac{3}{8} & +\frac{3}{4} & +\frac{3}{4}x^2 \\ 3\,e^{2x} & -\frac{3}{8} & -\frac{3}{4} & +\frac{3}{4}x^2 \end{vmatrix}$$



The single-line entry in row 3, column 1, is brailled on the first line of the row even though it is centered in its row in print.

16.16.4 Keying: When the array will not fit on the page even after trying the methods described above, a key may be devised. The technique of keying is discussed in the next lesson.

Instructions: In the first array, use spatial fractions. In the second array, use runovers with indentation. Reminder: The partial derivative symbol ∂ was introduced in **Lesson 13**.

(*Note:* The image appears dark but it is not printed in boldface.)

PRACTICE 16H

The derivative of T can be expressed as follows.

$$dT = \begin{bmatrix} \frac{\partial y_1}{\partial x_1} & \frac{\partial y_1}{\partial x_2} & \cdots & \frac{\partial y_1}{\partial x_n} \\ \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot & \cdot\ \cdot\ \cdot & \cdot\ \cdot \\ \frac{\partial y_m}{\partial x_1} & \frac{\partial y_m}{\partial x_2} & \cdots & \frac{\partial y_m}{\partial x_n} \end{bmatrix}$$

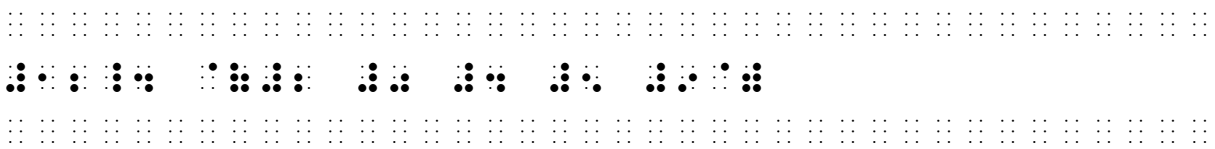
The Jacobian determinant of the mapping is shown below.

$$J(\rho, \theta, \phi) = \begin{vmatrix} \cos\theta \sin\phi & \sin\theta \sin\phi & \cos\phi \\ -\rho \sin\theta \sin\phi & \rho \cos\theta \sin\phi & 0 \\ \rho \cos\theta \cos\phi & \rho \sin\theta \cos\phi & -\rho \sin\phi \end{vmatrix}$$

$$= -\rho^2 \sin\phi$$

16.17 Row Matrix: A row matrix has only one row. Because it is a matrix, it is brailled as a spatial arrangement—that is, a line is left above and below the arrangement. Regular grouping symbols are used, not enlarged symbols, despite the larger appearance of the grouping symbols in print. Here is an example of a 1×5 row matrix.

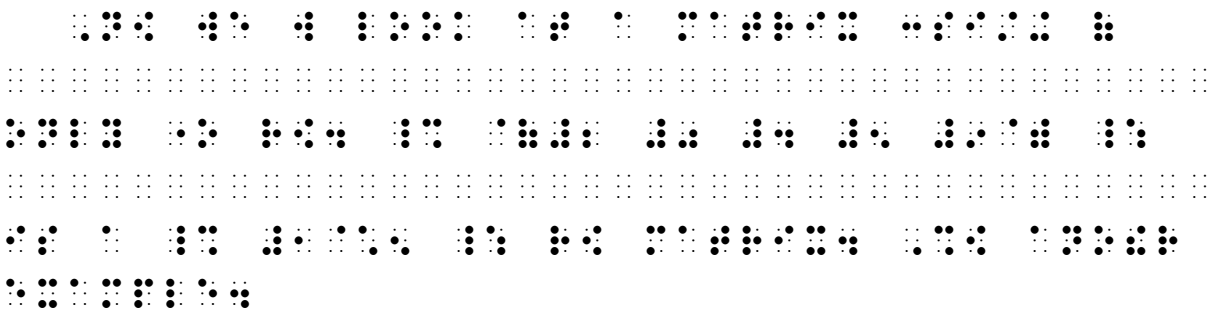
Example 16.17-1 12. $\begin{bmatrix} 2 & 0 & 4 & 5 & 9 \end{bmatrix}$



Observation: This is not an "enclosed list" because there are no commas between items. Context will make it clear that this is a row matrix.

16.18 Embedded Arrays: If the array is embedded within narrative text, the required blank line is inserted on the line before and the line after the line where the arrangement lies. Text is placed on the same line as the array if it fits.

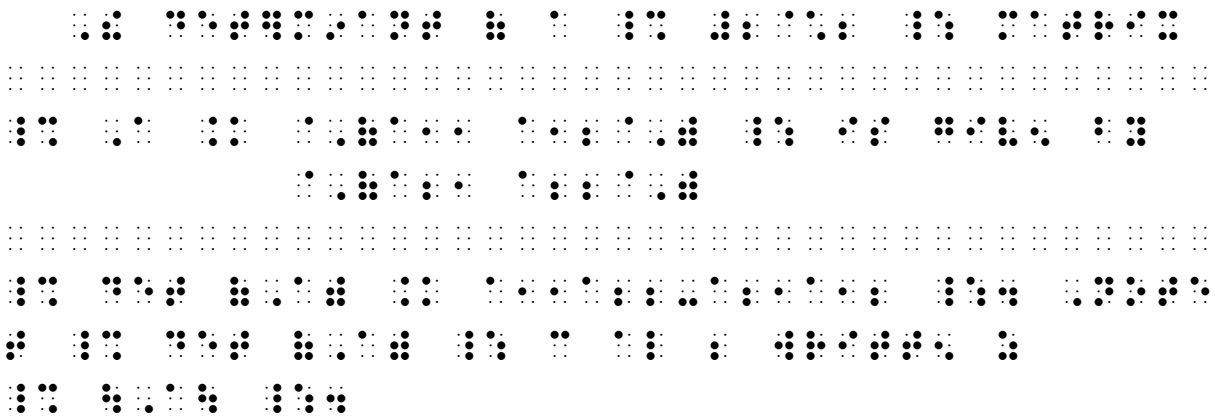
Example 16.18-1 Now we will look at a matrix consisting of only one row. $\begin{bmatrix} 2 & 0 & 4 & 5 & 9 \end{bmatrix}$ is a 1×5 row matrix. Show another example.



If the embedded array consists of more than one line, the surrounding text is placed on the top line of the arrangement only – before and/or after the top row of the array, as it fits in context of the narrative. Narrative continues in the runover cell of the paragraph. The opening Nemeth Code indicator and the Nemeth Code terminator are placed only on the top line of the embedded arrangement. The code switch indicators apply to the whole arrangement.

Example 16.18-2 The determinant of a 2×2 matrix $\mathrm{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ is given by $\det(A) = a_{11}a_{22} - a_{21}a_{12}$. Note that $\det(A)$ can also be written as $|A|$.

Print observation: The two rows of the embedded matrix are vertically centered to the equals sign in print.



*Braille observations: The first row of the embedded matrix is brailled on the same line as "*A *=". The second row is aligned beneath the first row; no text is brailled on that line.*

Note that "det" is a function abbreviation and therefore is followed by a space in the transcription.

PRACTICE 16I

1) If $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -9 & 2 \\ 5 & 7 & -6 \end{bmatrix}$, find AB.

   Answer: $AB = \begin{bmatrix} 1 & -25 & 10 \\ 29 & 1 & -18 \end{bmatrix}$.

2) Here are three examples of matrix operations.

   a) $\begin{bmatrix} 5 & 4 \end{bmatrix} + \begin{bmatrix} 20 & 30 \end{bmatrix} = \begin{bmatrix} 25 & 34 \end{bmatrix}$

   b) $\begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 1 & 3 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ 1 & 5 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 1 & 7 \\ 1 & 9 \end{bmatrix}$

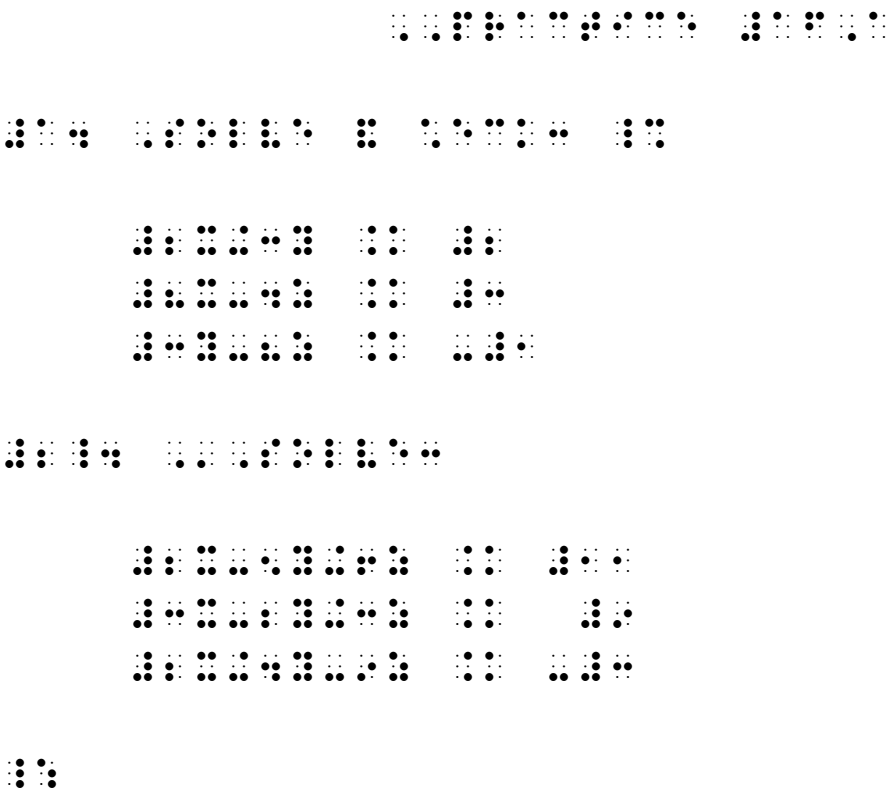
   c) $\begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 2 \end{bmatrix}$

16.19 Use of Tactile Graphics for Enlarged or Horizontal Grouping Signs: Enlarged grouping symbols may be drawn in place of the braille equivalents, especially when space saving is a factor or when a horizontal grouping sign is required. The horizontal brace shown below is best presented as a tactile graphic. Refer to BANA's *Guidelines and Standards for Tactile Graphics* for drawing techniques.

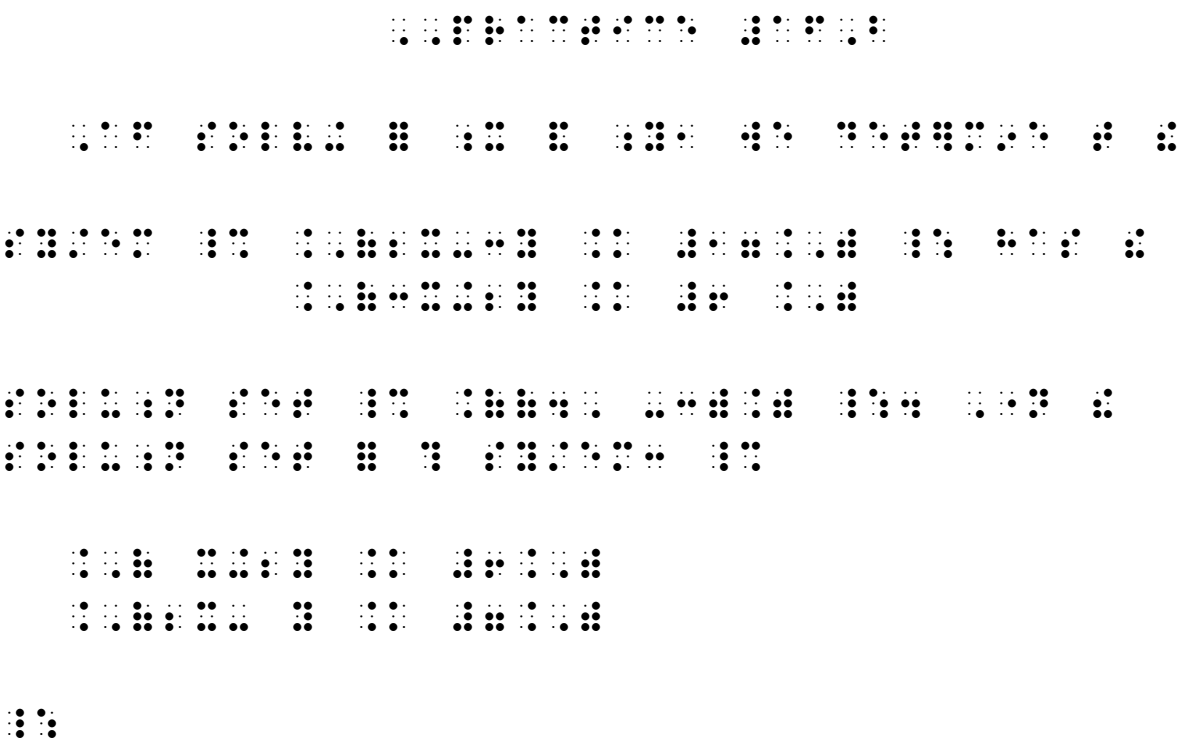
$$(2x+3)(x-5) = 2x^2 - 10x + 3x - 15$$
$$= 2x^2 \underbrace{\quad -7x \quad} - 15$$

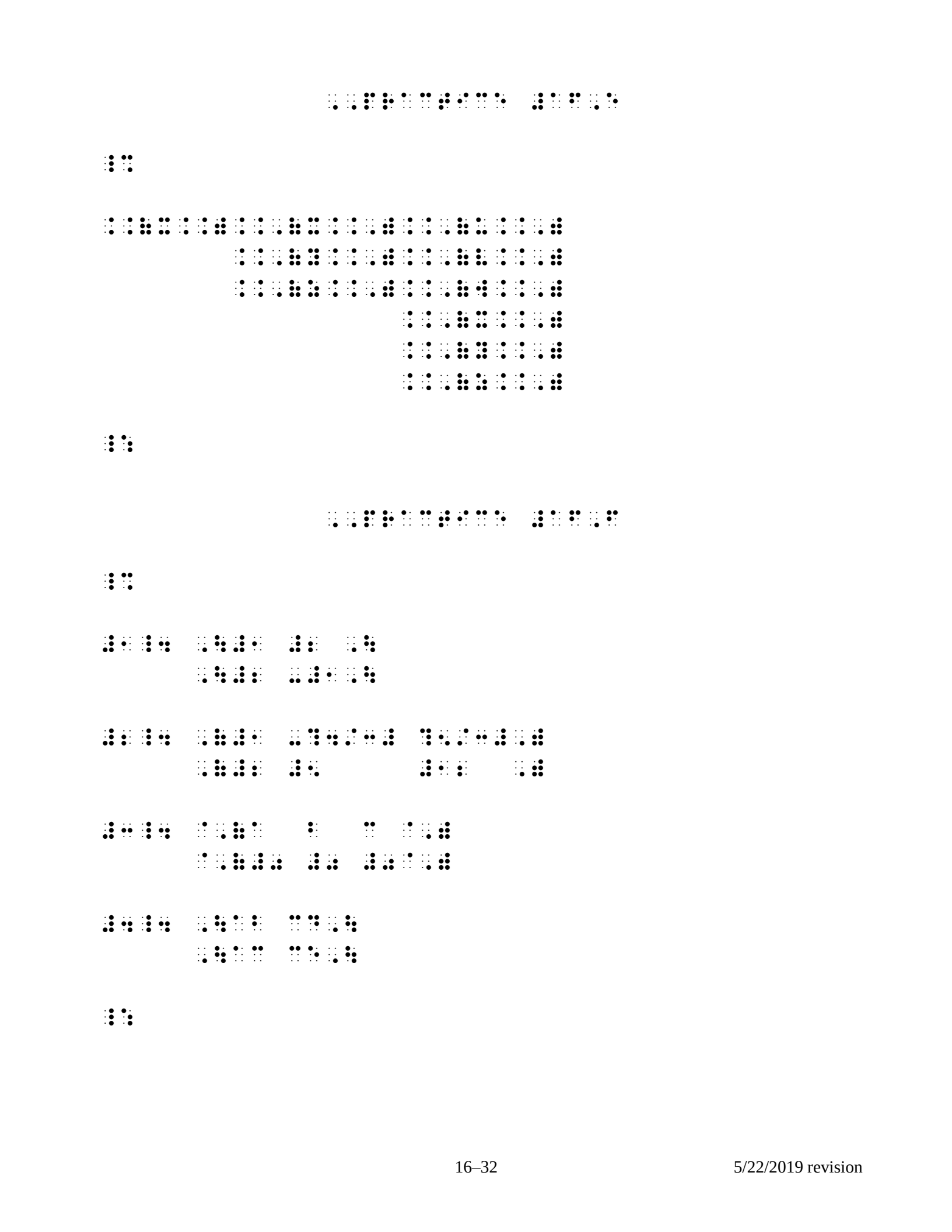
For further practice, see Appendix A—Reading Practice.

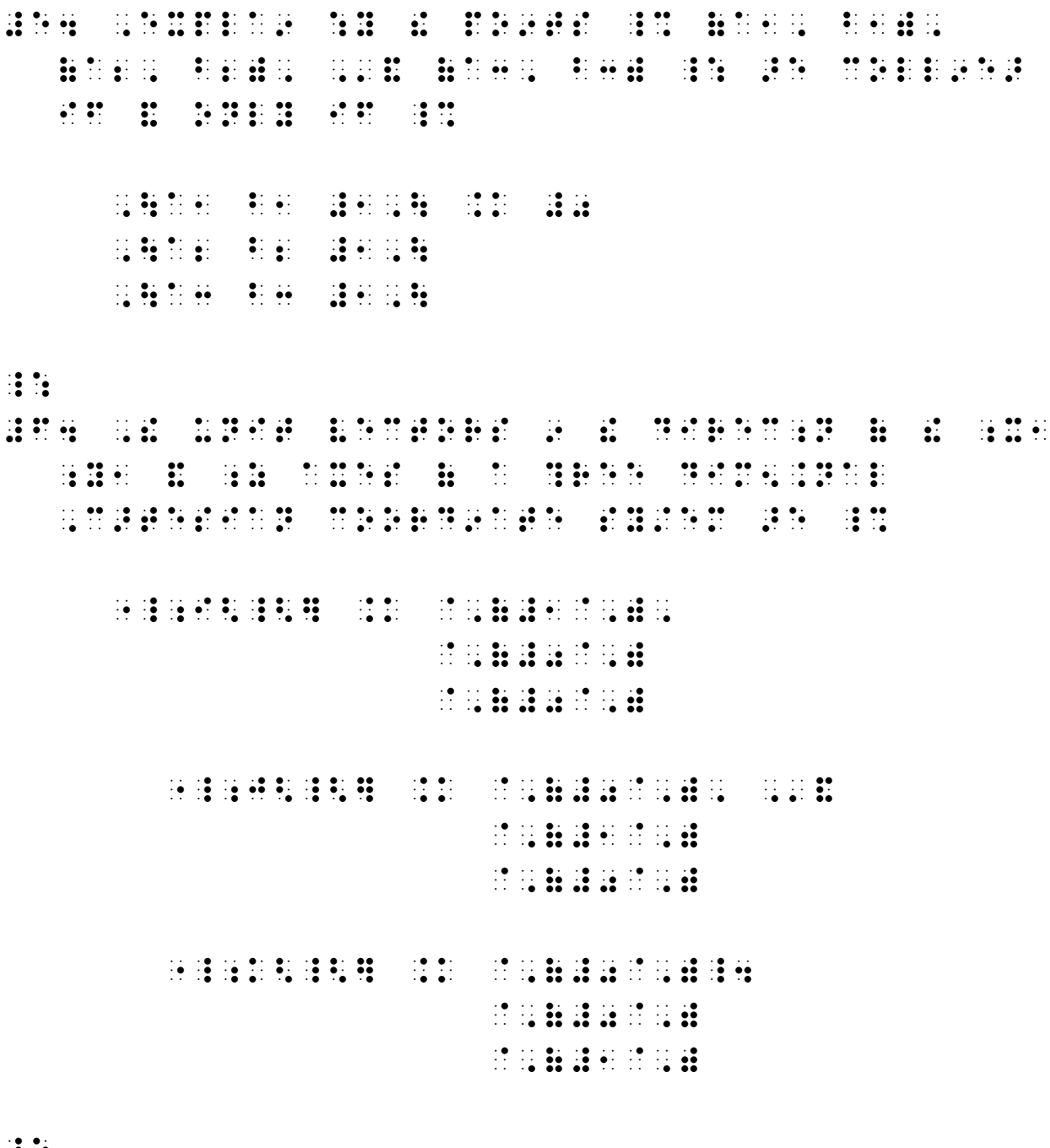
ANSWERS TO PRACTICE MATERIAL

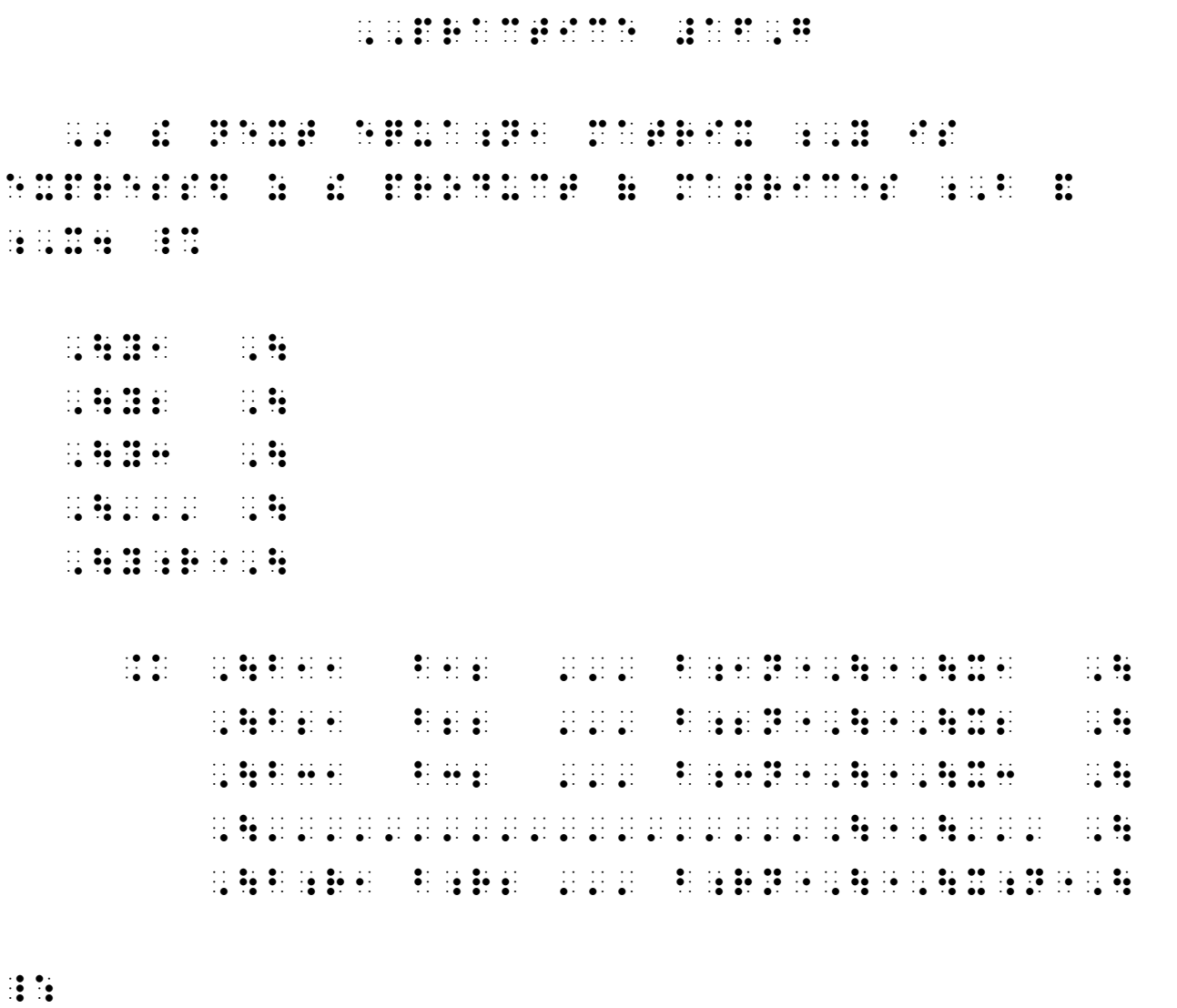


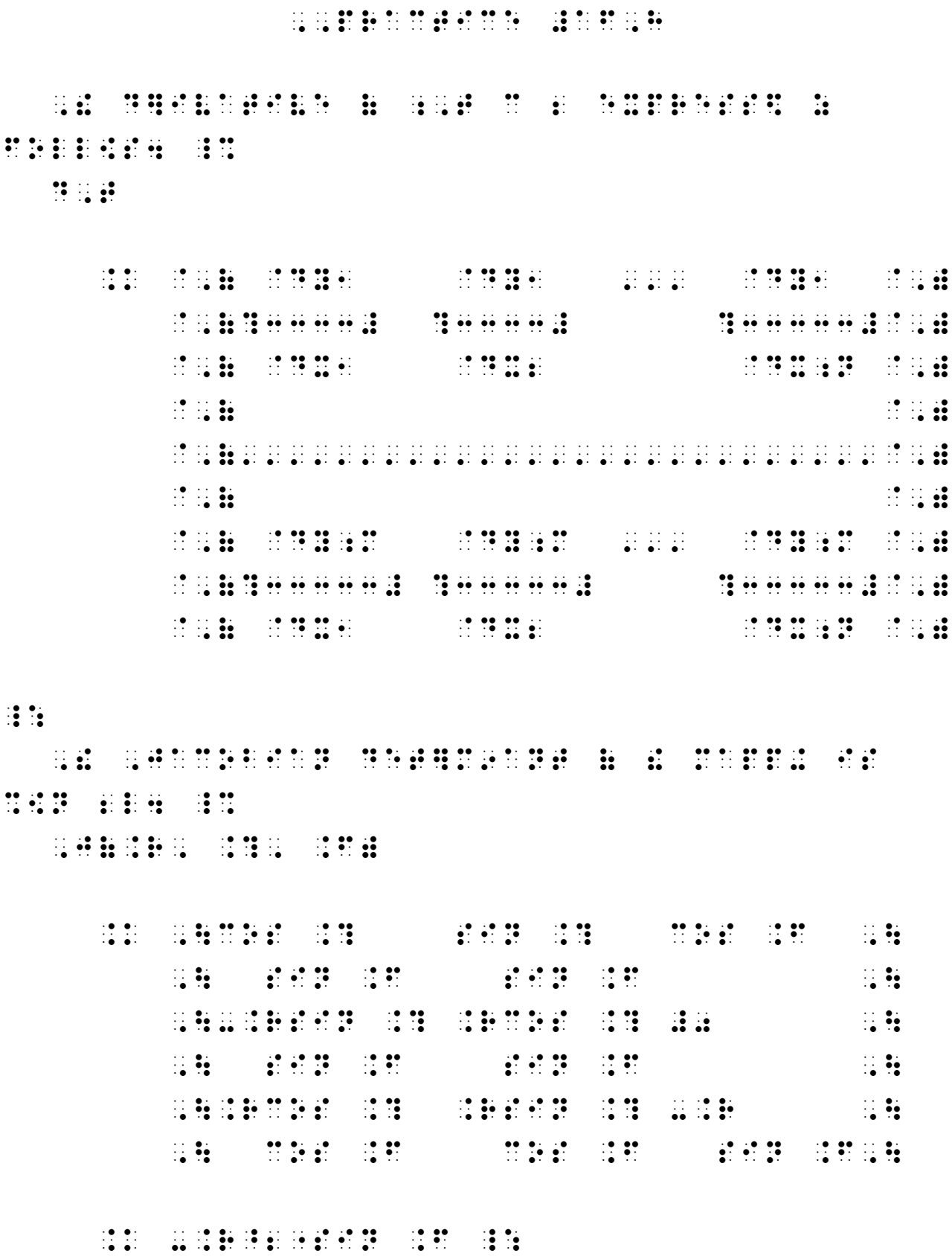
The margin for material displayed to itemized text is cell 5, even if the itemized text has no runovers.



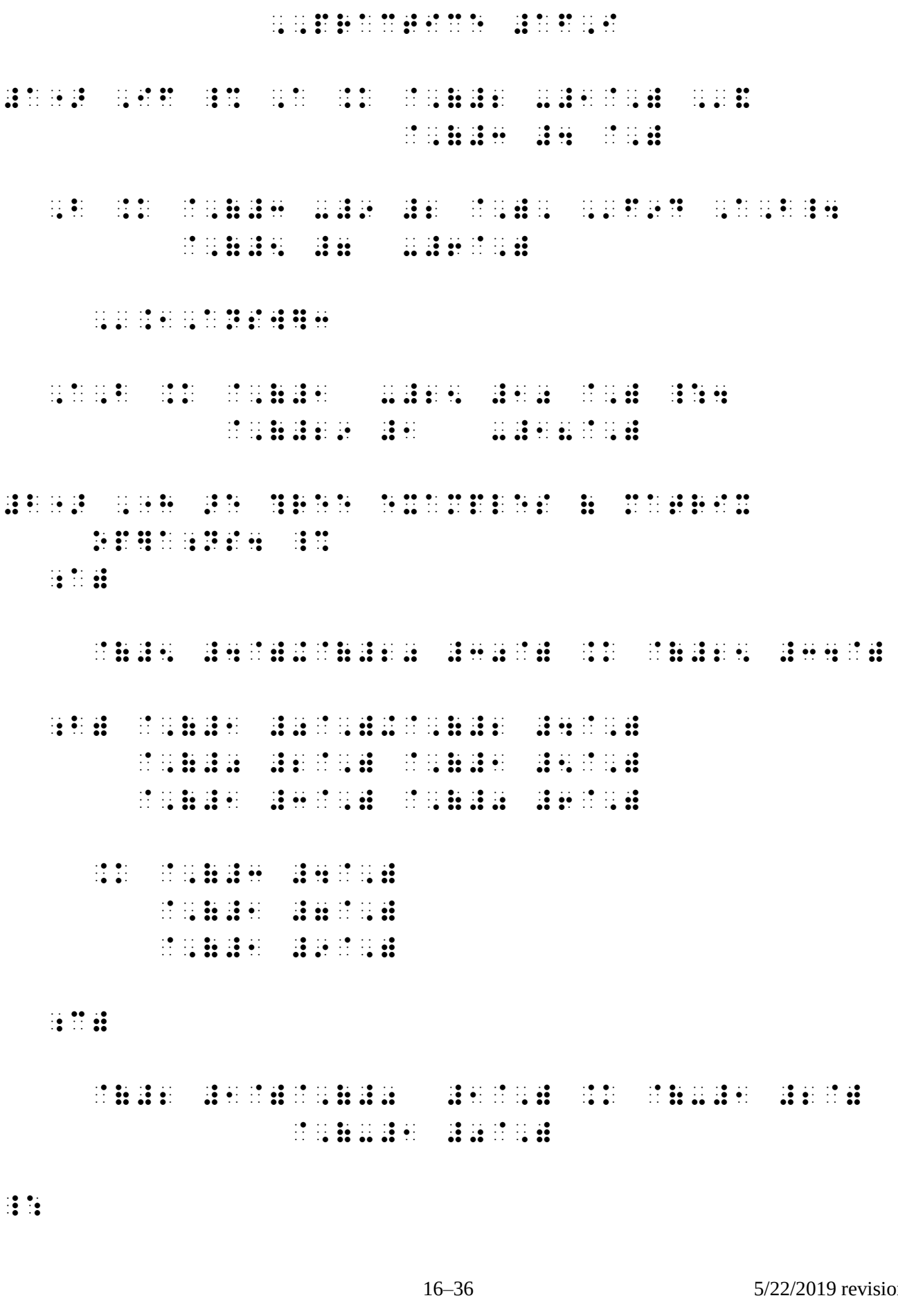








Note that blank lines are placed before and after the spatial portions of the problems only.



EXERCISE 16

Exercise 16 will be available when this course is finished being written and is no longer "Provisional".

Proceed to Lesson 17.

LESSON 17

Read about this PROVISIONAL EDITION in the front matter to this book.
Check the NFB website periodically for updates to this lesson.

- *TABLES*
  - *Boxed Tables*
- *FIGURES AND DIAGRAMS*
- *KEYING TECHNIQUE*
- *CHEMISTRY*
- *REFERENCE SYMBOLS*

TABLES

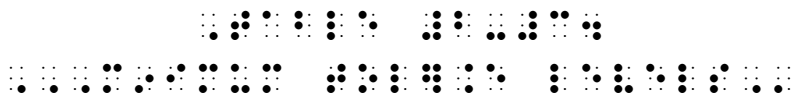
An introduction to table format in technical materials was given in **Lesson 6**. Stem-and-leaf plots were studied in **Lesson** 15. Further rules which apply to technical tables are illustrated in this lesson.

17.1 Structure of Tables: Tables consisting entirely of words are transcribed in UEB. When mathematical data occur in the table, code switching decisions depend upon the content of the entire table and the spacing restrictions encountered on the braille page. Details concerning the layout of the columns and rows are provided in *Braille Formats,* including strategies to consider when a table is too wide to fit within the margins of the braille page.

17.2 Table Label and Title: Follow appropriate rules according to the Nemeth Code or UEB in table labels and titles, switching to Nemeth Code only when necessary.

Example 17.2-1 (Table label and title only)

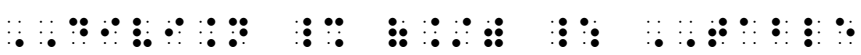
Table 2-3. MINIMUM TOLERANCE LEVELS



The table label and the table title are not mathematical. UEB is used.

Example 17.2-2 (Table title only)

DIVISION (÷) TABLE

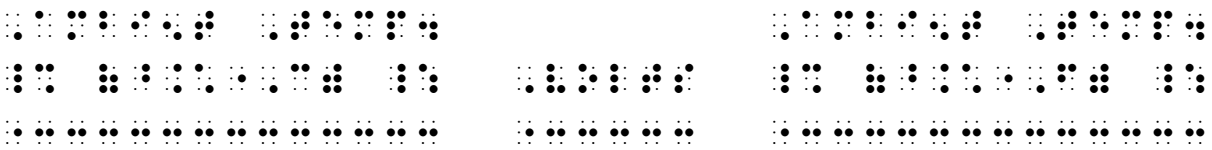


Only the math symbol requires a switch to Nemeth Code.

17.3 Column Headings: Follow appropriate rules according to the Nemeth Code or UEB for column headings, switching to Nemeth Code only when necessary.

Example 17.3-1 (Column headings only)

Ambient Temperature (°C)	Volts	Ambient Temperature (°F)



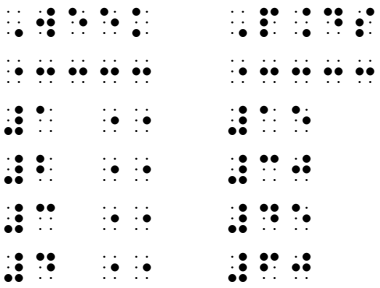
Only the degree designations are mathematical. (Column headings may be abbreviated in order to fit in the limited space. A transcriber's note is not required when the abbreviation is easily identifiable. See Braille Formats *for further details.)*

17.4 Table Entries: Table entries may not require a switch to Nemeth Code, or they may be entirely mathematical, or they may contain a mixture of UEB and Nemeth Code.

17.4.1 Tables in UEB: When entries do not require a switch to Nemeth Code, the table may be brailled entirely in UEB.

Example 17.4-1

Week	Pounds
1	15
2	30
3	45
4	60

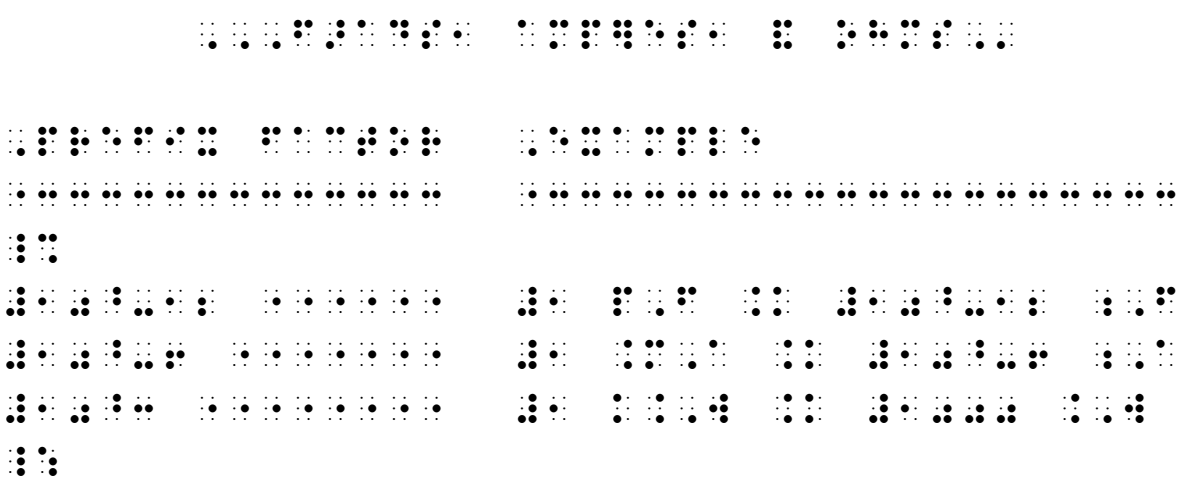


17.4.2 Tables in Nemeth Code: When entries are entirely mathematical, the opening Nemeth Code indicator is placed at the margin (cell 1) of the line following the column separation line. The entries begin on the next line. The Nemeth Code terminator follows the last line of entries, placed at the margin (cell 1).

Example 17.4-2

FARADS, AMPERES, AND OHMS

Prefix factor	Example
10^{-12}	$1\text{pF} = 10^{-12}\text{F}$
10^{-6}	$1\mu\text{A} = 10^{-6}\text{A}$
10^{3}	$1\text{k}\Omega = 1000\Omega$

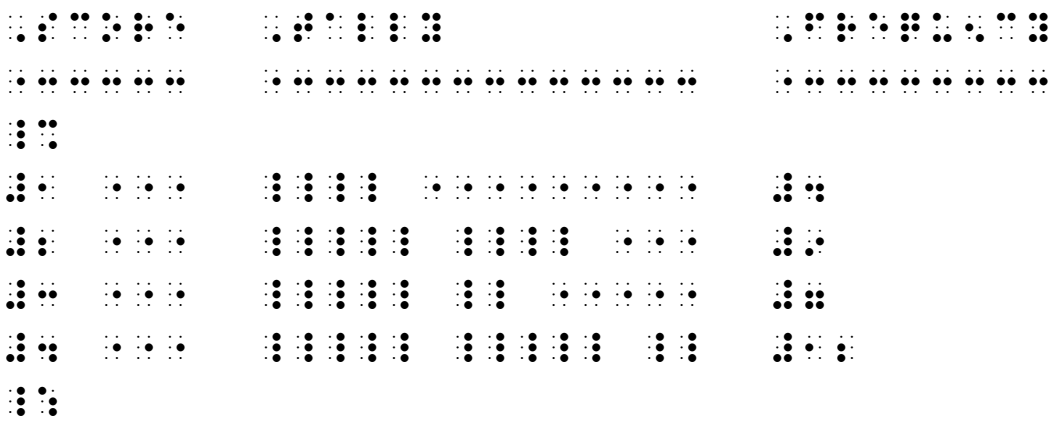


Each abbreviation is spaced according to the rules of the Nemeth Code.

17.4.3 Code Switching Decisions: A table may be more clearly presented by brailling it entirely in Nemeth Code even when some entries do not require a switch.

Example 17.4-3 (All table entries brailled in Nemeth Code)

Score	Tally	Frequency
1	||||	4
2	𝍸 ||||	9
3	𝍸 ||	7
4	𝍸𝍸 ||	12

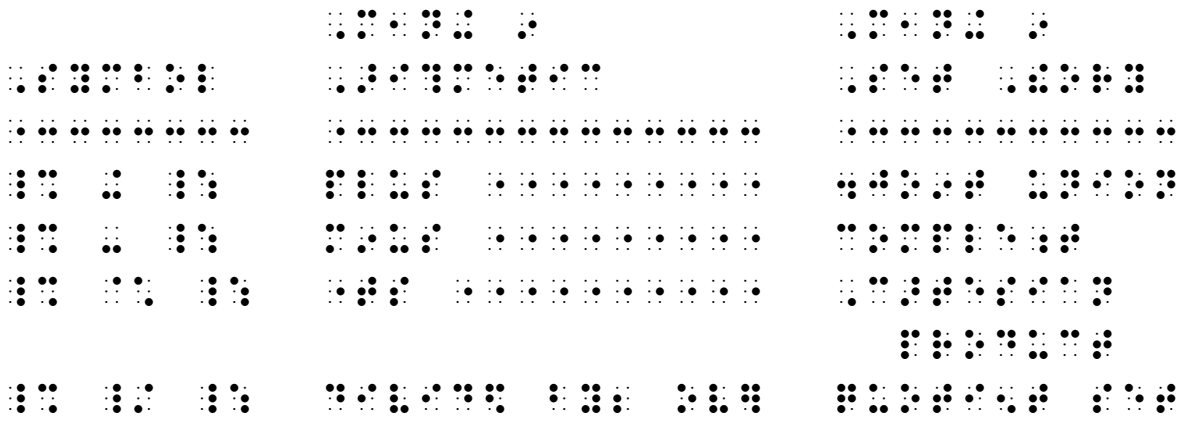


Digits in column three are left-adjusted in the print table; the same alignment is followed in braille.

When a mixture of narrative entries and mathematical data occur in the table, a switch to Nemeth Code may be applied only where needed.

Example 17.4-4 (Table entries brailled in both codes)

Symbol	**Meaning in Arithmetic**	**Meaning in Set Theory**
+	plus	disjoint union
–	minus	complement
×	times	Cartesian product
/	divided by; over	quotient set



Instructions: Do not braille tables side-by-side even though they are printed in this manner. The first table has a table label and a caption–start in cell 7 with runovers in cell 5. The second table has a heading–center the heading. The third table is introduced with narrative text. If the body of the table can be transcribed entirely in UEB, do so.

PRACTICE 17A

Table 17.1-5 Values and iterations of e.

e	e^2	S
1	1	6
2	4	24
3	9	54
4	16	96

RTD TABLE

R	T	D
30	t+2	30(t+2)
45	t	45t

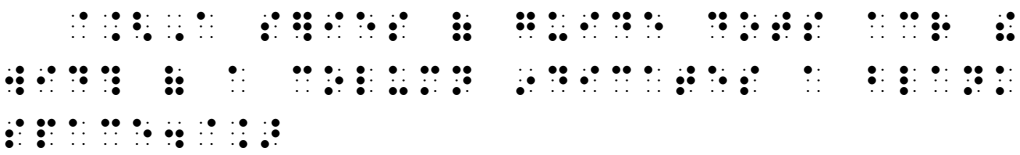
Here is a table of values for $y = x^2 - 3$.

x	*y*
−1	−2
0	−3
1	−2
2	1
3	6

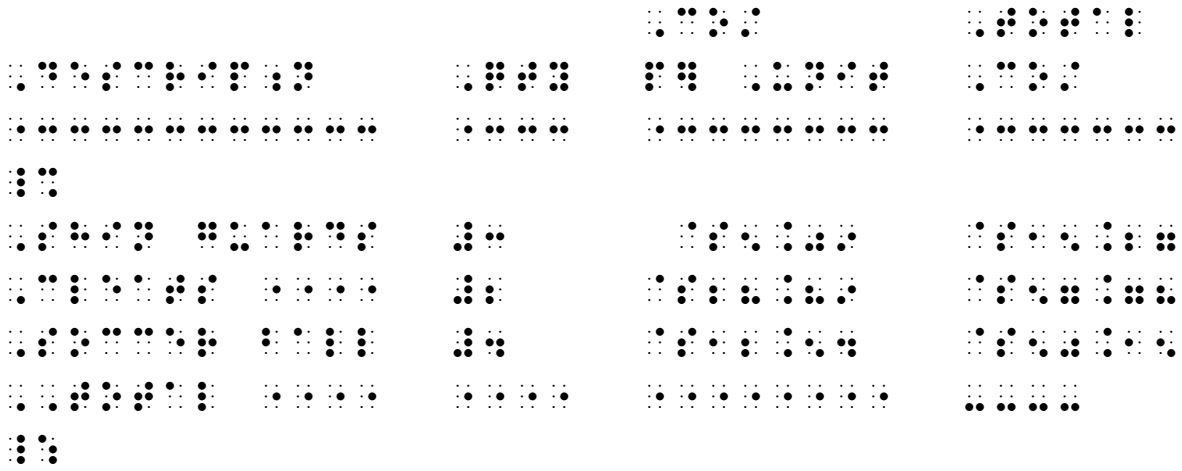
17.5 When Row Headings are Words: When table entries consist of technical material but the row headings are words, to minimize the use of code switch indicators the entire table (excluding the table title and column headings) is considered to be technical material. Words within the table are brailled without contractions. The single-word switch indicator is not used.

Example 17.5-1 (Row headings and all table entries brailled in Nemeth Code)

Description	Qty	Cost per Unit	Total Cost
Shin guards	3	$5.09	$15.27
Cleats	2	$28.89	$57.78
Soccer ball	4	$12.54	$50.15
TOTAL			____



(blank line)



(blank line)

Other rules of note in this table: The transcriber's note regarding blank entries is required. The table is preceded and followed by a blank line. Dollar amounts are aligned by place value in print; the same alignment is followed in braille. The long dash of the Nemeth Code is used for blank entries that are to be filled in.

PRACTICE 17B

Table A.7

Group	Light	5 days	10 days
Control	12	70.3 ± 2	90 ± 10.5
Test	12	60.4 ± 1.5	78 ± 7.9
Control	16	75.7 ± 8	100 ± 3
Test	16	52.2 ± 2	81 ± 6.7

Boxed Tables

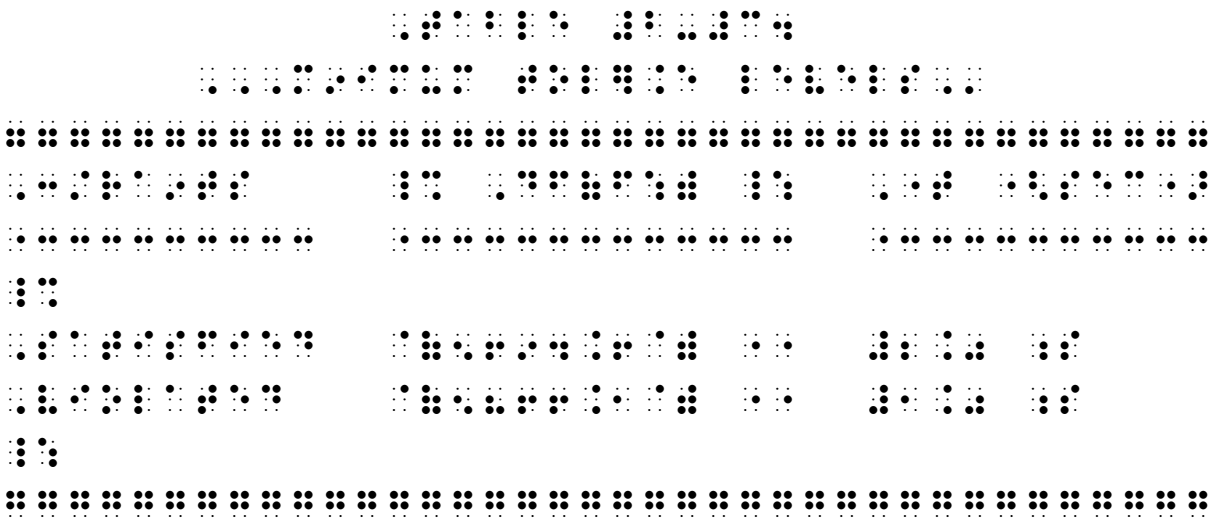
17.6 Code Switching and Box Lines: Box lines may be brailled in either code. Rules regarding box lines are given in *Braille Formats.* Code switching considerations are discussed below.

17.6.1 Switching Within the Table: When only the body of the table is in Nemeth Code, the bottom box line is brailled on the line following the Nemeth Code terminator.

Example 17.6-1

Table 2-3. MINIMUM TOLERANCE LEVELS

Constraints	$Df(\overline{f})$	Time (sec)
Satisfied	[5694.6]	2.0s
Violated	[5866.1]	1.0s

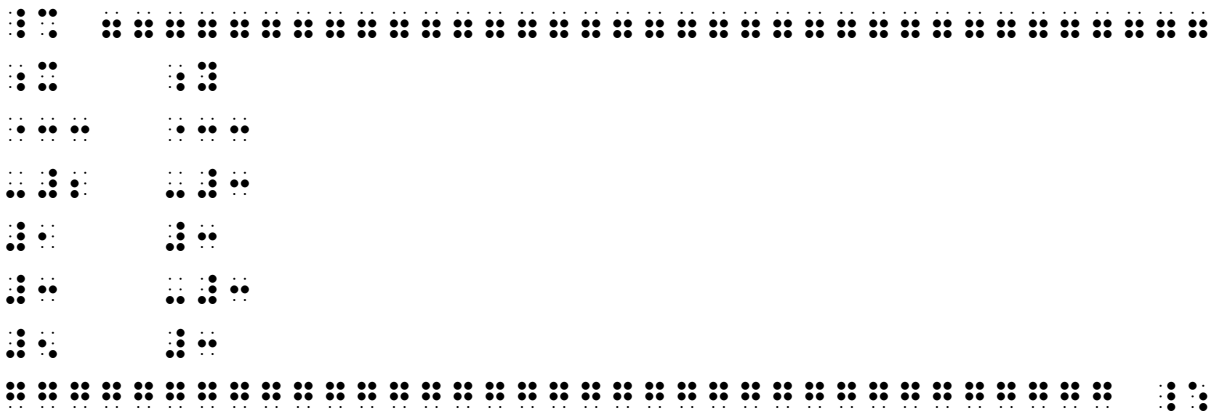


The abbreviation "s" in column 3 is spaced according to the rules of the Nemeth Code

17.6.2 Switching Within the Box Lines: For boxed material that is transcribed completely in Nemeth Code, the opening Nemeth Code indicator is included at the beginning of the top box line, followed by a blank space. The Nemeth Code terminator is included at the end of the bottom box line, preceded by a space.

Example 17.6-2

x	y
–2	–3
1	3
3	–3
5	3



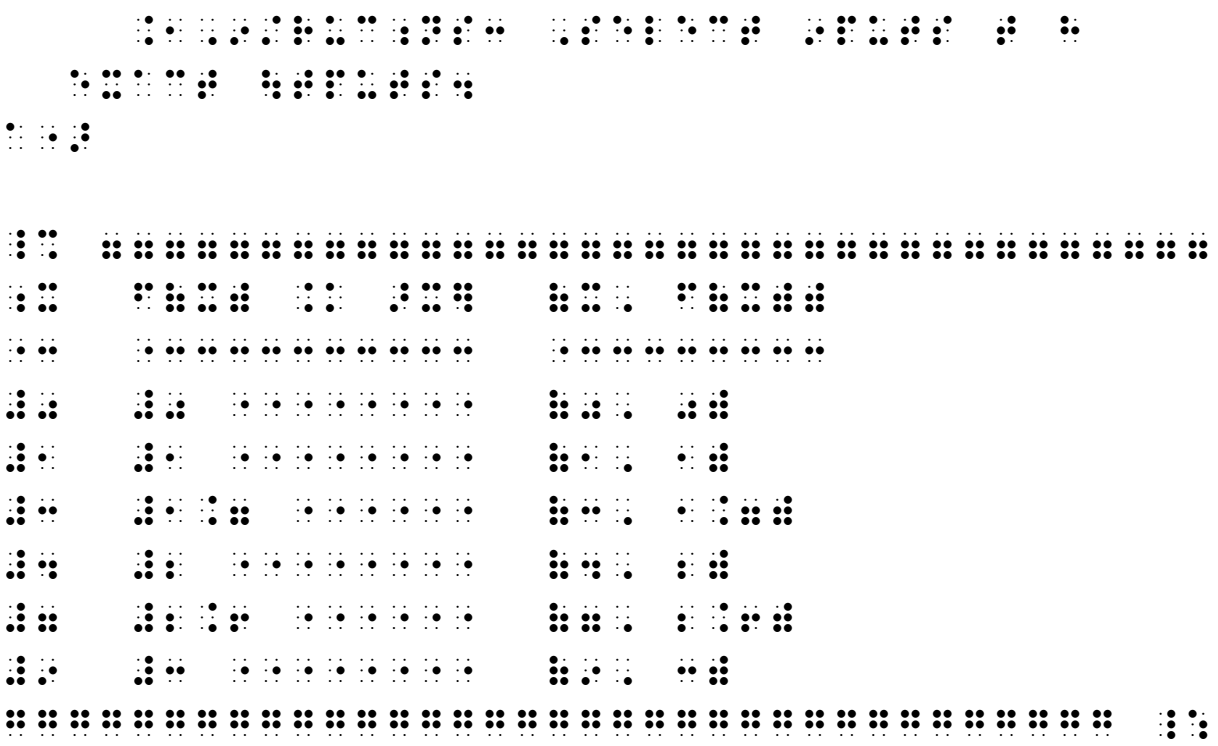
The negative sign dictates use of Nemeth Code in this table of values. Values are left-justified in each column.

Example 17.6-3

Instructions: Select inputs that have exact outputs.

a)

x	$f(x) = \sqrt{x}$	$(x, f(x))$
0	0	(0, 0)
1	1	(1, 1)
3	1.7	(3, 1.7)
4	2	(4, 2)
7	2.6	(7, 2.6)
9	3	(9, 3)

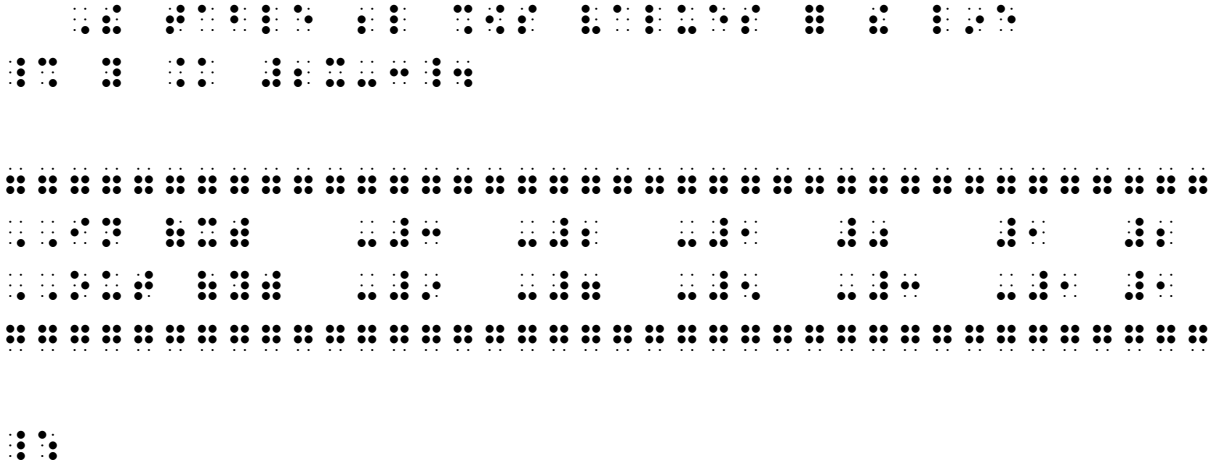


17.6.3 Technical Material Before or After a Box: If technical material immediately precedes or follows the box, the code currently in use continues through the box line.

Example 17.6-4 (Nemeth Code begins in material before the box.)

The table below shows values for the line $y = 2x - 3$.

IN (x)	– 3	– 2	– 1	0	1	2
OUT (y)	– 9	– 7	– 5	– 3	– 1	1



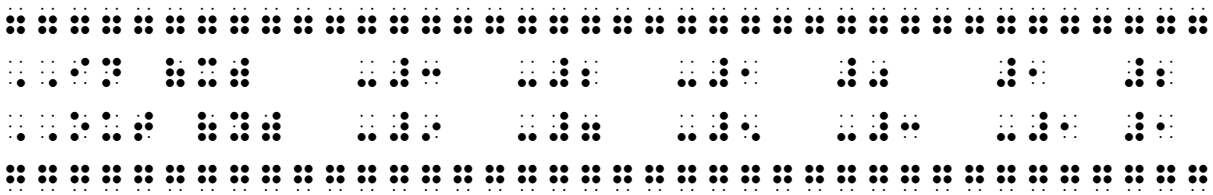
Nemeth Code begins before the technical material preceding the box and is terminated after the completion of the table. Box lines "match"—since the top box line does not incorporate a switch indicator, the Nemeth Code terminator is not part of the bottom box line.

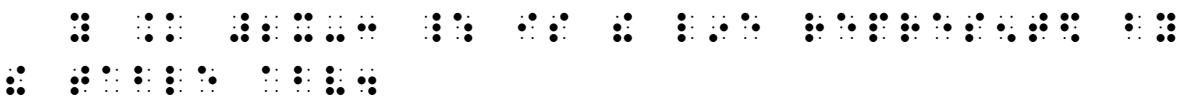
Example 17.6-5 (Nemeth Code continues after the box.)

IN (x)	– 3	– 2	– 1	0	1	2
OUT (y)	– 9	– 7	– 5	– 3	– 1	1

$y = 2x - 3$ is the line represented by the table above.







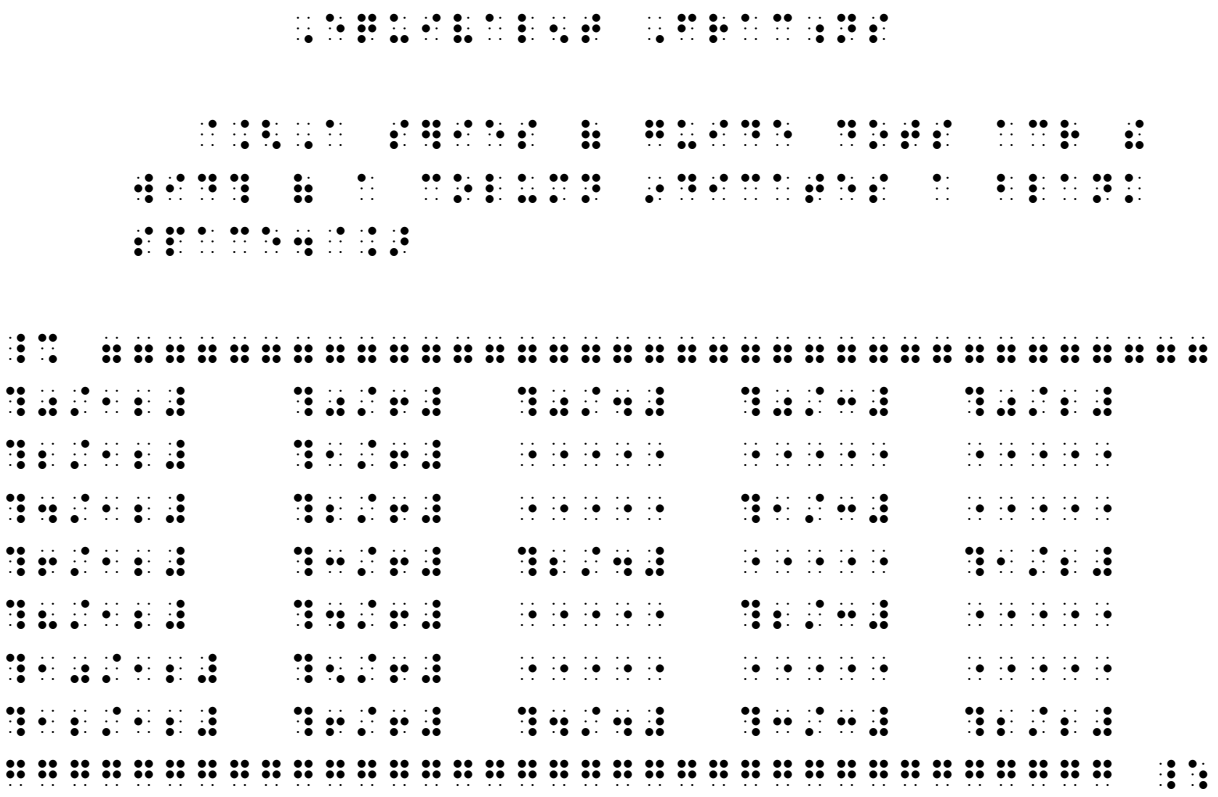
Nemeth Code is terminated after the technical material following the box. Box lines "match"—since the bottom box line does not incorporate a switch indicator, the opening Nemeth Code indicator is not part of the top box line.

17.6.4 Placement of Transcriber's Note: A transcriber's note that may be needed for boxed material that is transcribed completely in Nemeth Code should be transcribed outside the box.

Example 17.6-6

Equivalent Fractions

$\frac{0}{12}$	$\frac{0}{6}$	$\frac{0}{4}$	$\frac{0}{3}$	$\frac{0}{2}$
$\frac{2}{12}$	$\frac{1}{6}$			
$\frac{4}{12}$	$\frac{2}{6}$		$\frac{1}{3}$	
$\frac{6}{12}$	$\frac{3}{6}$	$\frac{2}{4}$		$\frac{1}{2}$
$\frac{8}{12}$	$\frac{4}{6}$		$\frac{2}{3}$	
$\frac{10}{12}$	$\frac{5}{6}$			
$\frac{12}{12}$	$\frac{6}{6}$	$\frac{4}{4}$	$\frac{3}{3}$	$\frac{2}{2}$



PRACTICE 17C

1) Given exponent *x*, compute the value of *y* by completing the tables.

a)

x	$2^x = y$	y
-1	$2^{-1} = y$	?
2	$2^2 = y$	?

b)

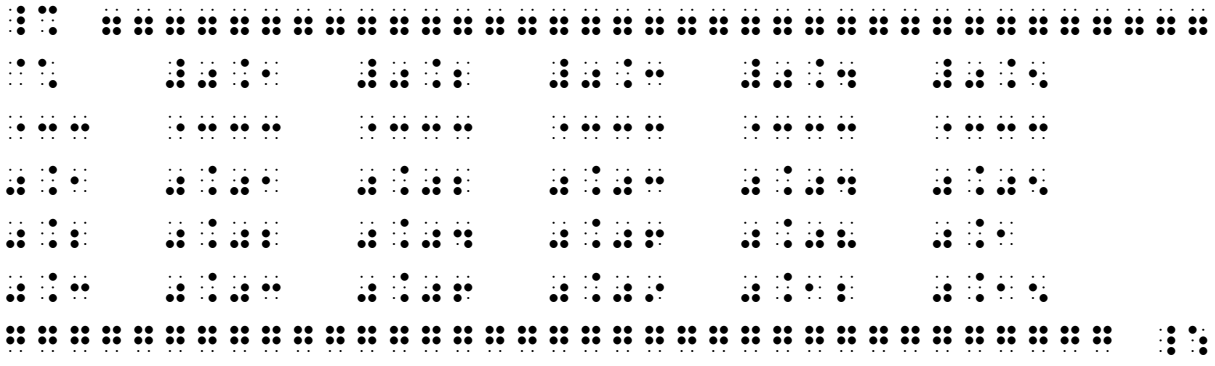
x	$2^{x+1} = y$	y
3	$2^4 = y$	?
6	$2^7 = y$	?

Table Rules Specific to the Nemeth Code

17.7 Table of Numbers: When row headings and entries in a table consist entirely of numerals the numeric indicator is omitted. The numerals may contain interior commas or decimal points but may not contain any other symbol. This rule applies only to the body of a table and not to the column headings. A transcriber's note is not required to explain the omitted numeric indicator.

Example 17.7-1

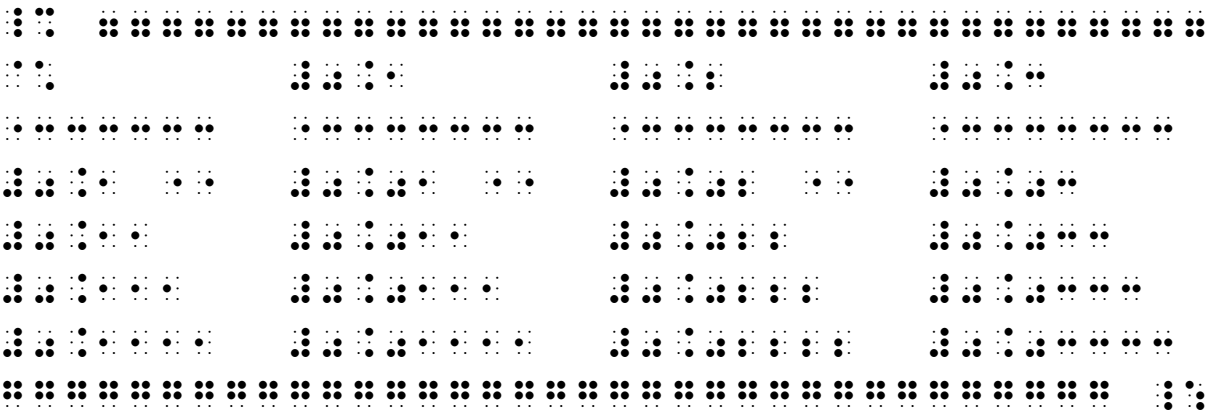
×	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**
0.1	0.01	0.02	0.03	0.04	0.05
0.2	0.02	0.04	0.06	0.08	0.1
0.3	0.03	0.06	0.09	0.12	0.15



Clarification: This rule does not apply to tables whose entries include words, letters, or other mathematical signs such as the dollar sign, percent sign, prime, fraction line, etc. In such tables, the numeric indicator must be used throughout the table according to the rules of the Nemeth Code. The minus symbol, general omission symbol, ellipsis, and long dash are not numeric. If any of those symbols occur in a table, the numeric indicator must be used throughout the table. If guide dots are needed within any column, the use of the numeric indicator for numeric entries in all columns must follow the rules of the Nemeth Code.

Example 17.7-2

×	**0.1**	**0.2**	**0.3**
0.1	0.01	0.02	0.03
0.11	0.011	0.022	0.033
0.111	0.0111	0.0222	0.0333
0.1111	0.01111	0.02222	0.03333



Because entries include a decimal point, the table is brailled in Nemeth Code. Because guide dots are needed, numeric indicators must be brailled.

Instructions: Apply the rules regarding use/nonuse of the numeric indicator as taught in this lesson. If the body of the table can be transcribed entirely in UEB, do so. Do not braille tables side-by-side even though they are printed in this manner.

PRACTICE 17D

Create one table which combines data from the three tables below.

Age	Blood Pressure
24	108
26	104
30	122
34	119
35	128

Age	Height
24	5'3"
26	5'9"
30	6'1"
34	5'10"
35	5'4"

Age	BMI
24	18.4
26	33.5
30	23.8
34	19.7
35	25.0

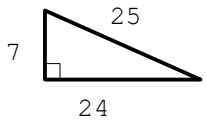
FIGURES AND DIAGRAMS

Instructions for producing figures, diagrams, and number lines are given in the BANA publication *Guidelines and Standards for Tactile Graphics.* A statement is required on the Transcriber's Notes page saying that *Guidelines and Standards for Tactile Graphics* is used in the preparation of the tactile graphics, giving the year the guidelines were adopted.

17.8 Which Code? Titles for figures and diagrams are transcribed in UEB. When diagram labels do not require a switch to Nemeth Code, UEB continues through the diagram.

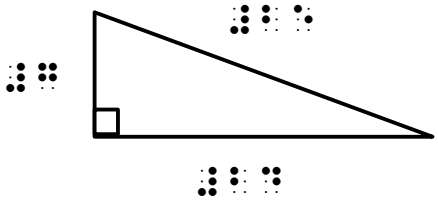
Example 17.8-1

Figure 1.4-2.





(blank line)

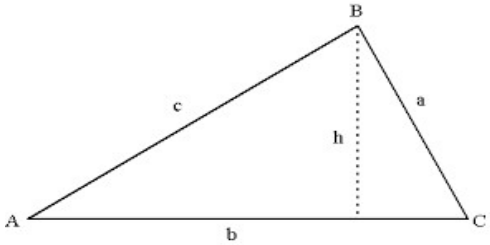


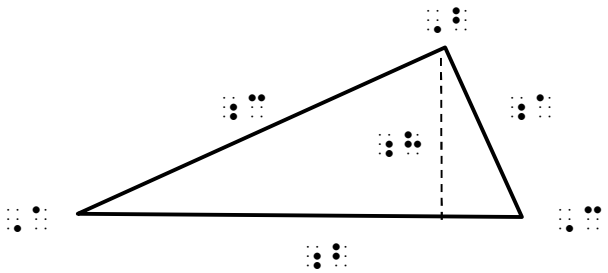
(blank line)

These figure labels are brailled in UEB because they are freestanding, unmodified numbers. A blank line precedes and follows the diagram.

17.8.1 Letters Used as Diagram Labels: When a single English letter in regular type is used as a label in a diagram, and when the labels are brailled in UEB, the Grade 1 indicator is required if the letter is in lowercase (even letters a i and o). If the letter is capitalized, the Grade 1 indicator is omitted.

Example 17.8-2

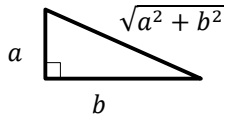


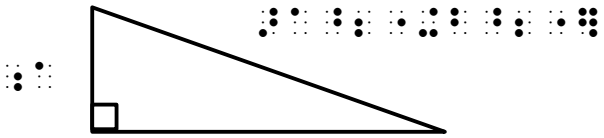


These figure labels are brailled in UEB because they are freestanding, unmodified letters.

17.9 Switch Indicators and Tactile Graphics: When a tactile graphic contains material that requires Nemeth Code, and when the preceding text is already in Nemeth Code, Nemeth Code continues to be in effect for the graphic.

Example 17.9-1 Explain how the diagram illustrates the Pythagorean equation, $a^2 + b^2 = c^2$.

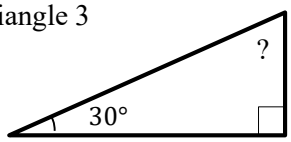






If the preceding text is in UEB and if a switch to Nemeth Code must be made for the tactile graphic, the opening switch indicator may be placed in cell 1 on the line before the required blank line ...

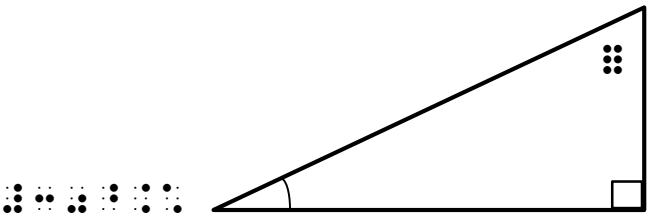
Example 17.9-2 Triangle 3







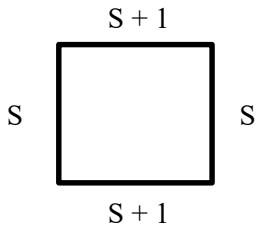
(blank line)

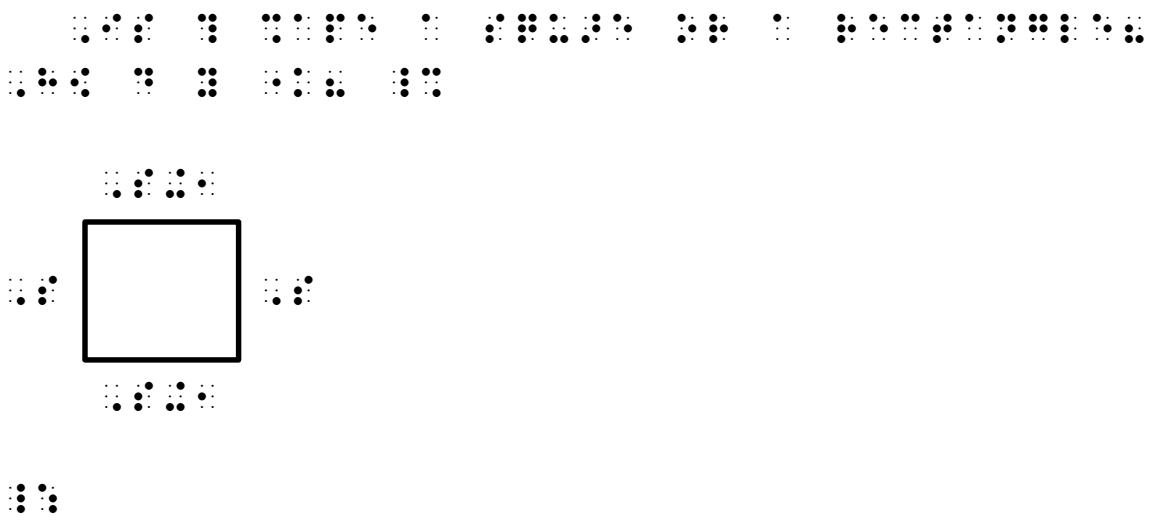




... or the switch indicator may be placed at the end of the preceding text.

Example 17.9-3 Is this shape a square or a rectangle? How do you know?





In Nemeth Code, when a diagram label is a single English letter in regular type, a letter indicator is required only if the letter is in lowercase (see Example 17.9-1). If the letter is capitalized, the English letter indicator is omitted (see Example 17.9-3).

When a diagram does not contain technical content such as mathematical expressions or modified numbers, do not transcribe the labels in Nemeth Code even if the body of text before and/or after the diagram is in Nemeth Code. Terminate Nemeth Code before the graphic, and reestablish it at the beginning of the following text.

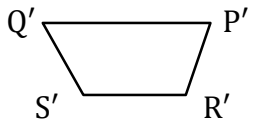
17.10 Graphic Number Lines: Symbols used in graphic number lines are required to be listed in a separate category on the Special Symbols page, listed in braille order. Suggested wording for the symbols is given in "Guidance for Transcription Using the Nemeth Code within UEB Contexts".

17.11 Diagrams in Exercise Material: If a diagram, number line, or other graphic is placed between instructions and the itemized exercise material which follows, apply the spacing and margin rules for the graphic as outlined in *Guidelines and Standards for Tactile Graphics*. Then continue Nemeth Code formatting for the exercise material.

Instructions: Leave blank space for tooling the lines.

PRACTICE 17E

Raj transformed quadrilateral PQRS to form quadrilateral P′Q′R′S′.



KEYING TECHNIQUE

17.12 Keying: When space does not permit the inclusion of labels, headings, entries, etc., in a figure, determinant, matrix, or table as shown in the print copy, one or more of the labels, headings, entries, etc., may be replaced by an alphabetic or numeric key. In addition to the keying guidelines outlined in *Braille Formats,* the following rules apply in Nemeth Code.

17.12.1 Alphabetic Key: An alphabetic key consists of two lowercase English letters, one of which contains a dot 3 or dot 6. Letter combinations that correspond to shortforms may not be used. The combination should be suggestive of the item it represents, if possible. Quoting *Braille Formats,* "Keys work best when they are related to the terms used in the text to help the reader remember what they are. Typically a letter key will be more memorable for the reader."

Two items which are identical should have the same key assigned to them. If any of the print entries in the table are made up of two lowercase letters, an alphabetic key cannot be used. In such cases, a numeric key is the only keying option.

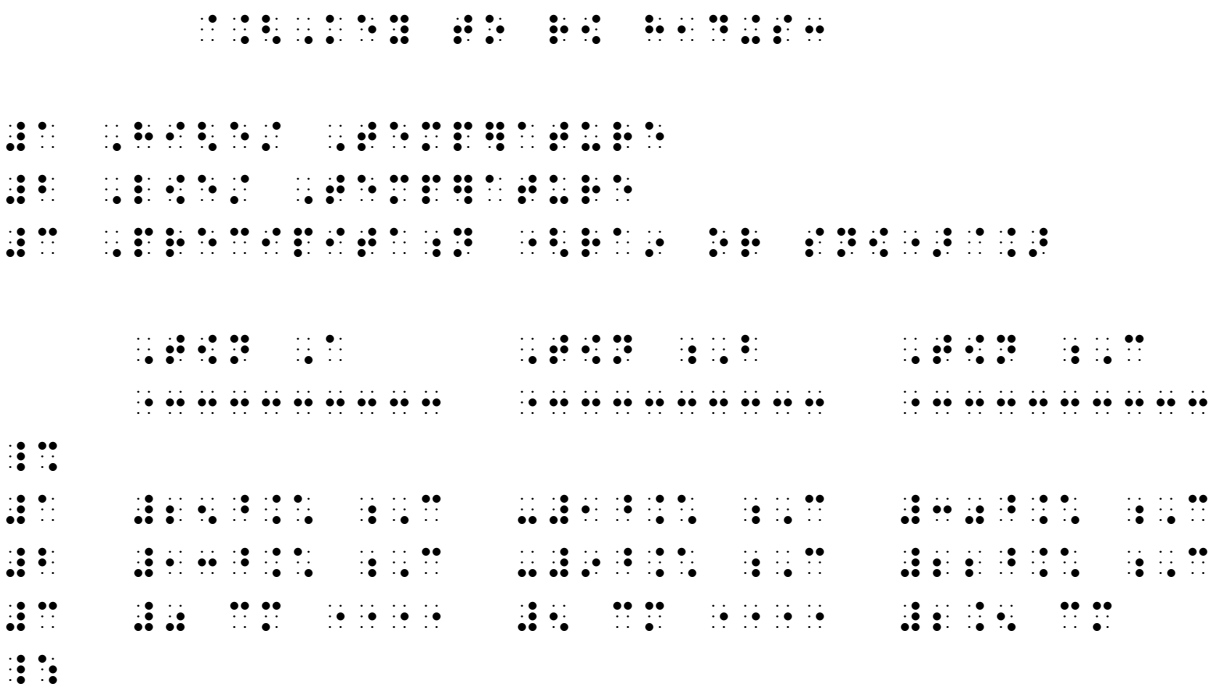
17.12.2 Numeric Key: A numeric key consists of one or more numerals brailled in the upper part of the braille cell. This number is preceded by the numeric indicator and must not be punctuated. The key numbers are placed in the figure, determinant, matrix, or table in the same position as the material which they replace. Two items which are identical should have the same key assigned to them.

17.12.3 The Key List: A list of numeric and/or alphabetic keys and their meanings must be included in a transcriber's note. Numeric keys are listed in numeric order. Alphabetic keys are listed in alphabetical order. Refer to *Braille Formats* for guidelines regarding the transcriber's note, margins, placement on the page, etc.

If the last listed item ends in Nemeth Code, Nemeth Code must be terminated before closing the transcriber's note.

Example 17.12-1

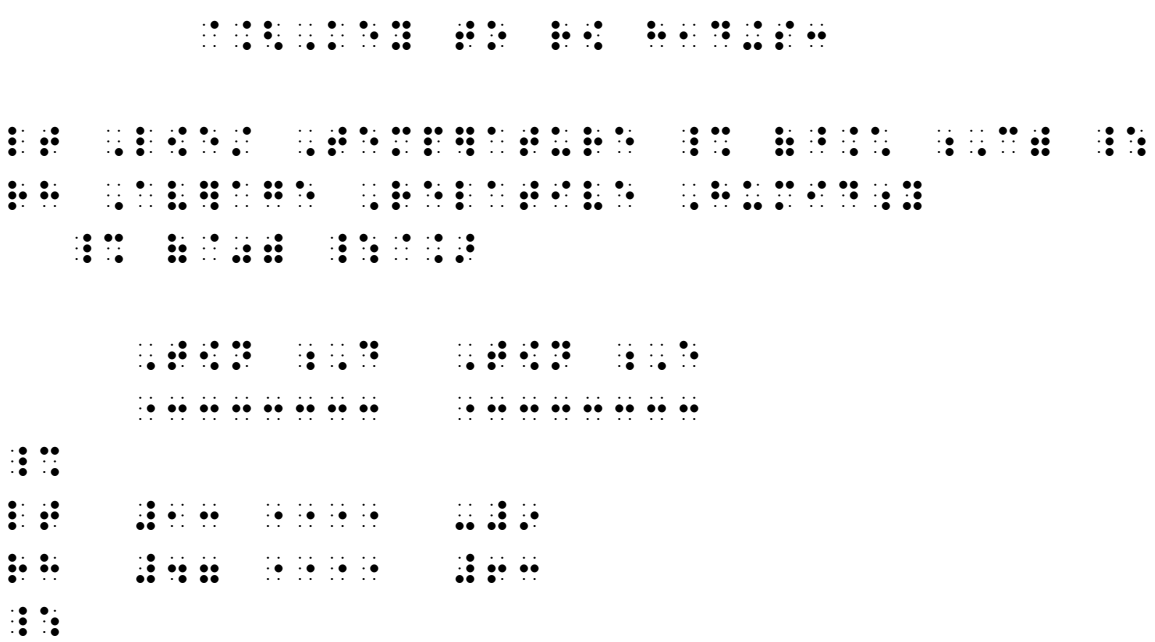
	Town A	Town B	Town C
Highest Temperature	25°C	–1°C	30°C
Lowest Temperature	13°C	–9°C	22°C
Precipitation (rain or snow)	0 cm	5 cm	2.5 cm



By keying the row headings, the layout of the print table is maintained. Because there are entries in the table consisting of two lowercase letters ("cm"), a numeric key must be used.

Example 17.12-2

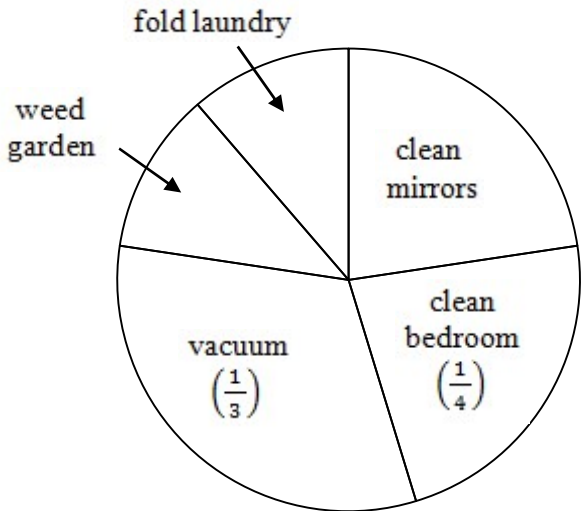
	Town D	Town E
Lowest Temperature (°C)	13	–9
Average Relative Humidity (%)	47	63



An alphabetic key provides the reader with clues regarding the item's meaning.

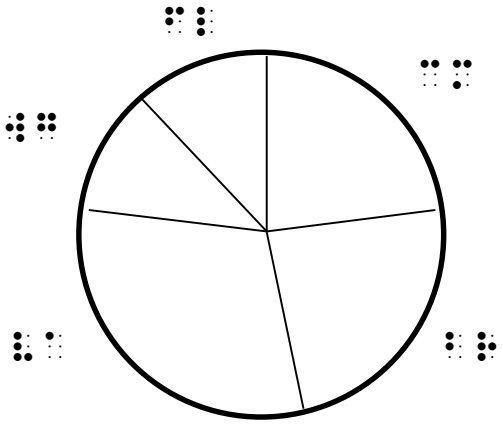
Example 17.12-3

ADYLYN'S CHORE SPINNER



Adylyn hopes she will spin *either* "vacuum" *or* "weed garden" today. What is the probability that she will spin one of these chores?

- What is P(vacuum)?
- What is P(weed garden)?
- What is P(vacuum) OR P(weed garden)?



The key list is in alphabetical order. As outlined in Guidelines and Standards for Tactile Graphics, *a blank line precedes and follows the graphic, and the keyed labels are placed outside of the circle. Bulleted items follow guidelines given in* Braille Formats*. The probability notation "P(vacuum)", "P(weed garden)", and "OR" is mathematical – Nemeth Code switch indicators are used and words are not contracted.*

Instructions: Key only the first and third column heading using an alphabetic key. *Note:* "dim" is a function abbreviation. Review function notation in **Lesson 14**.

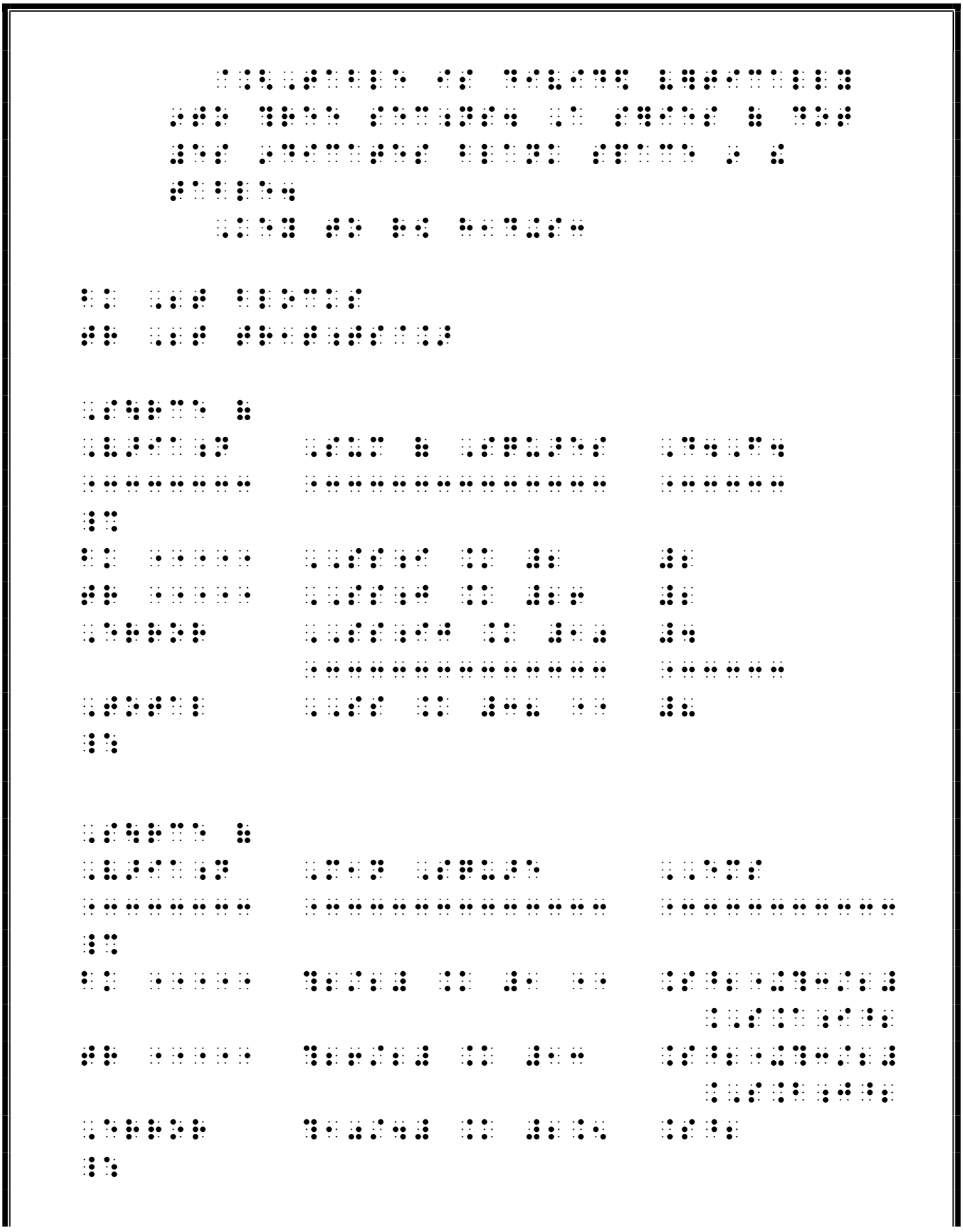
PRACTICE 17F

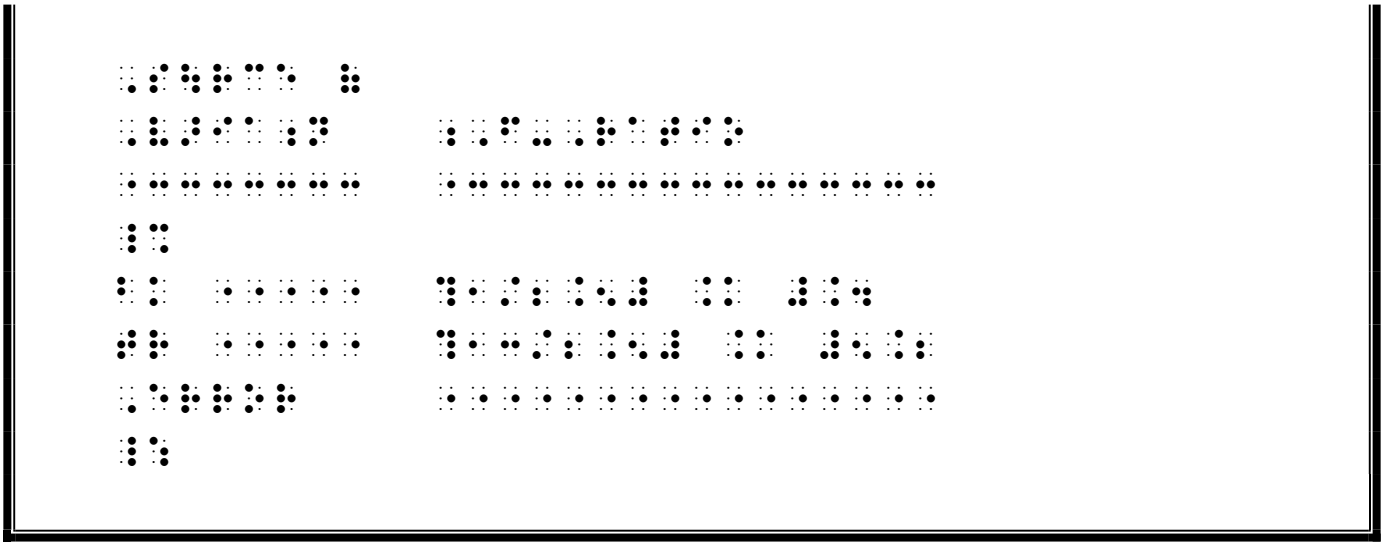
Eigenvalue	Eigenvectors	dim E(λ)
$1, 1$	$(a, b) \neq (0, 0)$	2
$1, 1$	$t(1, 0), t \neq 0$	1
$1, 1$	$t(0, 1), t \neq 0$	1
2	$t(1, 1), t \neq 0$	1
0	$t(1, -1), t \neq 0$	1

CHALLENGE PROBLEM

Source of Variation	Sums of Squares	D.F.	Mean Square	EMS	F-Ratio
Between blocks	$SS_i = 2$	2	$\frac{2}{2} = 1$	$\sigma^2 + \frac{3}{2}\Sigma\alpha_i^2$	$\frac{1}{2.5} = .4$
Between treatments	$SS_j = 26$	2	$\frac{26}{2} = 13$	$\sigma^2 + \frac{3}{2}\Sigma\beta_j^2$	$\frac{13}{2.5} = 5.2$
Error	$SS_{ij} = 10$	4	$\frac{10}{4} = 2.5$	σ^2	
Total	$SS = 38$	8			

This table is too wide to fit on a 40-cell-wide braille page. Although the column headings and entries could be keyed in order to maintain the same layout as shown in print, reading a table that is highly keyed is abstract and difficult to use. A solution is shown below. It combines the use of vertical division of the table, division of the long expressions in the EMS column, and keying two of the row headings.





CHEMISTRY

17.13 Two BANA Publications: Transcribing chemical notation requires further study and is beyond the scope of this lesson manual. The following publications offer rules and guidance:

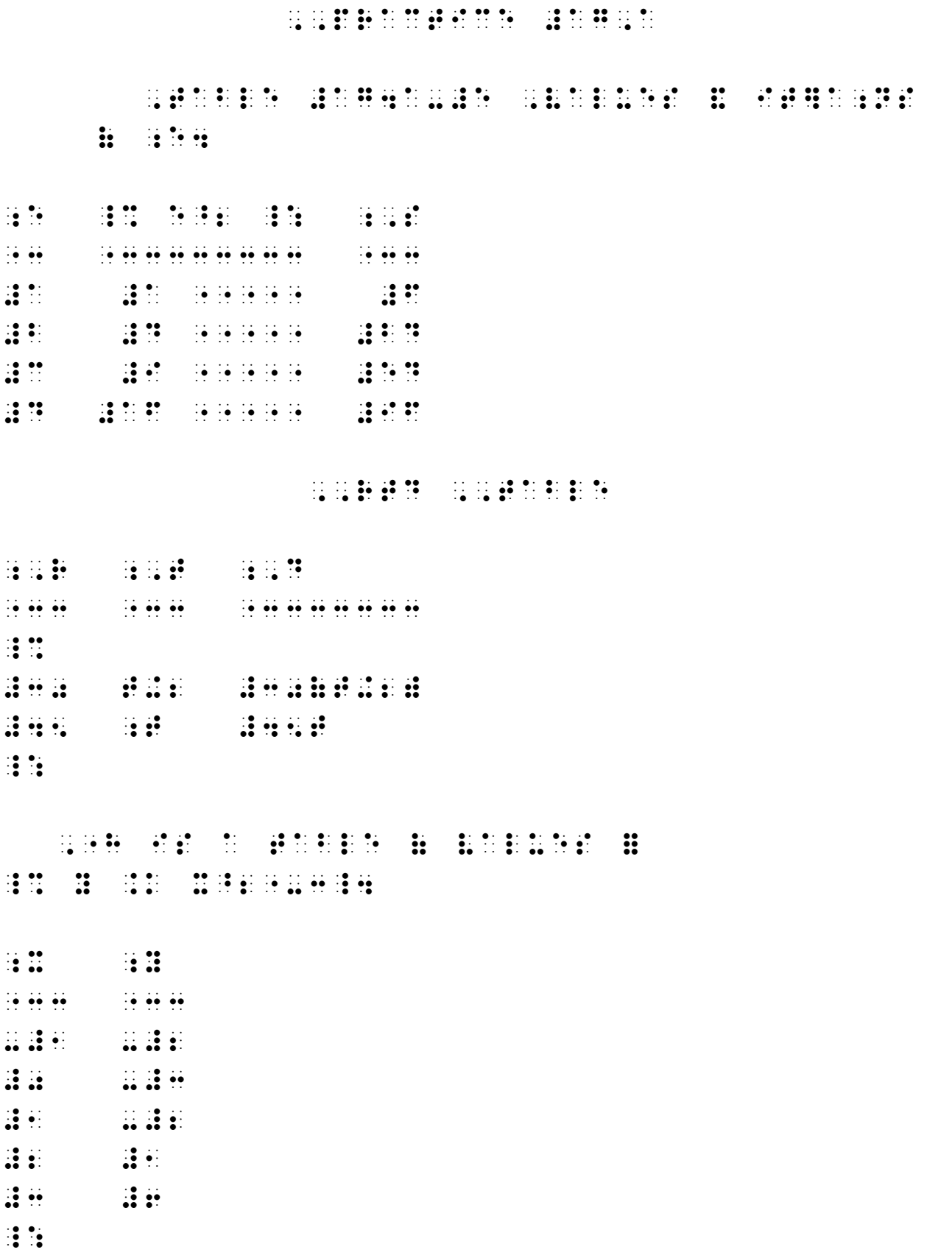
- *Braille Code for Chemical Notation 1997*
- ″Provisional Guidance for Chemistry Notation Using Nemeth in UEB Contexts" (2016)

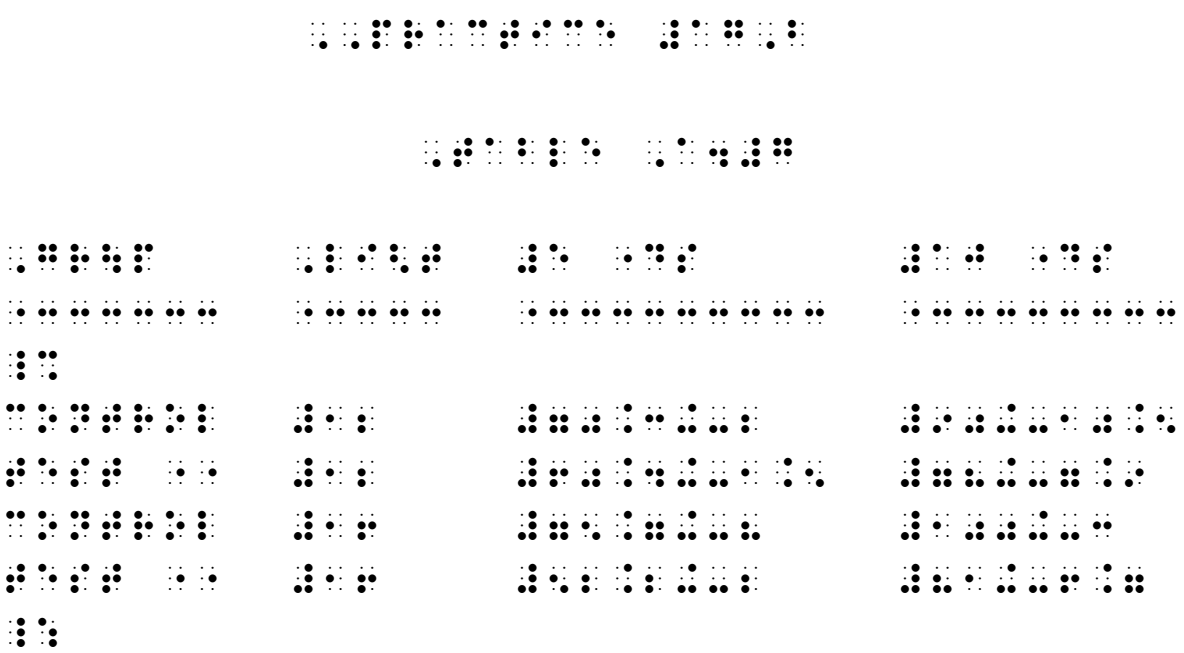
REFERENCE SYMBOLS

The BANA Nemeth Code Technical Committee is discussing details regarding the transcription of reference symbols within mathematical context. This section will be completed after decisions are made.

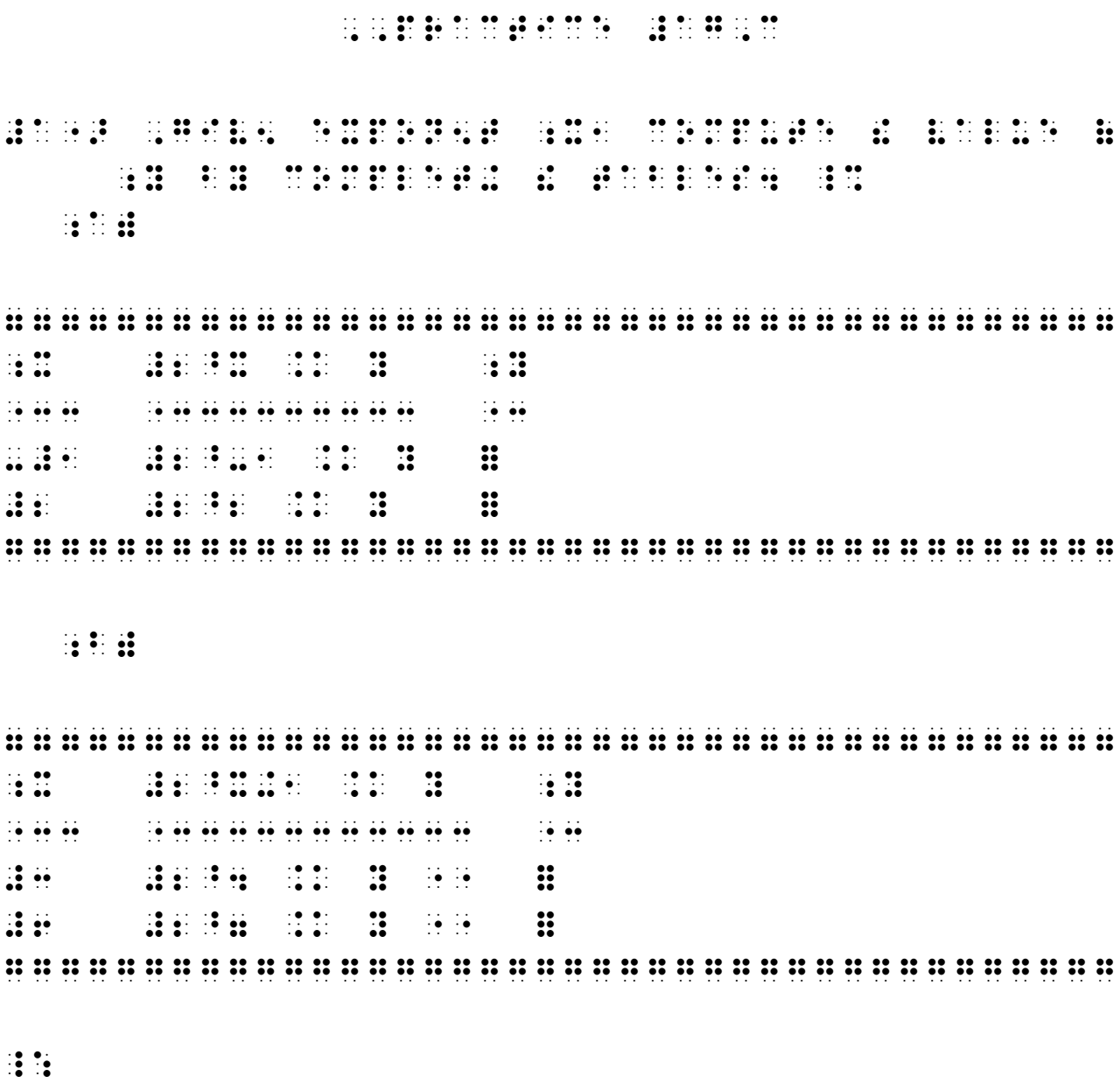
For further practice, see Appendix A—Reading Practice.

ANSWERS TO PRACTICE MATERIAL

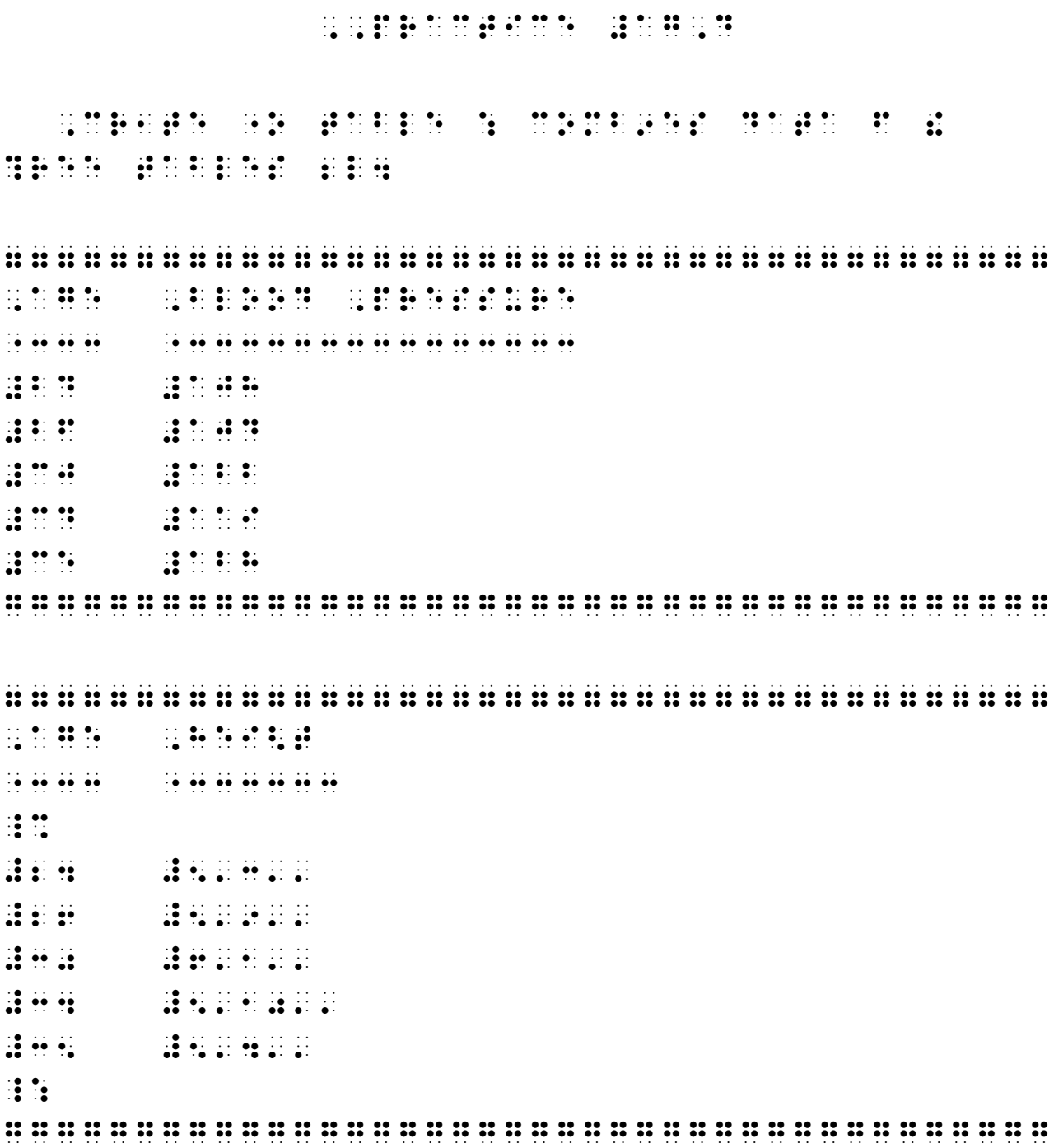




The table label and column headings are transcribed in UEB. The row headings are part of the technical material and are uncontracted. The single-word switch indicator is not used.



Formatting Notes: Subitem identifiers begin in cell 3. Column 1 begins in cell 1. Box lines use the full width of the page. Boxes are preceded and followed by a blank line. In column 3, the general omission symbol represents the question mark omission sign in Nemeth Code. The opening Nemeth Code switch indicator precedes the identifier "a)". Nemeth Code is terminated after the completion of the table. Box lines "match"—since the top box line does not incorporate a switch indicator, the Nemeth Code terminator is not part of the bottom box line.

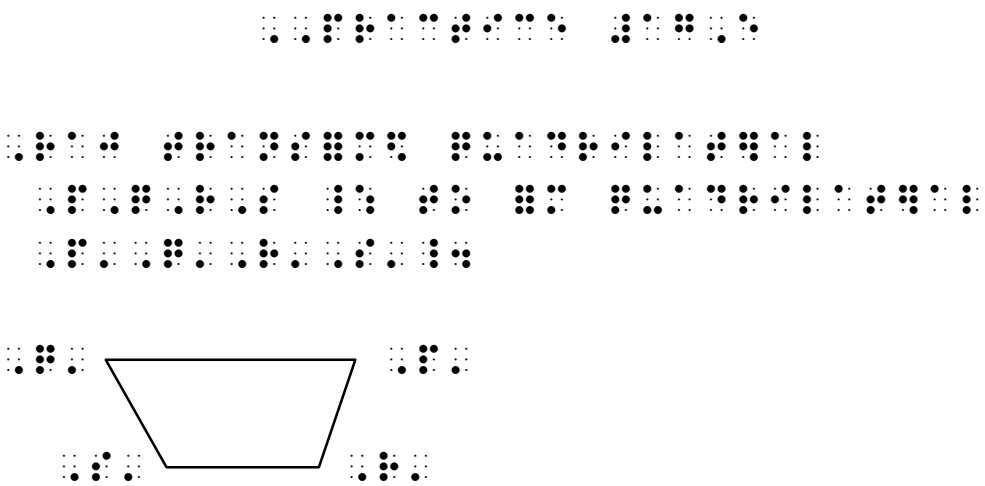


Formatting Notes: Although the text is "instructional", it is formatted as a narrative paragraph (3-1) because it is not followed by itemized material.

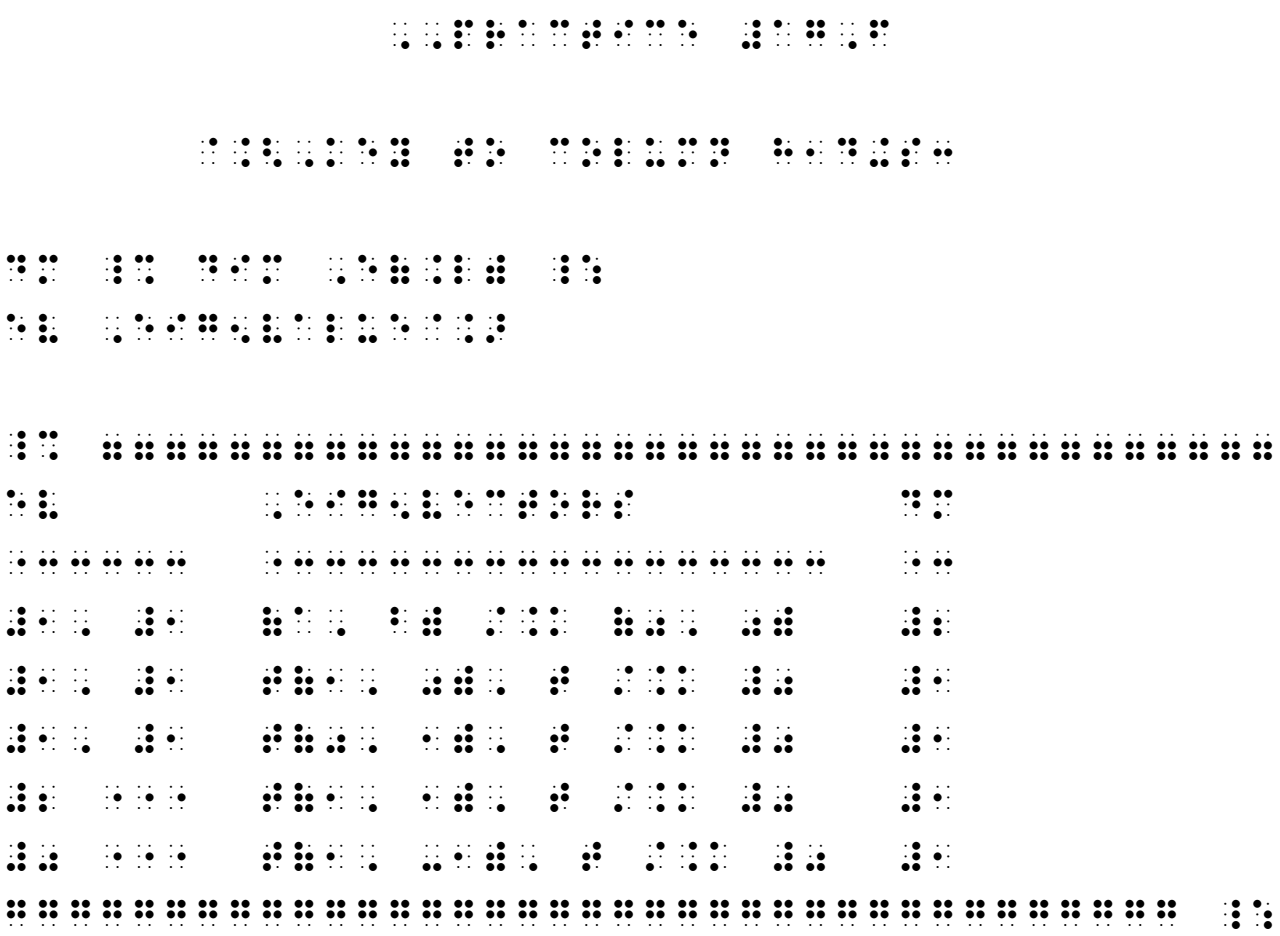
First Table: The numbers can be transcribed in UEB. The second column heading will fit on one line, which is the preferred braille format.

Second Table: The presence of prime signs dictates Nemeth Code. The code switch indicator precedes the first data entry.

Nemeth Code is indicated because the entries contain decimal points. Since the data is composed entirely of numbers. numeric indicators are not used. The Nemeth Code terminator is placed alone on the line following the last entry, before the bottom box line.



The presence of prime symbols in the labels requires Nemeth Code. Nemeth Code is terminated after the required blank line following the graphic.



The alphabetic key chosen may consist of any two lowercase English letters, as long as one letter in each key item contains a dot 3 or dot 6 and as long as they do not correspond to a shortform. The key listing in the transcriber's note should be in alphabetical order.

EXERCISE 17

Exercise 17 will be available when this course is finished being written and is no longer "Provisional".

Proceed to Lesson 18.

LESSON 18

Read about this PROVISIONAL EDITION in the front matter to this book.
Check the NFB website periodically for updates to this lesson.

This is the final lesson but in no way is it the end of your learning. As this book's title states, this is an INTRODUCTION to the Nemeth Braille Code. In your work you will encounter symbols and usage that will require creative and consistent application of the rules. Each new assignment will present challenges. As you research answers, your understanding of the rules and guidelines will develop.

18.1 Preparing for the Certification Exam: A certification exam for brailling Nemeth Code in UEB context is not yet available. In the meantime, topics addressed in this lesson are offered for the transcriber who is preparing materials for publication.

18.2 The Nemeth Codebook: *The Nemeth Braille Code For Mathematics and Science Notation* should become your primary source for transcribing technical materials. Subject matter from several of the lessons in this training manual may be grouped into one section in the codebook, giving new perspective and understanding of a topic or rule. Be sure you have the BANA updates for the Nemeth Code. Ask your teacher if you do not know how to obtain the updates.

To get started, read the following "use/nonuse" sections in the Nemeth Codebook. Review the rules, study the examples, and follow the cross-references.

	Use of	Non-use of	Other Considerations
Capitalization indicator	§20	§21	§22, §50
English letter indicator	§26	§27	§28, §29, §30, §51
Enlarged grouping symbol	§126	§127	
Level indicator	§80	§81	§82
Numeric indicator	§9	§11	§10, §12, §13
Punctuation indicator	§37	§38	§52
Simple fraction indicator	§62	§63	
Typeform indicator	§32, §33	§34	§35

Review situations when we do not follow print layout Here is a sampling of situations for which the Nemeth Code has specific rules regarding spacing and arrangement on the braille line. *Centering*: cancellation, ditto marks, spatially-arranged fractions. *Spacing*: abbreviations, factors in math expressions, math symbols, functions, alignment of items in spatial arrangements. *Linage*: "keep together" rules (an abbreviation and its associated numeral or letter, function abbreviations, hyphenated expressions, signs of shape), division of long math expressions, side-by-side arrangements (e.g., itemized, unitemized, displayed, spatial). *Margins:* formal proofs, instructions, itemized material, labeled mathematical statements, linked expressions, paragraphing.

Revisit rules which you find to be troublesome For example, review the various forms of fractions, the many uses of the multipurpose indicator, correct assessment of ambiguous mathematical signs, and the use of literary punctuation for words and abbreviations in mathematical context.

Back matter Familiarize yourself with Appendix B of the Nemeth Code which offers a useful index of braille symbols. Skimming the General Index is a good way to reinforce the vocabulary and terminology used in the Nemeth Braille code.

Review code switching guidelines The only time Nemeth symbols and indicators may be used is within the Nemeth Code switch indicators. Review guidelines regarding when code switching is optional and when it is required, as well as considerations concerning placement of the switch indicators in embedded material, displayed material, spatial material, and at page turns. A summary appears in **Appendix C** of this course.

18.3 Beyond the Nemeth Code: Every aspect of the Nemeth Code has been introduced in this course. Certification in this code implies that you are prepared to transcribe a textbook which contains mathematical notation. This requires knowledge of the structure of a braille textbook as well as how the Nemeth Code cooperates with textbook formatting and the rules of Unified English Braille. Format guidelines which apply to the structuring of a textbook are in the BANA publication *Braille Formats: Principles of Print-to-Braille Transcription.* The **2016** edition is applicable to UEB transcriptions. The transcriber should be thoroughly familiar with that resource as well as the other sources listed below. These documents are available online at www.brailleauthority.org. Keep up to date as newer editions or updates are posted by the Braille Authority of North America.

The dates shown are the current edition at the time of this writing.

The Rules of Unified English Braille, Second Edition 2013

Braille Formats: Principles of Print-to-Braille Transcription, 2016

The Nemeth Braille Code for Mathematics and Science Notation, 1972 Revision, 2011-2015 Updates

"Guidance for Transcription Using the Nemeth Code within UEB Contexts" (2016)

Watch for ″Application of the Formats Guidelines 2016 to Nemeth Transcriptions within UEB Contexts"

This course does not address topics regarding creating a tactile graphic. A thorough reading of the most recent edition of *Guidelines and Standards for Tactile Graphics* is recommended before undertaking a technical transcription. Mathematical and scientific diagram strategies presented in this guidebook include 2-D and 3-D drawings, clocks (analog and digital), complex geometric shapes, counting symbols, graphs (circle graphs, bar graphs, line graphs, histograms, Cartesian graphs, pictographs, pie charts, scatter plots, line or dot plots, box-and-whisker plots), measurement tools, money, nets, number lines, orthographic drawings, spinners, tessellations, thermometers, and Venn diagrams.

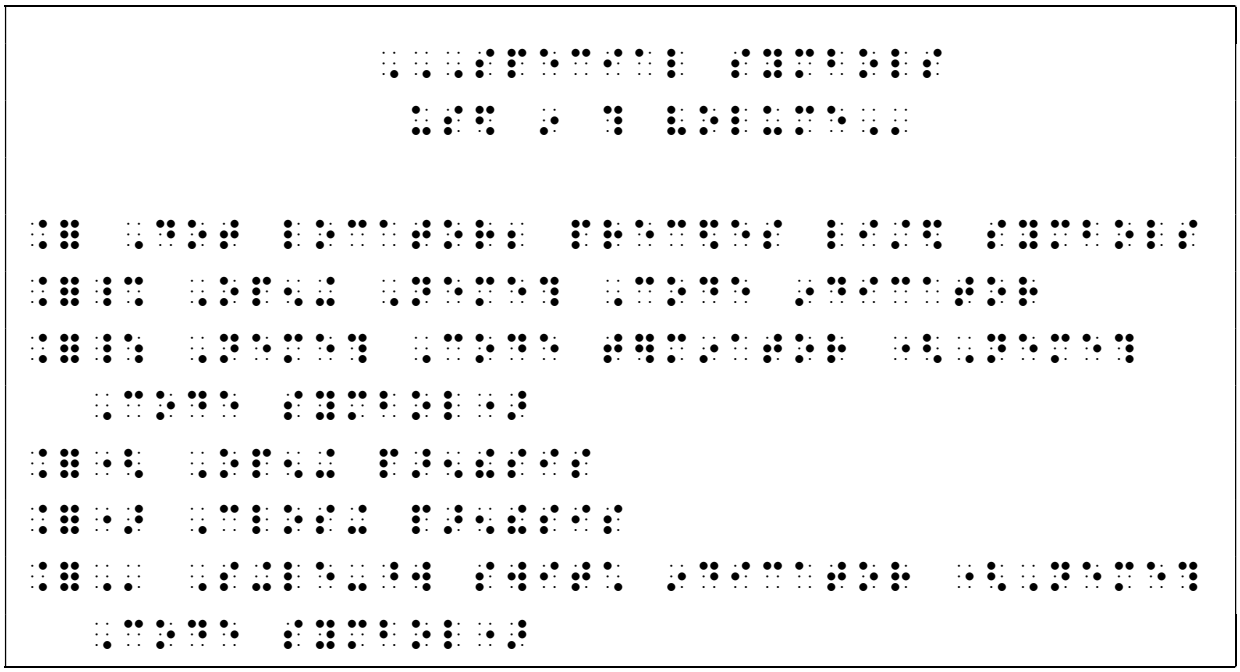
Symbols, arrangements, and structures not covered in the courses, codebooks, and other guidance documents are frequently encountered when preparing assignments. Professional development resources and online forums can provide illustrations and discussions of many topics in the fields of mathematics and science. As comprehension and understanding of the rules develops, one can find a solution to just about any transcribing challenge.

18.4 Transcriber-Generated Pages and Front Matter: Before brailling the first page of text, certain transcriber-generated and front matter pages are required. Review the guidelines for the transcriber-generated pages (title pages, special symbols page, and transcriber's notes page) in Section 2 of *Braille Formats*. You may also wish to review the lesson on "Braille Book Format" in the most recent edition of the *Instruction Manual for Braille Transcribing – UEB Edition.*

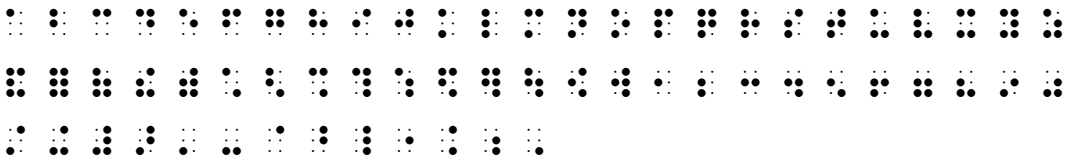
18.4.1 Special Symbols Page: The heading for this "t" page is SPECIAL SYMBOLS USED IN THIS VOLUME. Symbols from *The Rules of Unified English Braille* are to be listed as required in that rulebook. List UEB symbols that appear throughout the volume, including any that may occur on the title page. Follow the guidelines in Appendix G of *Braille Formats* to select symbols that should be included. (Disregard the UEB math symbols given in that list since UEB math symbols are not used in a Nemeth transcription.)

The code switch indicators for Nemeth Code are to be included in the list, following "braille order" as explained in Section 1.1.2 of *The Rules of Unified English Braille.* The phrase "(Nemeth Code symbol)" should follow the definition of both the Nemeth Code terminator and the single-word switch indicator. Other Nemeth Code symbols are not listed on the Special Symbols page unless they are symbols devised by the transcriber.

Here is a model, showing a few UEB symbols as well as the three Nemeth Code switch indicators. Additional symbols will be inserted into the model as needed in each braille volume.



18.4.1.a Braille Order: The "braille order" chart is reproduced here (in three rows) for your convenience. See Section 1.1.2 of *The Rules of Unified English Braille* for further details.



18.4.2 Transcriber's Notes Page: The heading for this "t" page is TRANSCRIBER'S NOTES. The purpose of this page is to identify special formats or usage found throughout a braille volume, and to cite sourcebooks. When Nemeth Code is used in a transcription, the Transcriber's Notes page should include the following note: "Mathematical content is transcribed according to *The Nemeth Braille Code for Mathematics and Science Notation, 1972 Revision,* 2007-(year of latest update) Updates including the *Guidance for Transcription Using the Nemeth Code within UEB Contexts*." You can find items that require explanation listed throughout *Braille Formats*. In addition to those requirements, the Nemeth Code identifies the following as items which require explanation either at the point in the transcription where they appear or on the Transcriber's Notes page:

- Continental usage of the comma and/or decimal point in the print copy
- Use of capitalized letters in the print copy for digits in non-decimal bases
- Use of alternative forms of Greek letters in the print copy
- Omission of vector arrows which appear in the print copy
- Description of the shapes used in print to depict calculator or computer keys
- When identifiers to displayed expression are moved to the left in the braille edition
- Changing the column format of a formal proof to an itemized list in the braille edition

---

PRACTICE 18A

Instructions: Prepare transcriber-generated pages and one front matter "contents" page according to the guidelines in Section 2 of *Braille Formats.* Do not use a running head.

Title Page: Use the following information to prepare a title page.

Book title: *ADVENTURES IN Y2K MATHEMATICS*
Book subtitle: *Math in the New Millennium*
Authors: Monica and Matías Cruz
Publisher information: Math4You Publications, Inc., Antelope Valley, CA, www.M4YPub.edu
Copyright information: © 2018 by M&M Publishers
ISBN: 9-6230-99228x
Transcriber segment: Use your name, your city, and your state
Volume Information segment: Assume this is the second volume of a three-volume transcription. The braille page designation is "t1-t3, p1, and 1-120" and the print page designation is "v and 87-a123"

Special Symbols Page: List the symbols from the title page that are required according to Appendix G of *Braille Formats*, including symbols from the "may be included" list. Also list the UEB symbols which appear in the other practice drills in this lesson. Those symbols are given below.

Grade 1 symbol indicator; Nemeth Code switch indicators; parentheses; typeform indicators for boldface passage, boldface terminator, boldface word, italic passage, italic terminator, italic word, underlined passage, and underlined terminator.

You will need to arrange the symbols in braille order as explained in Section 1.1.2 of *The Rules of Unified English Braille*.

PRACTICE 18A, cont.

Transcriber's Notes Page: In addition to the required statement citing the use of the Nemeth Code, write a description of the change made to the column format of the formal proof which appears in Practice E.

Front Matter: Include a transcription of this contents page:

ADVENTURES IN Y2K MATHEMATICS v

CONTENTS

CHAPTER 18 87

18.1 Roman Numerals

18.2 Arabic Numerals

CHAPTER 19 95

19.1 Binary Code

19.2 Hexadecimal Code

CHAPTER 20 101

20.1 Exponential Functions

20.2 Logarithmic Functions

CHAPTER 21 106

21.1 Inductive Thinking

21.2 Conjecture

18.5 Body of Text: The entire transcription follows a collaborative pattern between *Braille Formats* 2016 and formats provided for in the *Nemeth Braille Code*. When a format is specified in the Nemeth Code, those rules are applied not only to the technical material but also to the UEB material. Some examples are given below.

18.5.1 Follow Nemeth Code Formatting Rules: The following matters are governed by Nemeth Code format and apply both to the technical material and to UEB portions of text.

- Exercises with subentry levels are treated individually regarding runover margins.
- Instructions preceding itemized material begin in cell 5 and runover in cell 3.
- "Keep together" rules for hyphenated expressions and for abbreviations and a numeral or letter associated with it.
- Labeled mathematical statements—Nemeth Code rules apply regarding paragraphing, blank lines, and typeface.
- Margins applied to itemized material and their subparagraphs.
- Paragraphs begin in cell 3 and runover in cell 1. (Blocked paragraphing is not allowed.)

18.5.2 Follow Braille Formats Formatting Guidelines: Items which are not addressed in the Nemeth Code rely on *Braille Formats* for positioning. Some items governed by *Braille Formats* include:

- Blank lines
- Box lines
- Content and structure of the transcriber-generated pages and front matter
- Displayed literary text
- Exercise examples
- Headings
- Indented list format for nontechnical text
- Lists
- Margins for captions, notes, sidebars, and transcriber's notes
- Page numbering

18.5.3 Context-Dependent Formats

These formats occur only in Nemeth Code context.

- Blank lines with spatial arrangements
- Division of mathematical expressions
- Margins for displayed mathematical expressions

These items follow Nemeth Code formatting between the switches, and follow *Braille Formats* outside of the switches.

- Blank table entries to be filled in
- Keying of long entries
- Tables consisting only of numbers

Final Word Thank you for making the extra effort to learn the Nemeth braille code. We hope you have noticed your transcribing and proofreading skills improve over the course of the lessons. As you take on assignments, check the NFB website periodically for changes to this lesson material and check the BANA website for updates to the braille codes. We also encourage you to take advantage of opportunities to stay informed and connected to other transcribers. The National Braille Association (NBA) publishes a quarterly Bulletin and hosts several learning opportunities—an online forum "Ask An Expert", monthly webinars, and professional development conferences. Local groups offer similar support, for example, the California Transcribers and Educators for the Blind and Visually Impaired (CTEBVI), Visual Aid Volunteers of Florida (VAVF), and the Midwest Regional Braille Conference.

We wish you success and satisfaction providing much-needed mathematics and science materials for braille readers.

This course concludes with four practices which are characteristic of K-12 grade level topics.

Four Practices

Instructions: Use a 40-cell line and a 25-line page. Show print and braille page numbers on every page. Use the number shown in the upper right corner as the print page number. Do not use a running head. Begin numbering with braille page 1, showing the book title at the top of the page as required (see 1.8.1 in *Braille Formats).* Continue numbering the braille pages consecutively throughout the four practices. Begin each practice on a new braille page, but within each practice do not force a new braille page unless a rule supports doing so.

PRACTICE 18B

ADVENTURES IN Y2K MATHEMATICS 87

ADDING AND SUBTRACTING

53 + 36	14 + 44	33 + 21	98 + 1	2 + 2	456 + 13
4 + 84	23 + 32	71 + 1	33 − 22	150 − 40	9 − 6
826 − 16	55 − 2	77 − 27	420 + 519	95 − 62	93 + 6

ADVENTURES IN Y2K MATHEMATICS 88

COMPARING FRACTIONS

Directions: Write the correct comparison symbol (>, <, or =) in each box.

1) $\frac{1}{6} \square \frac{3}{4}$ 2) $\frac{1}{2} \square \frac{3}{6}$ 3) $\frac{2}{4} \square \frac{1}{3}$

4) $\frac{2}{5} \square \frac{2}{3}$ 5) $\frac{9}{10} \square \frac{4}{5}$ 6) $\frac{1}{6} \square \frac{2}{12}$

PRACTICE 18C

Exercise Set 18-6

1. **Home Economics** The cost of using a 60-watt light bulb is given by the function $y = 0.0036x$. The cost is in dollars, and x represents the number of hours the bulb is lit.
   a. How much does it cost to use a 60-watt light bulb 8 hours a day for a week?
   b. If the total cost of using a 60-watt bulb is $1.98, for how many hours can it be used?
2. **What's the Question?** The following set of points defines a function: $\{(3, 6), (-4, 1), (5, -5), (9, -6), (10, -2), (-2, 10)\}$. If the answer is 6, 1, –5, –6, –2, and 10, what is the question?
3. **Physics** Ohm's law can be described by the simple formula

   $$I = \frac{V}{R}$$

   where I = current (in amps, A), V = voltage (in volts, V), and R = resistance (in ohms, Ω). What equation would you use to solve for voltage?
   a. V = I/R
   b. V = IR
   c. V = R/I

ADDING, SUBTRACTING, MULTIPLYING, AND DIVIDING INTEGERS

Find the sum, product, or quotient as indicated by the signs $+, \times, \div$.

1) $-6 + -5 =$ ____
2) $-2 \times -1 =$ ____
3) $35 \div -5 =$ ____
4) $5 + -19 =$ ____
5) $-24 \div 4 =$ ____
6) $-132 \div -11 =$ ____
7) $9 \times 9 \times -5 =$ ____

PRACTICE 18D

Instructions: Format the italicized text as Nemeth Code instructions.

UNIT 6 REVIEW

Fill in the correct answers.

1. If $7n = 0$, then $n =$ ____.
2. Replace the ☐ with a numeral to make a true sentence: $(4 + 8) + ☐ = 4 + (8 + 3)$.
3. $\left(\frac{1}{3} + \frac{5}{6}\right) - \frac{5}{12} = ?$
4. $5 \times 5 \times 5 = 5^{?}$

Study the equivalencies.

5. To find the mixed-number name for $\frac{154}{9}$, divide 9 into 154.

$$\begin{array}{r} 17\text{ R}1 \\ 9\overline{)\,154} \\ \underline{9} \\ 64 \\ \underline{63} \\ 1 \end{array}$$

$$\frac{154}{9} = 17\frac{1}{7}$$

6. Multiply $4\frac{2}{3}$ by $\frac{3}{14}$. $4\frac{2}{3} = \frac{12+2}{3} = \frac{14}{3}$. $\frac{14}{3} \times \frac{3}{14} = 1$!

PRACTICE 18D, cont.

Review set notation, Roman numerals, and miscellaneous topics.

7. If R = {a, b, c, d} and S = {a, c, e, g, h}, then the intersection of sets R and S is {a, c}.
8. Only the following pairs of symbols may occur out of the natural order: I before V or X, X before L or C, C before D or M. For example, XL = L–X = 50–10 = 40.
9. Use the distributive property to multiply 14 by 3.

$$\begin{aligned} 3 \cdot 14 &= 3\,(10 + 4) \\ &= 3 \cdot 10 + 3 \cdot 4 \\ &= 30 + 12 = 42 \end{aligned}$$

10. "$24_{(\text{five})}$" is read "two four, base five."
11. A gain of 5 yd followed by a loss of 2 yd gives a total gain of 3 yd. Expressed mathematically, $(^{+}5) + (^{-}2) = {}^{+}3$.

Explain the relationships.

12. $\overline{AB}$ is congruent to $\overline{DE}$.
13. $\overline{RS} \approx \overline{EF}$; $\overline{RS} \neq \overline{LK}$.
14. $\overrightarrow{DA} \cup \overrightarrow{DB} = \overleftrightarrow{AB}$
15. $\frac{7}{\cancel{12}\,6} \times \frac{\overset{11}{\cancel{22}}}{5} = \frac{77}{30}$

Simplify each radical expression.

16. $\sqrt[4]{x} \cdot \sqrt{x}$ 17. $\sqrt[5]{\sqrt[3]{a^2}}$ 18. $\frac{\sqrt{36x}}{\sqrt[3]{8x^2}}$

PRACTICE 18E

THEOREM The sum of the angles in a triangle is 180 degrees.

Given: $\triangle$ABC

Prove: $\angle a + \angle b + \angle c = 180°$

STATEMENT	REASON
1. Let BD be a line through B parallel to AC.	1. Parallel postulate.
2. $\angle a = \angle d$.	2. Corresponding angles are equal.
3. $\angle b = \angle b$.	3. Identity.
4. $\angle c = \angle e$.	4. Alternate-interior angles are equal.
5. $\angle a + \angle b + \angle c = \angle d + \angle b + \angle e$.	5. Sum of equal quantities are equal.
6. $\angle d + \angle b + \angle c = 180°$.	6. A straight angle equals 180°.
7. $\therefore \angle a + \angle b + \angle c = 180°$.	7. Quantities equal to the same quantity are equal to each other. ∎

Use $\pi/12 = \pi/3 - \pi/4$ and the identity for the tangent of a difference to solve $\mathbf{tan}\left(\frac{\pi}{12}\right)$.

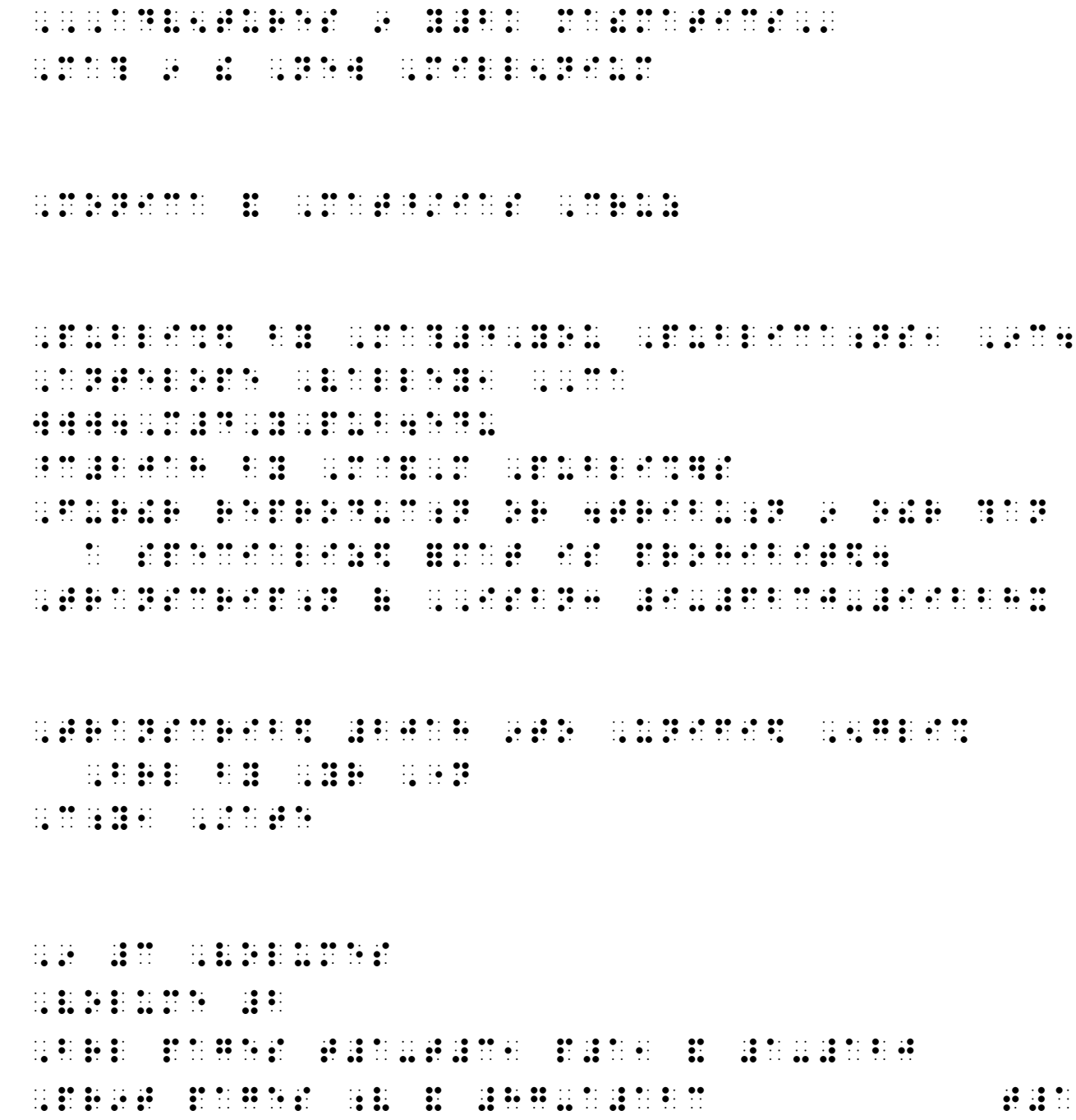
$$\tan\left(\frac{\pi}{12}\right) = \tan\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \frac{\tan\frac{\pi}{3} - \tan\frac{\pi}{4}}{1 + \tan\frac{\pi}{3}\tan\frac{\pi}{4}} = \frac{\sqrt{3}-1}{1+\sqrt{3}\cdot 1} = \frac{(\sqrt{3}-1)(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)} =$$
$$\frac{3-2\sqrt{3}+1}{2} = \frac{4-2\sqrt{3}}{2} = 2 - \sqrt{3}$$

The answer is $\tan\left(\frac{\pi}{12}\right) = 2 - \sqrt{3}$.

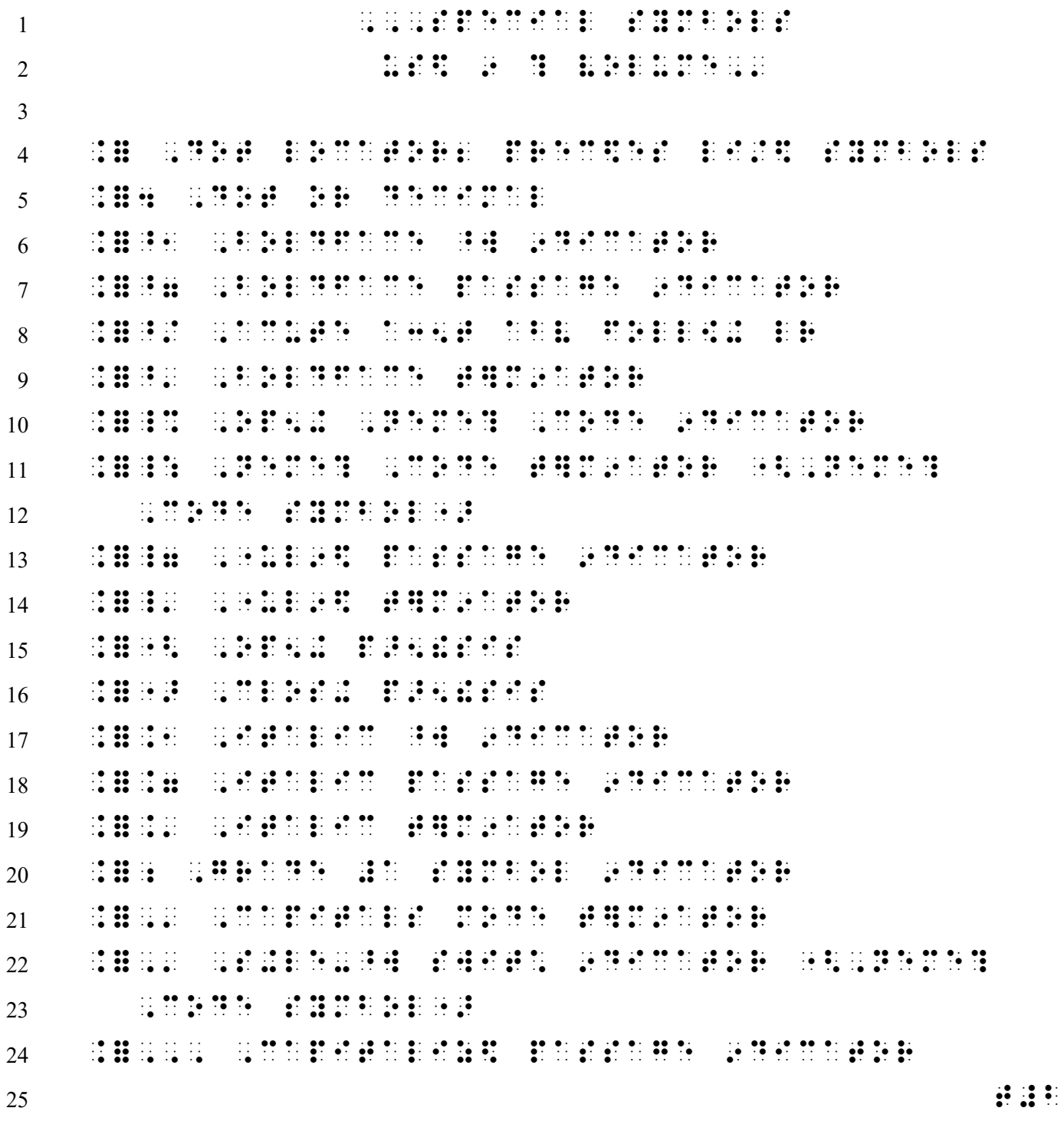
ANSWERS TO PRACTICE MATERIAL

PRACTICE 18A

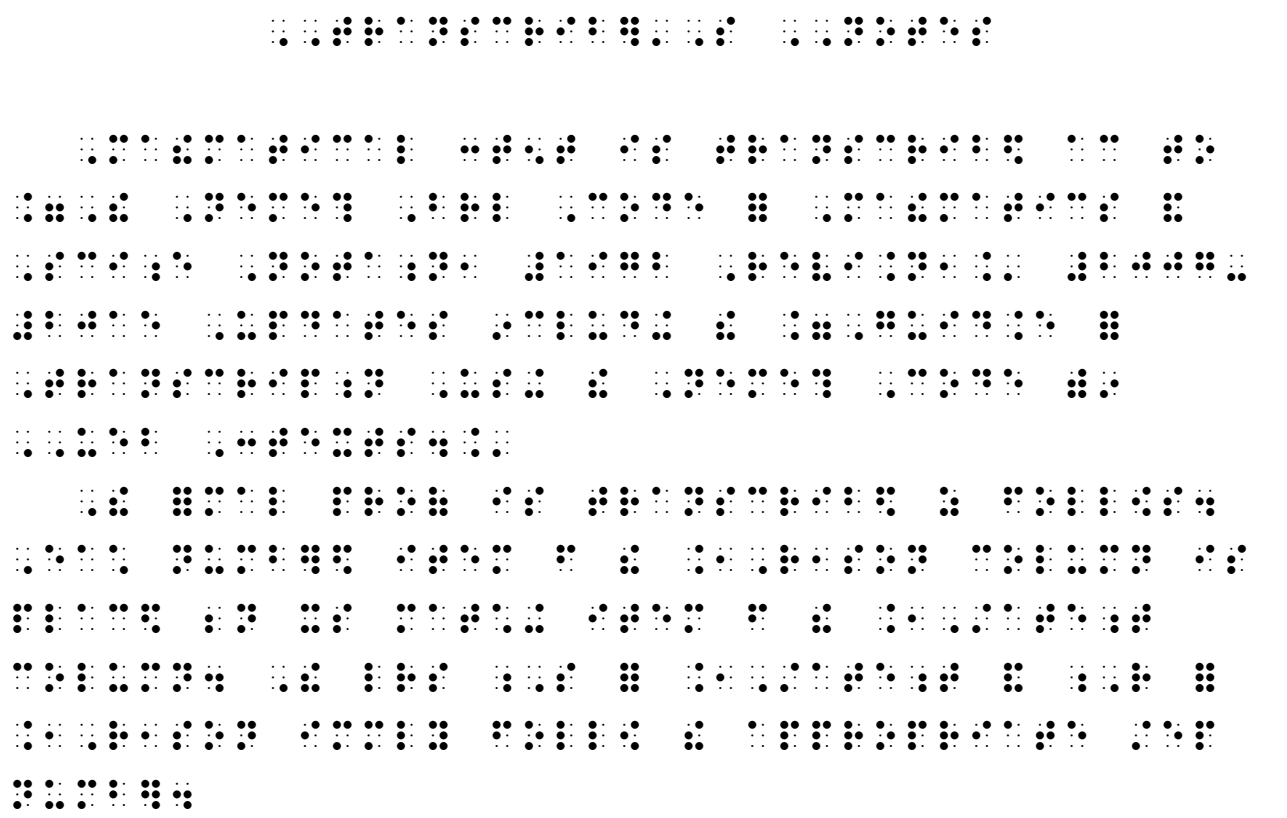
Title Page



PRACTICE 18A, cont.
Special Symbols Page

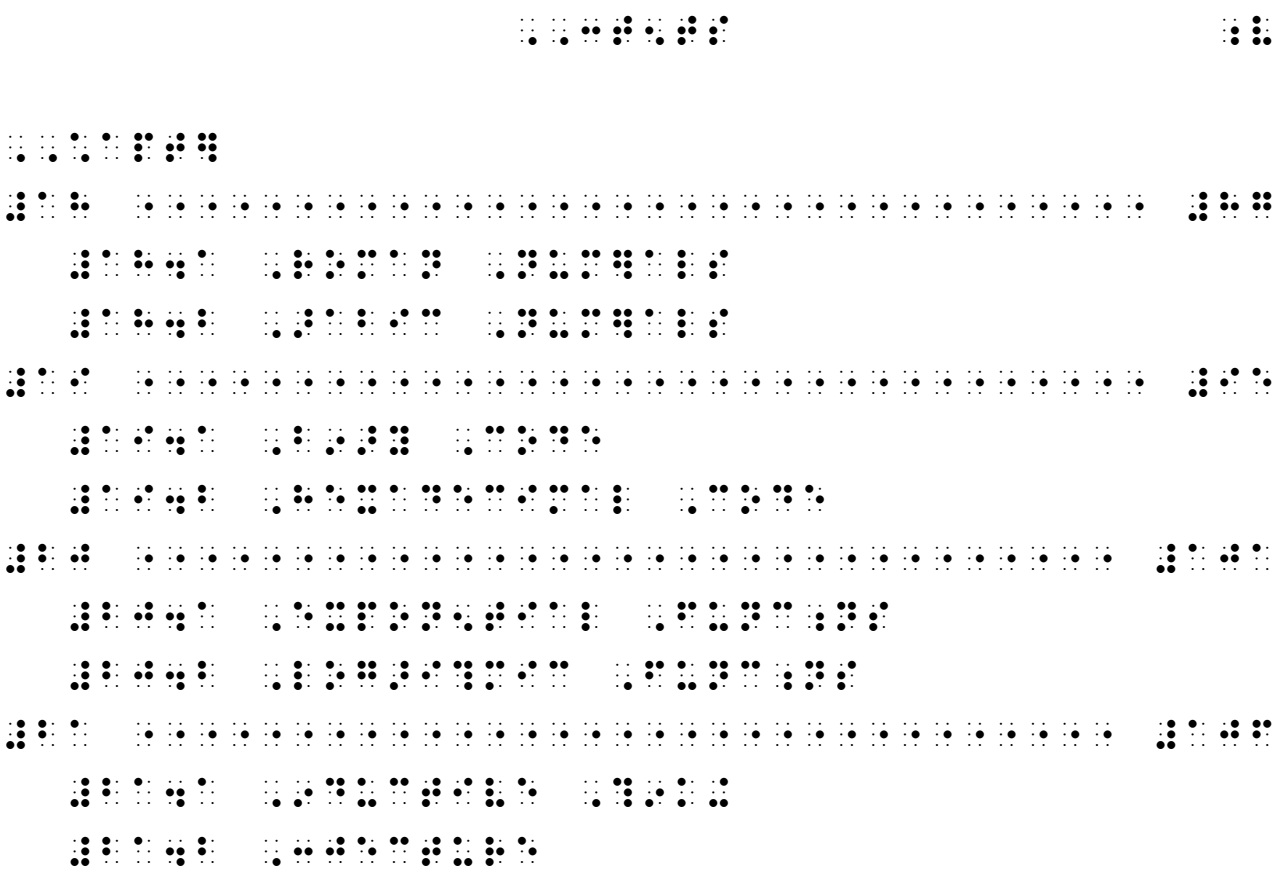


In addition to the symbols from Practices B-E, these symbols appear on the title page: dot in web address (line 4), acute accent (line 8), capitals mode terminator (line 17), and capitalized passage indicator (line 18). Note that the ampersand and copyright symbol need not be listed.



(line 25)

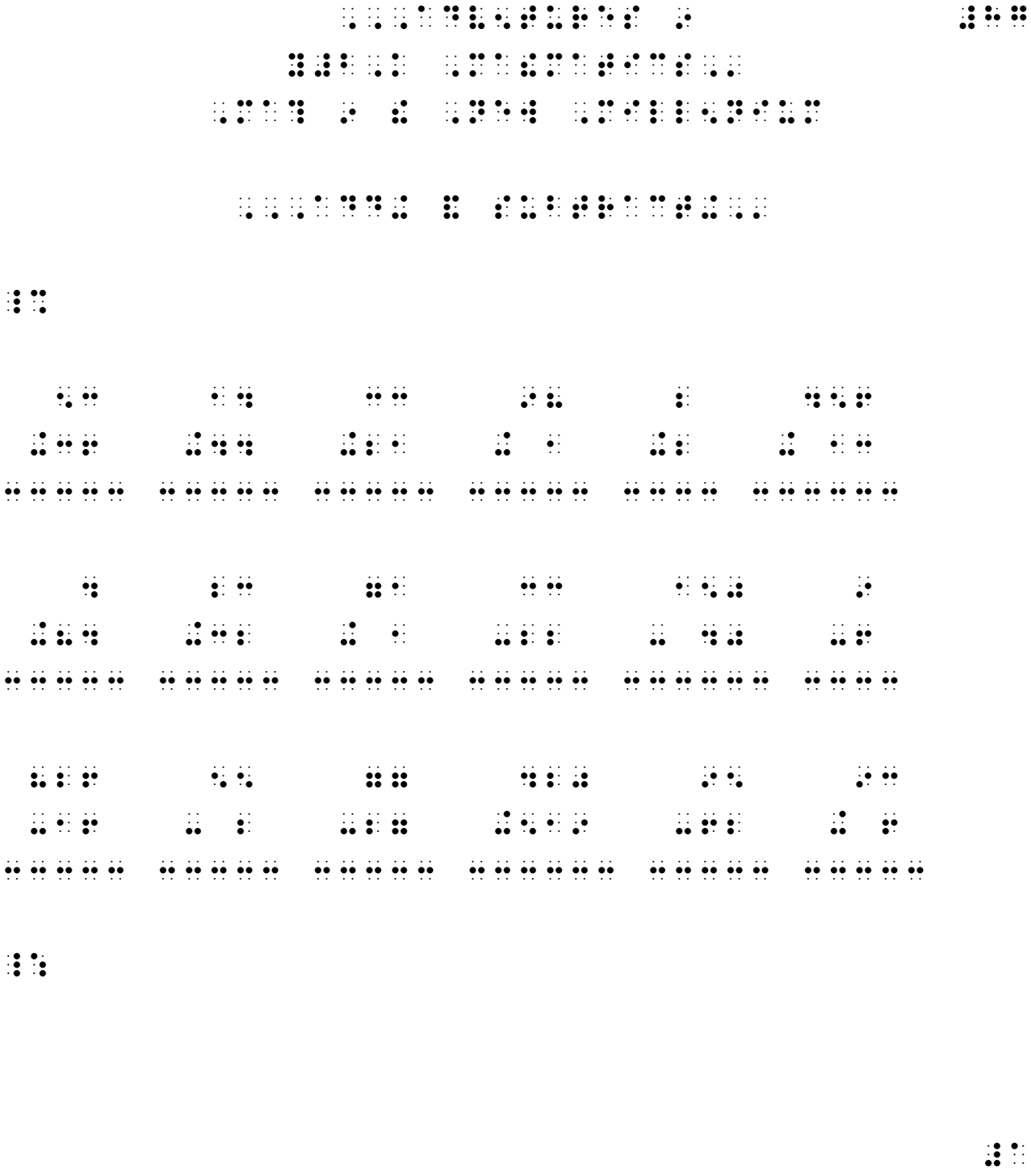
PRACTICE 18A, cont.
Contents Page



(line 25)

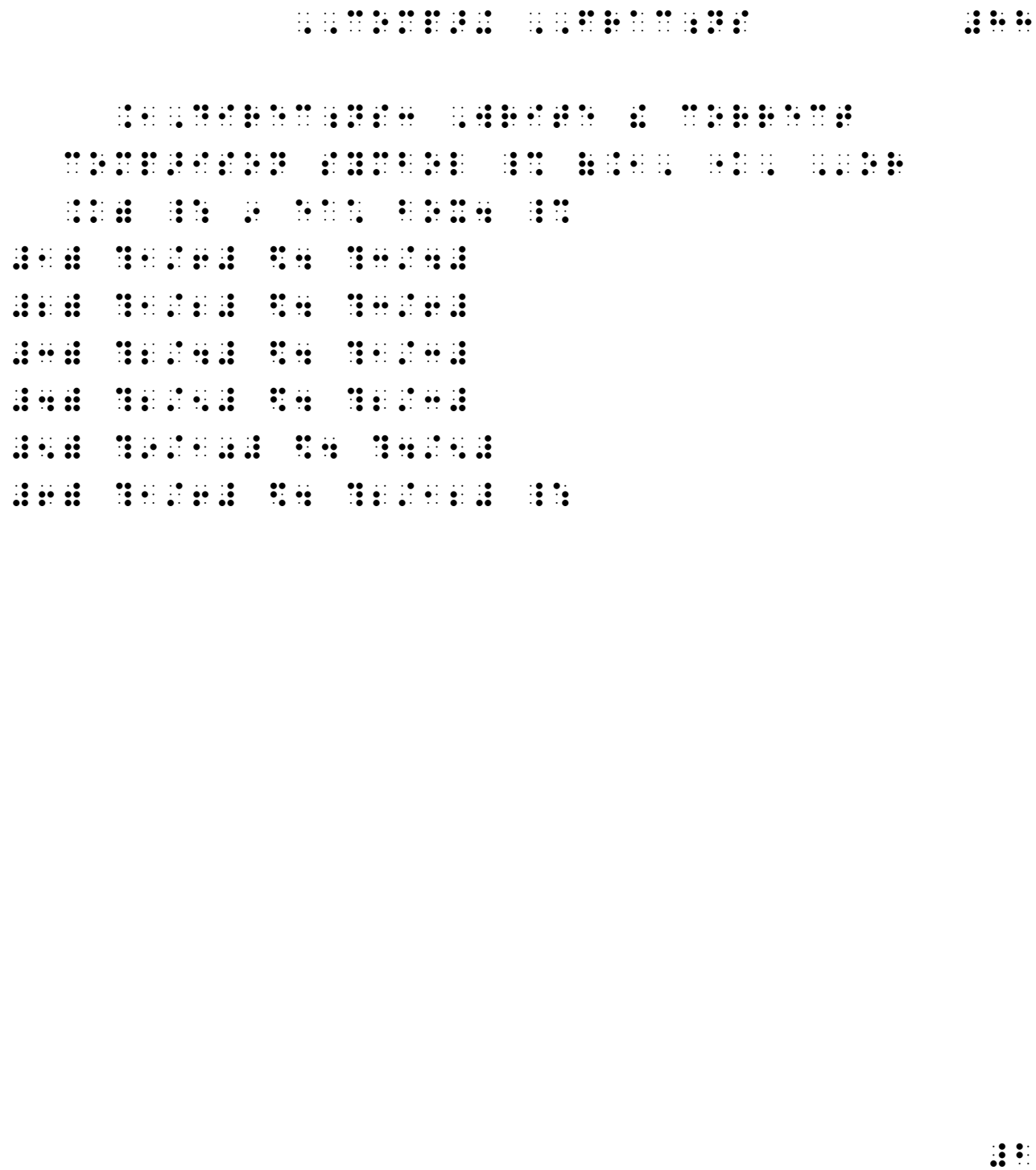
Line 1: Instructions for this page say to assume this is the second volume of a transcription. Implied page numbers i-iv are brailled in Volume 1 only. See Braille Formats *2.7.1 for details.*

PRACTICE 18B



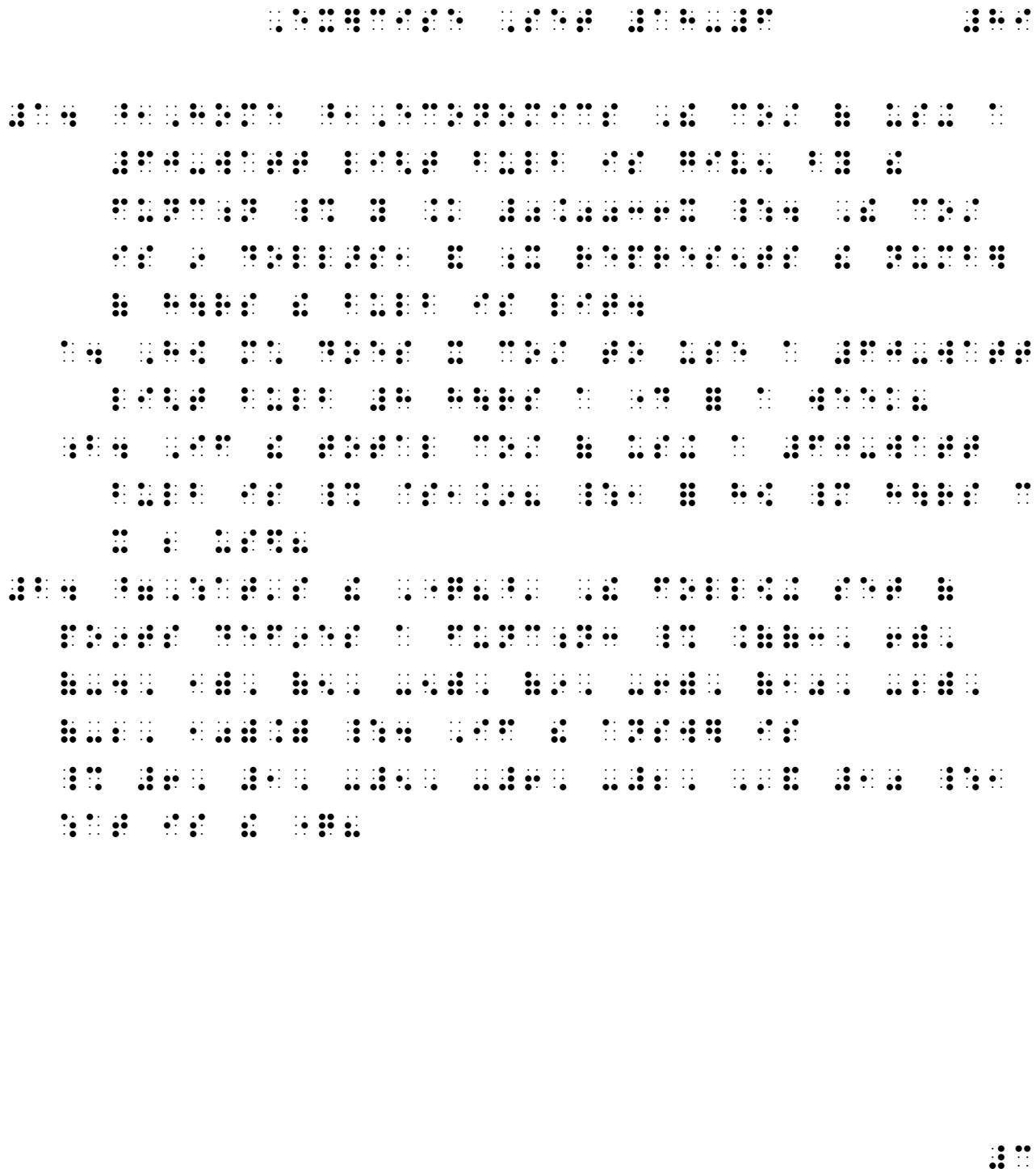
Format Comment: The Nemeth Code switch indicators do not take the place of required blank lines. A blank line follows the centered heading; a blank line precedes and follows the spatial material.

PRACTICE 18B, cont.



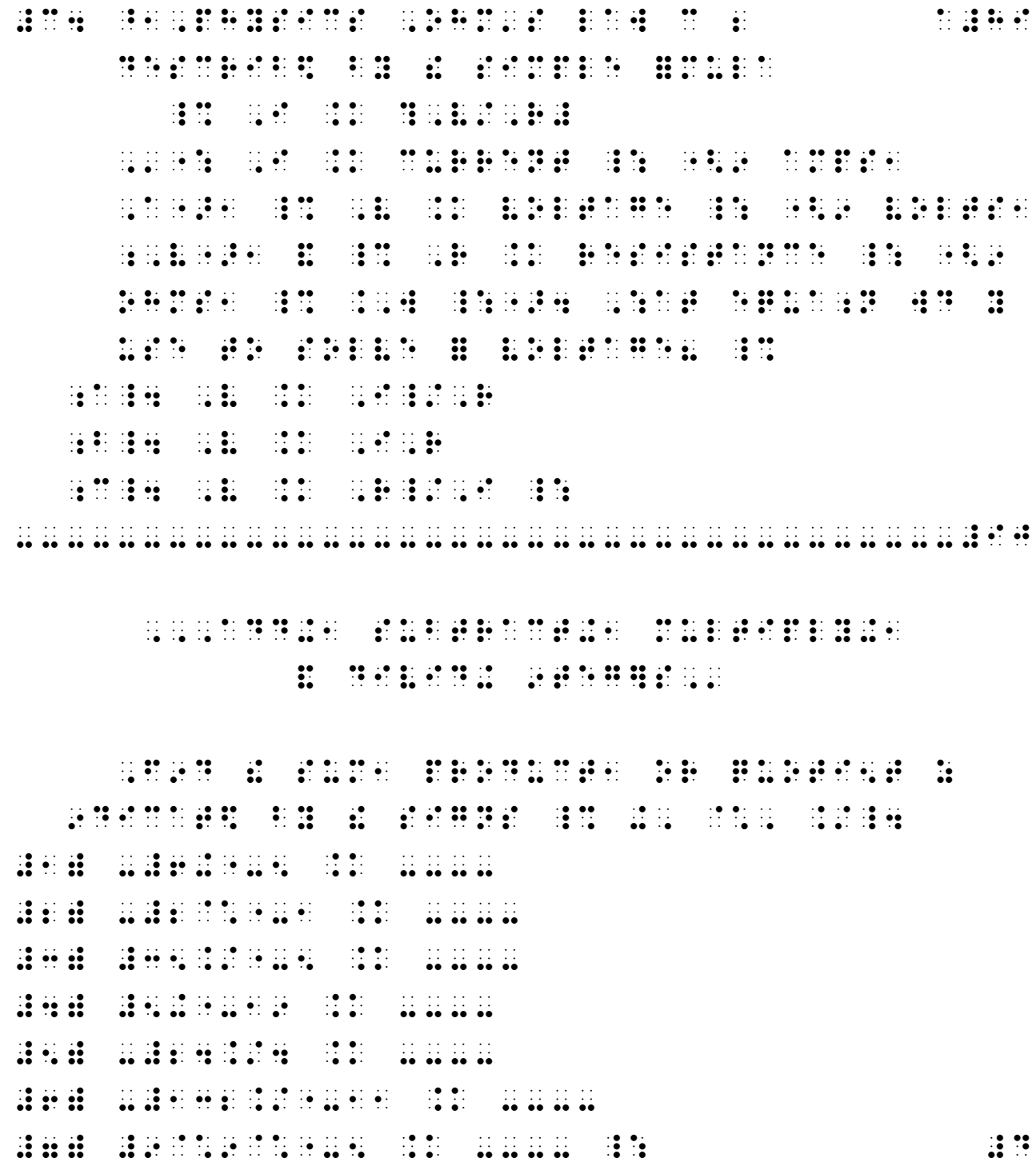
Lines 5-10: Side-by-side arrangement of nonspatial itemized material is not allowed in the braille transcription. Each identifier starts in cell 1. [NC]

PRACTICE 18C



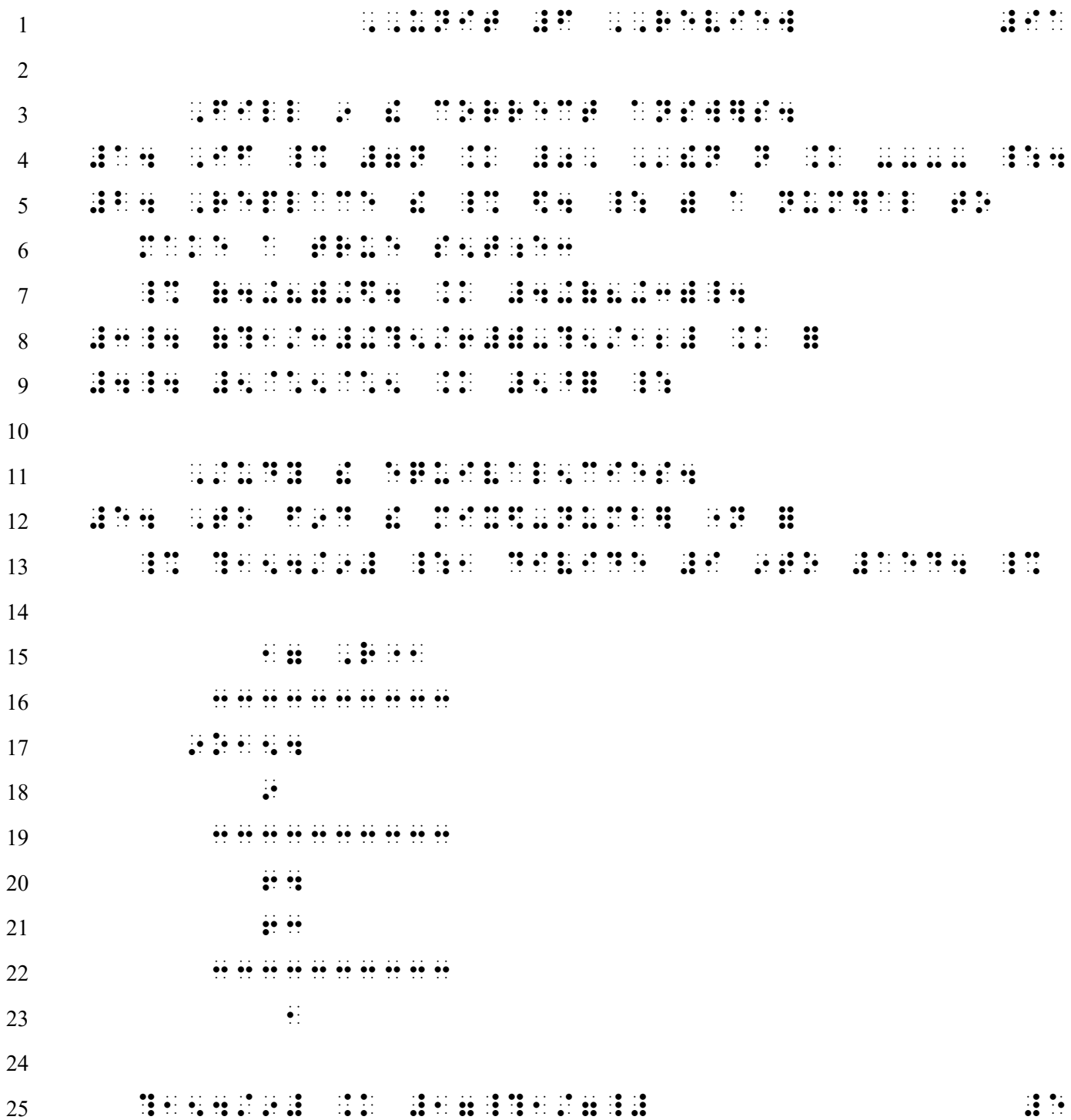
Line 1: The heading could be formatted as a cell-5 heading, in which case line 2 would not be blank. [BF]
Lines 14-18: Runover margins for itemized material are determined individually for each question. Problem #2 has no subitems, so its runovers are in cell 3. [NC]
Lines 19-25: The next problem set starts on the next braille page because question and answer choices should appear on a single page whenever space permits, regardless of the amount of blank space resulting on a page. [BF]

PRACTICE 18C, cont.

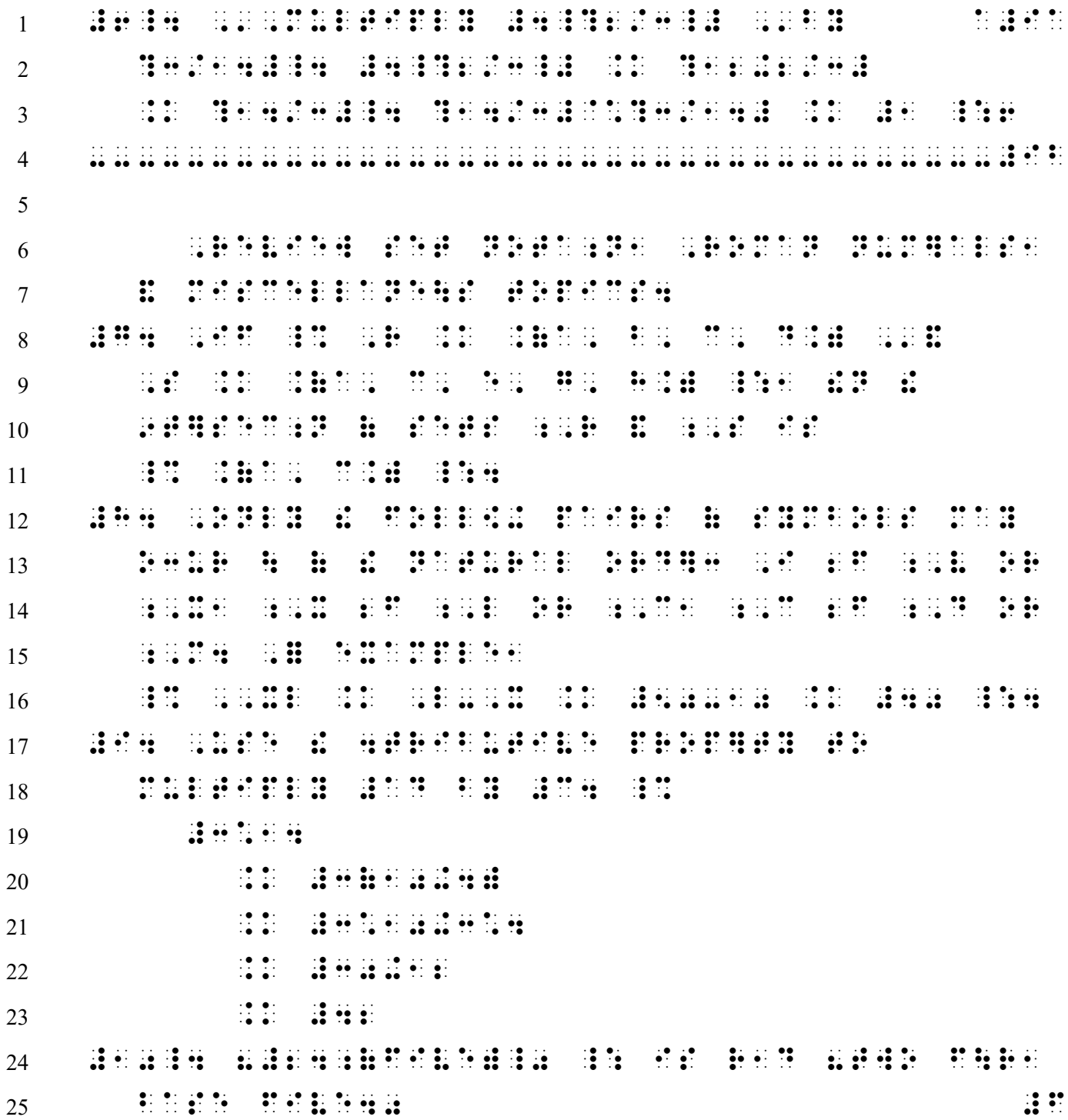


Lines 19-25: Side-by-side arrangement of nonspatial itemized material is not allowed in the braille transcription. Each identifier starts in cell 1. [NC]

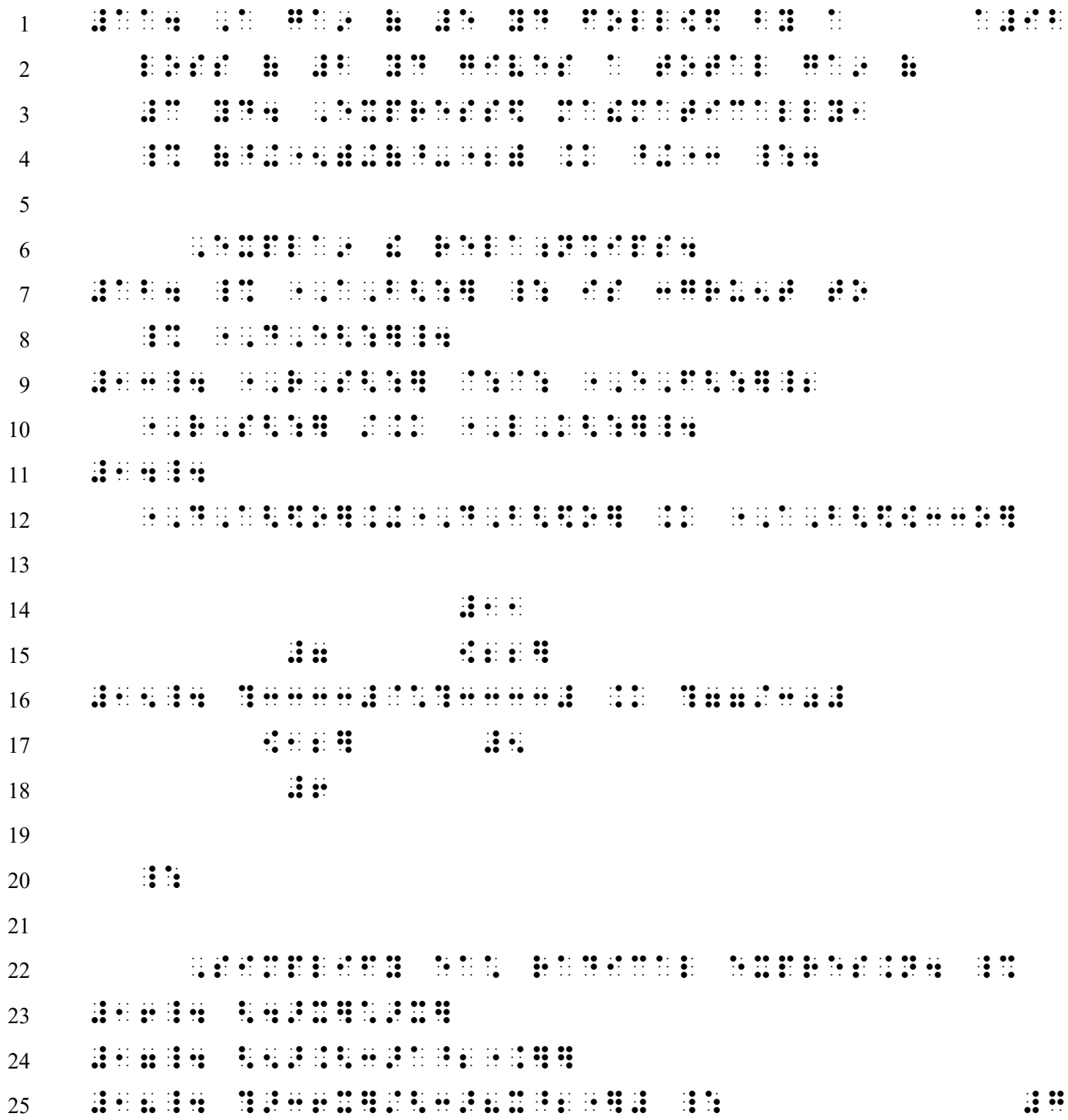
PRACTICE 18D



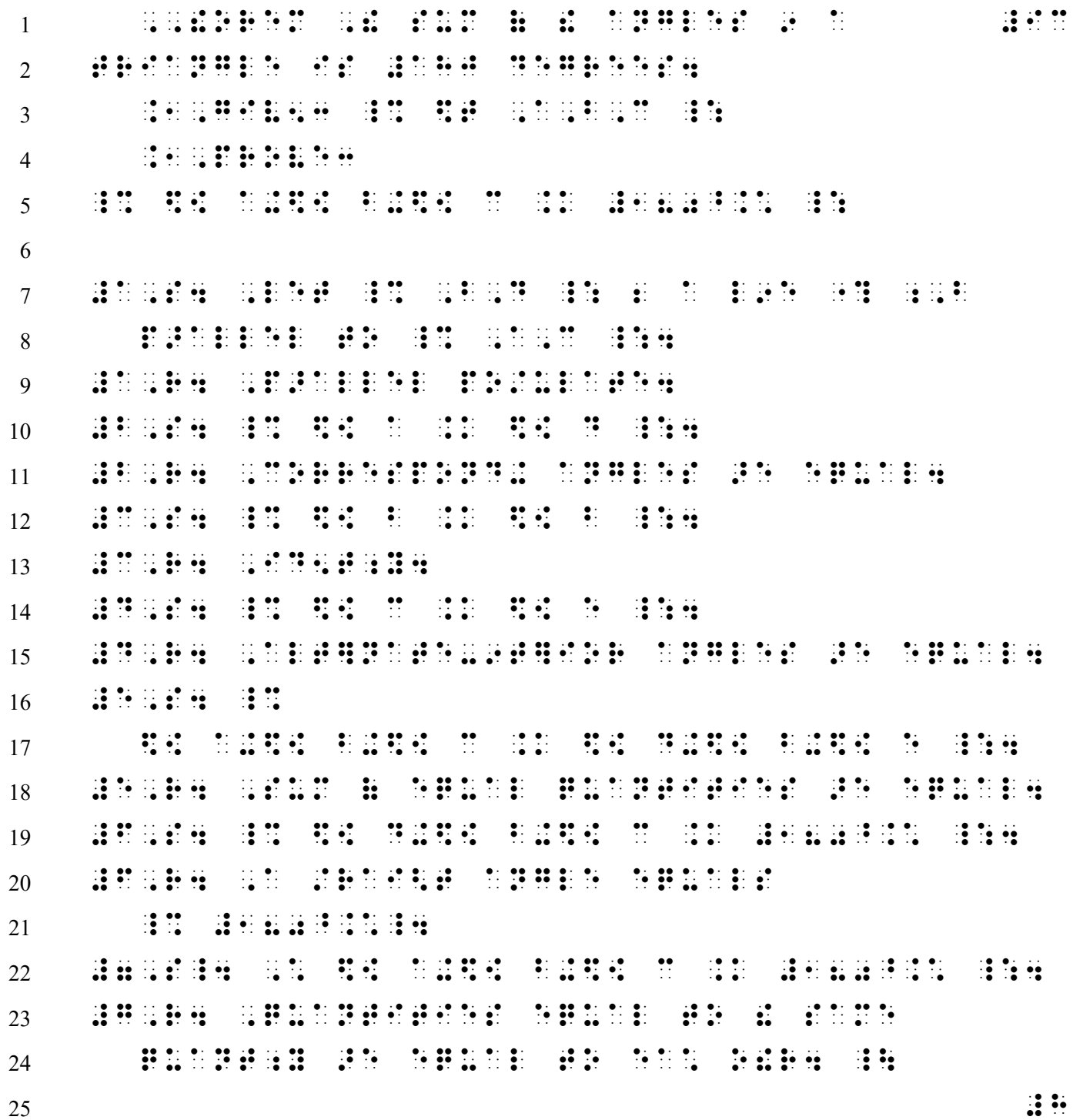
PRACTICE 18D, cont.



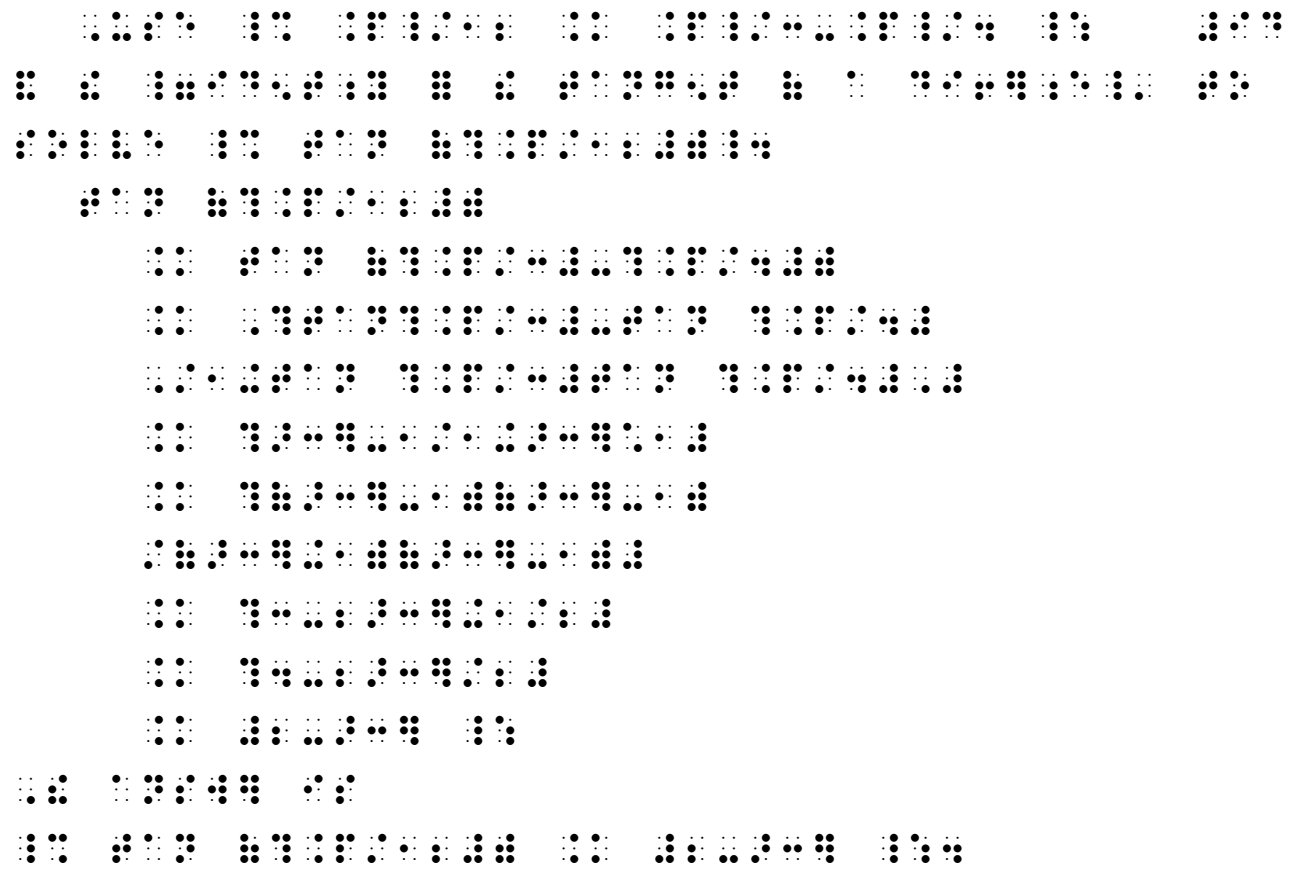
PRACTICE 18D, cont.



PRACTICE 18E



PRACTICE 18E, cont.



(line 25)

Notes: The displayed expression begins in cell 3 with runovers in cell 5. Because two of the links require division, each link begins on a new line in the runover cell. The divided links begin a new line with the main fraction line, and are not indented further because in print the signs of comparison are not vertically aligned.

EXERCISE 18

Exercise 18 will be available when this course is finished being written and is no longer "Provisional".

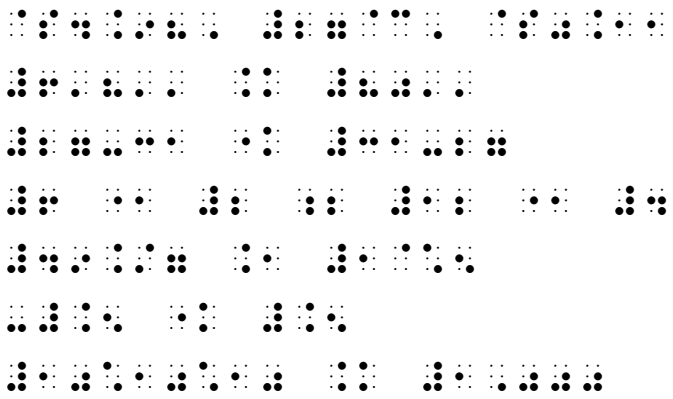
APPENDIX A — READING PRACTICE

Proofreading your work will improve as you become more proficient at recognizing the braille symbols. The exercises in this appendix will develop your braille reading skills.

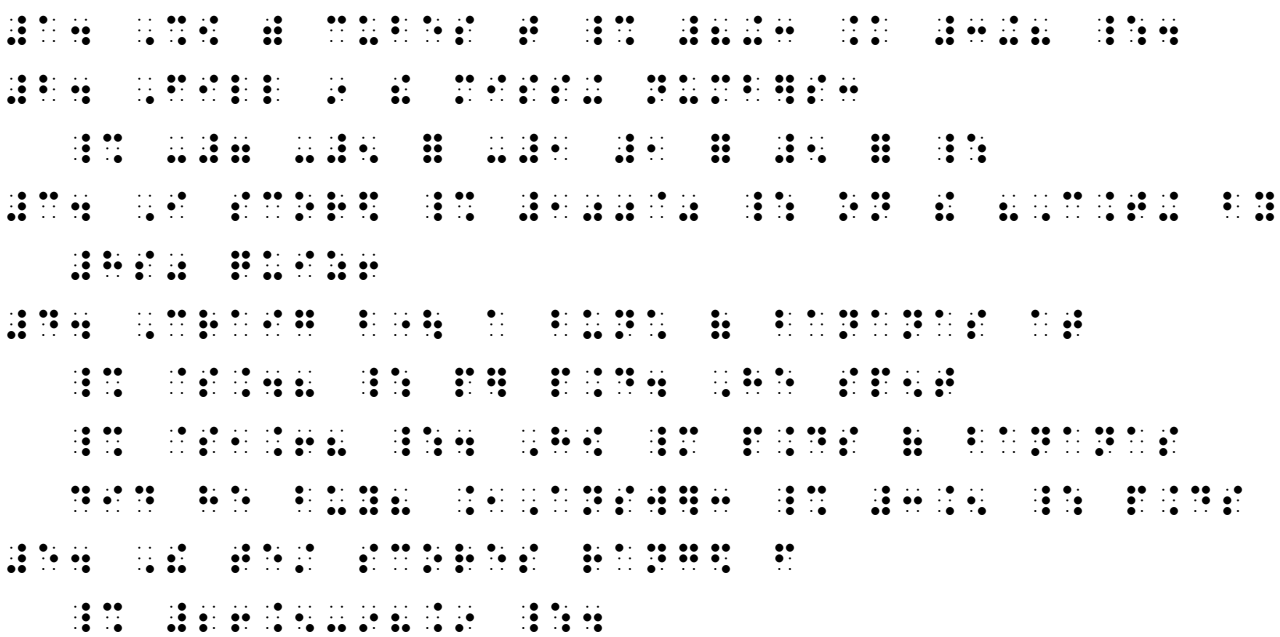
Instructions: After studying each lesson, practice writing out a translation of the simulated braille. Check your interpretation by comparing to the printed version found in the second half of this appendix.

These readings contain only a few of the topics covered in the course. You can gain more reading practice by back-translating the ANSWERS TO PRACTICE MATERIAL at the end of each lesson.

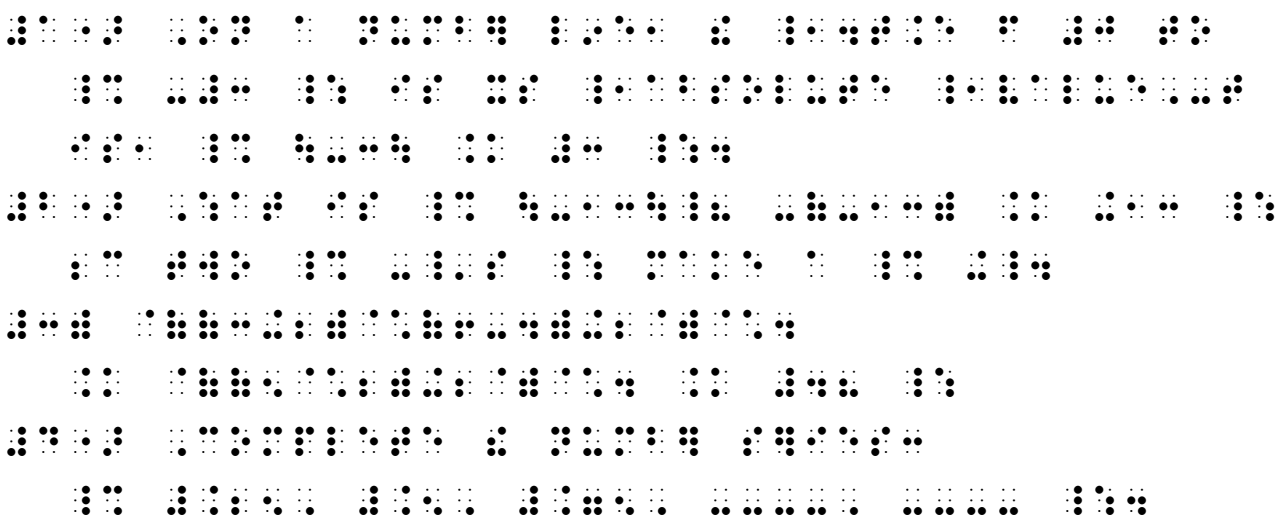
Lesson 1 Reading Practice



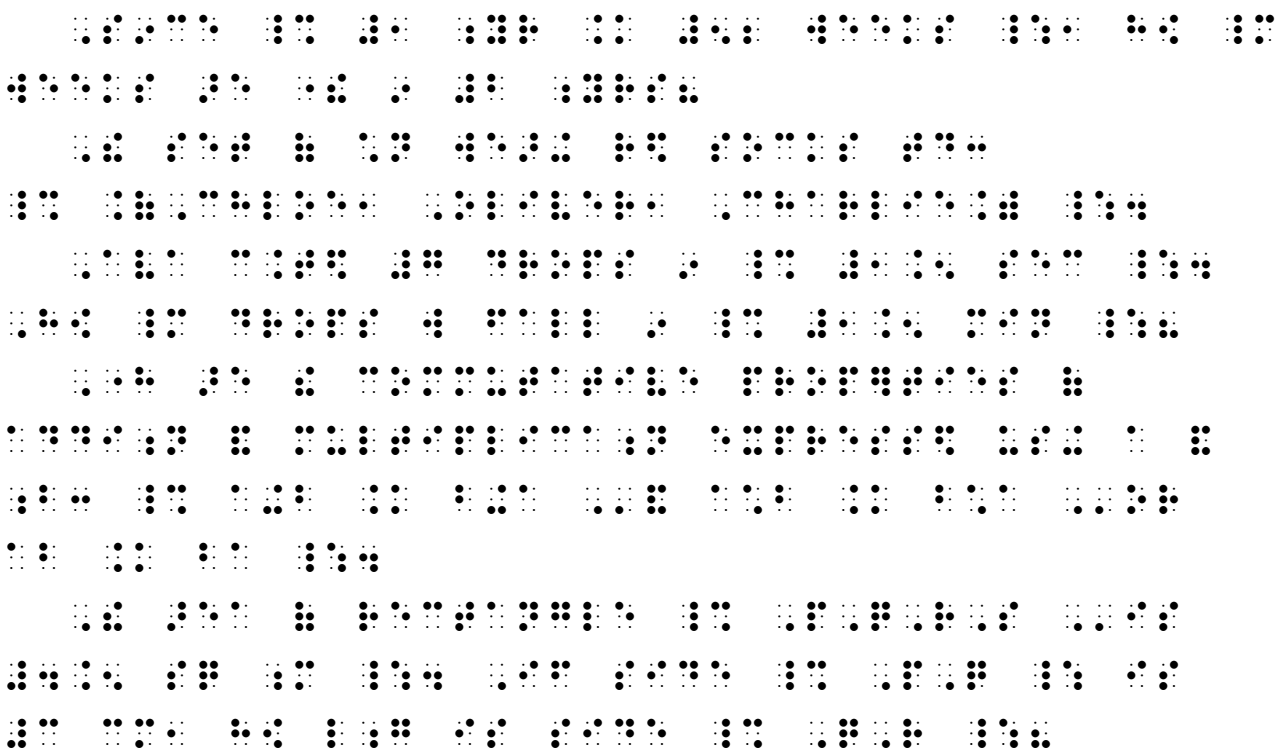
Lesson 2 Reading Practice



Lesson 3 Reading Practice



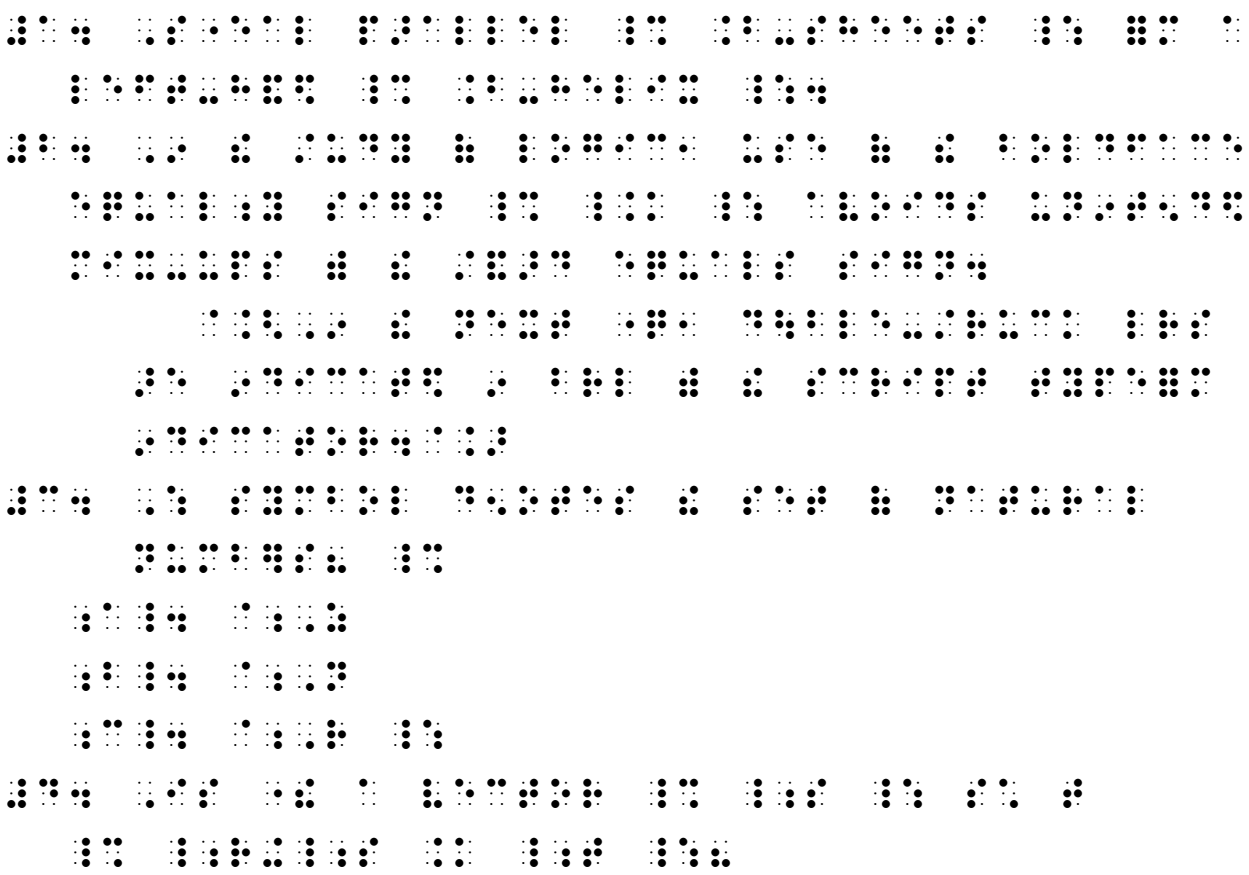
Lesson 4 Reading Practice



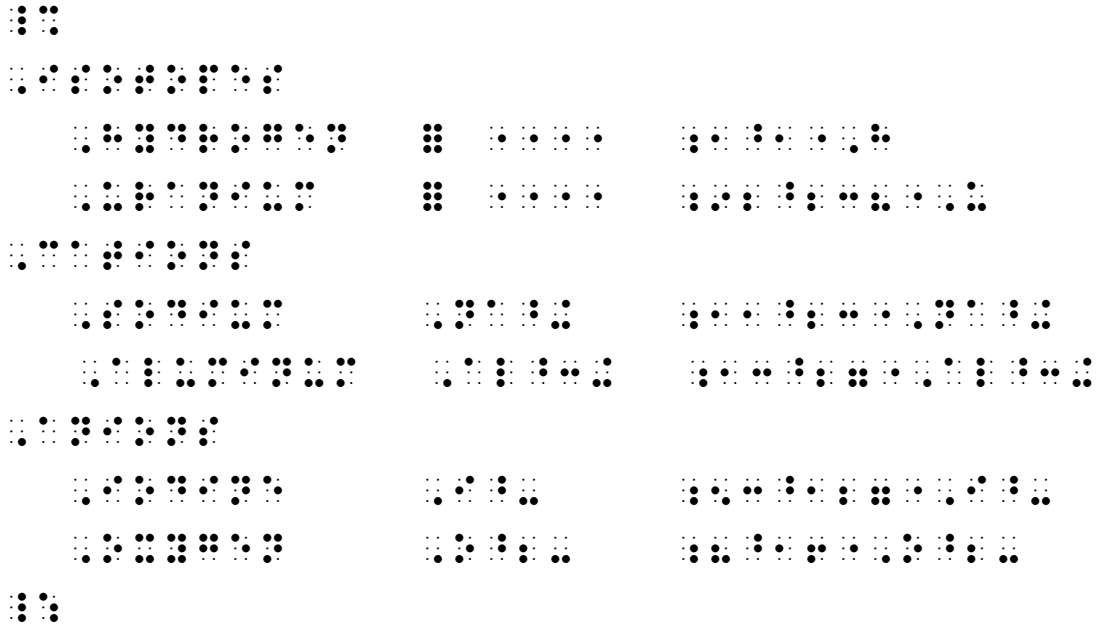
Lesson 5 Reading Practice

Lesson 6 Reading Practice

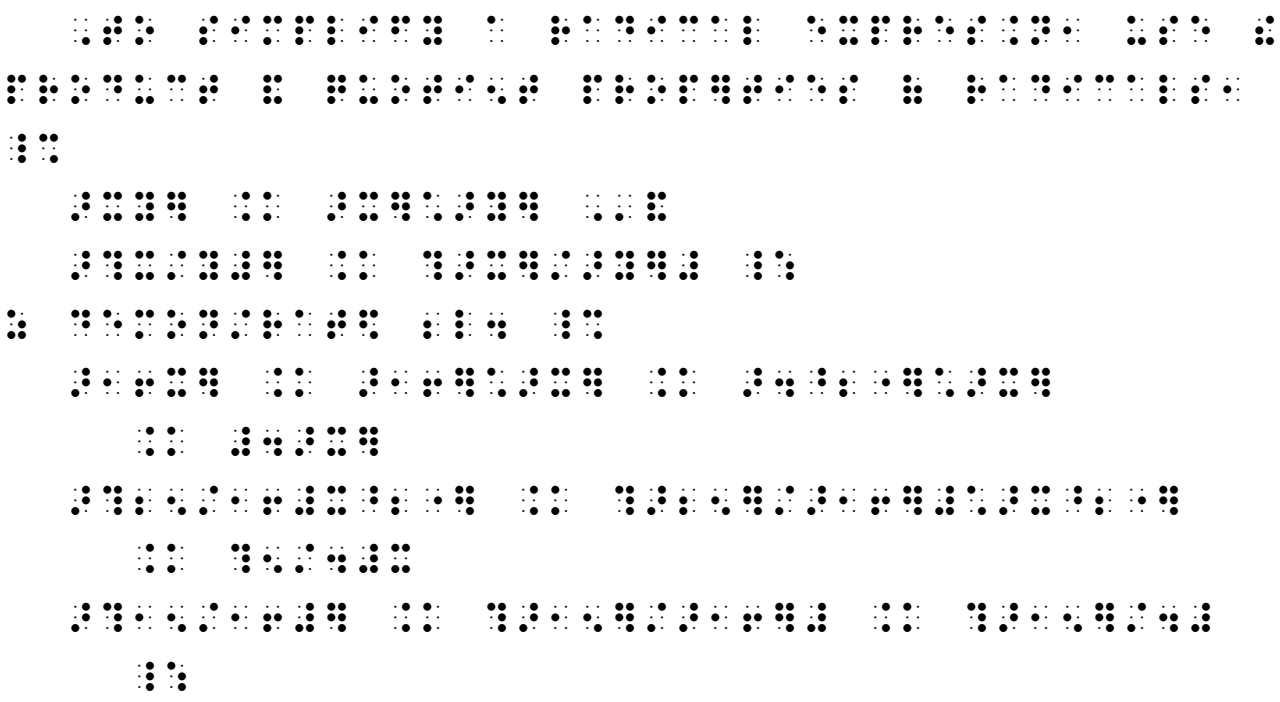
Lesson 7 Reading Practice



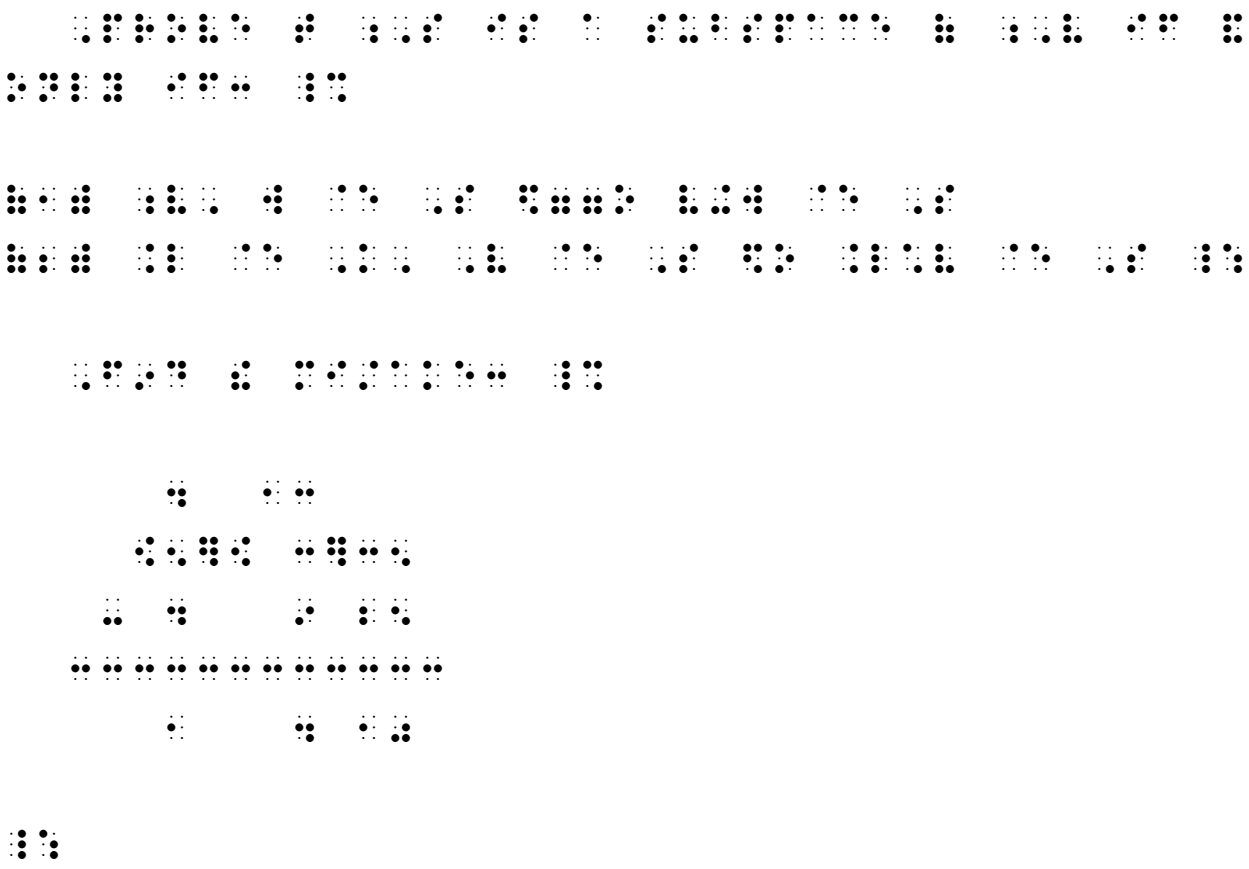
Lesson 8 Reading Practice



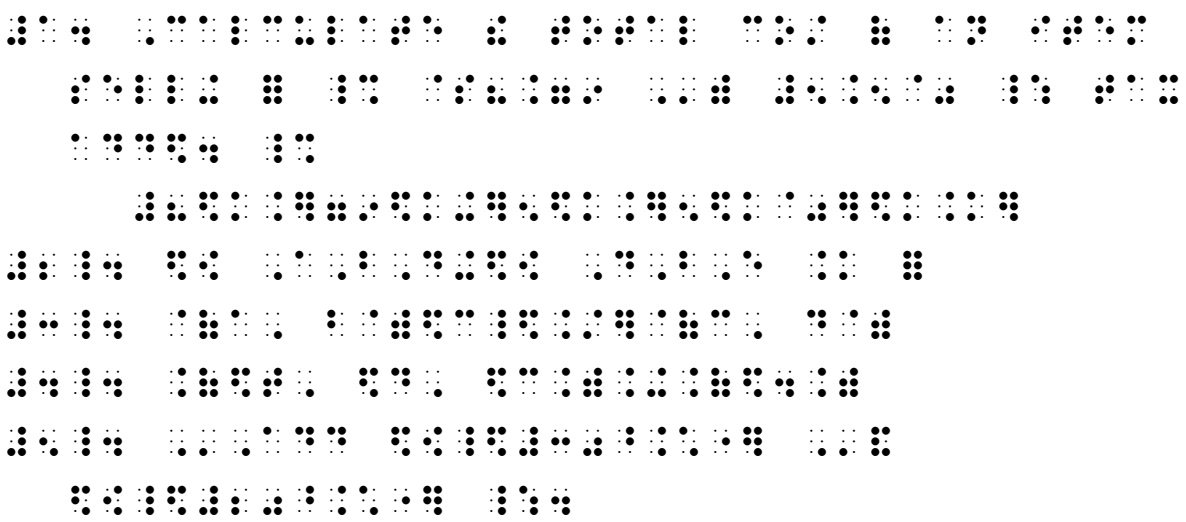
Lesson 9 Reading Practice



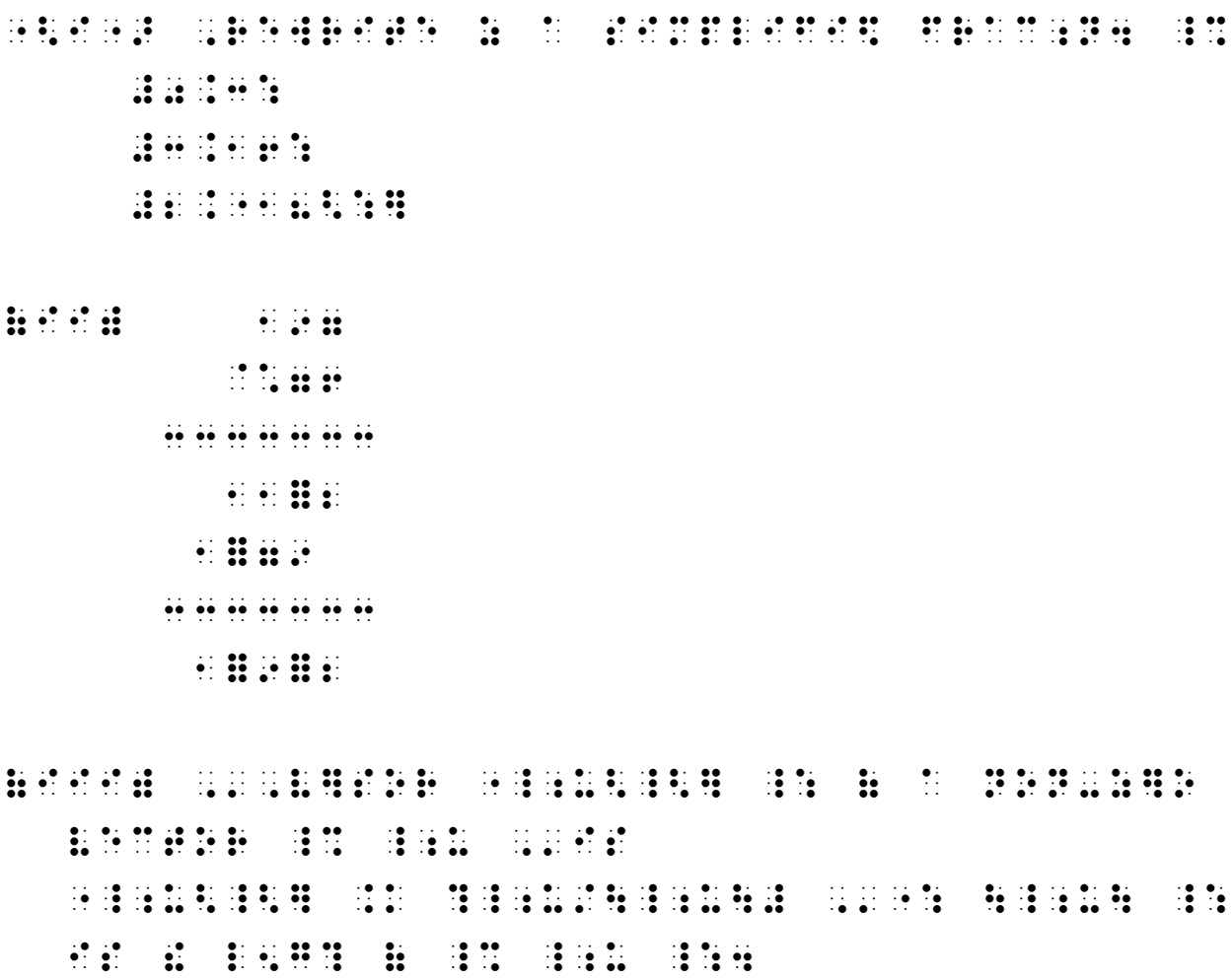
Lesson 10 Reading Practice



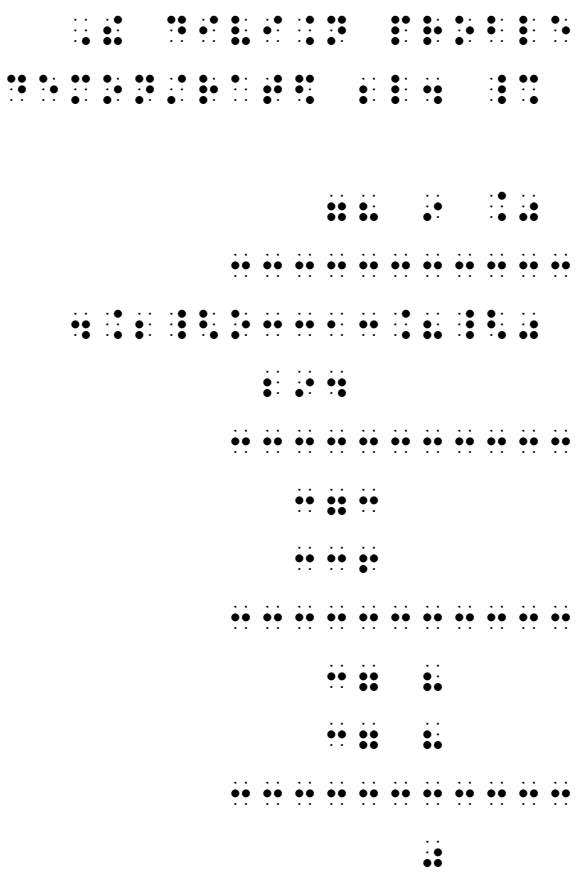
Lesson 11 Reading Practice

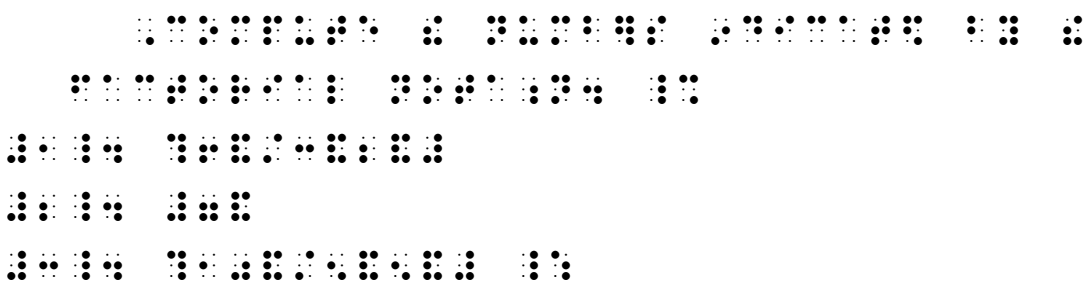


Lesson 12 Reading Practice

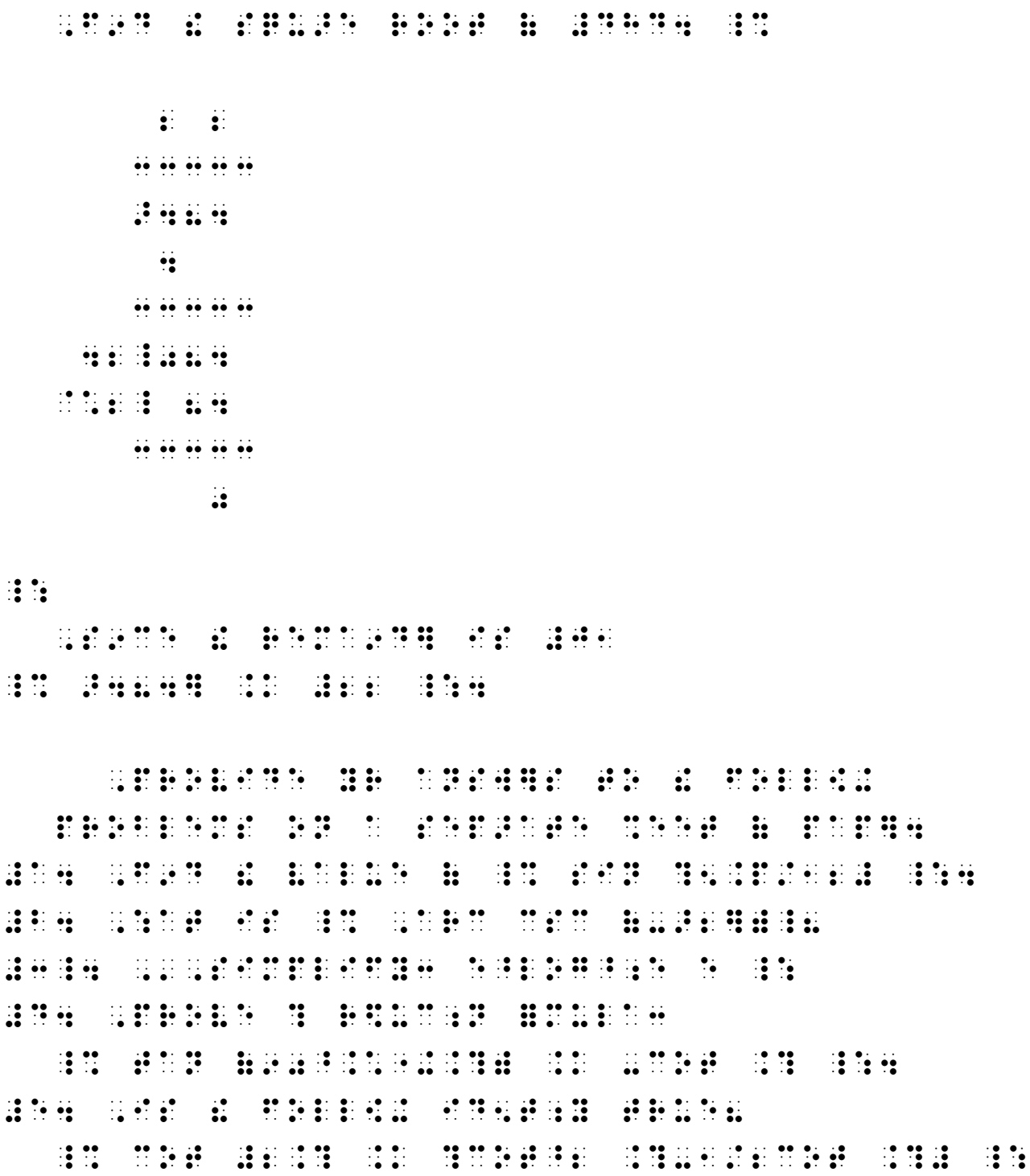


Lesson 13 Reading Practice

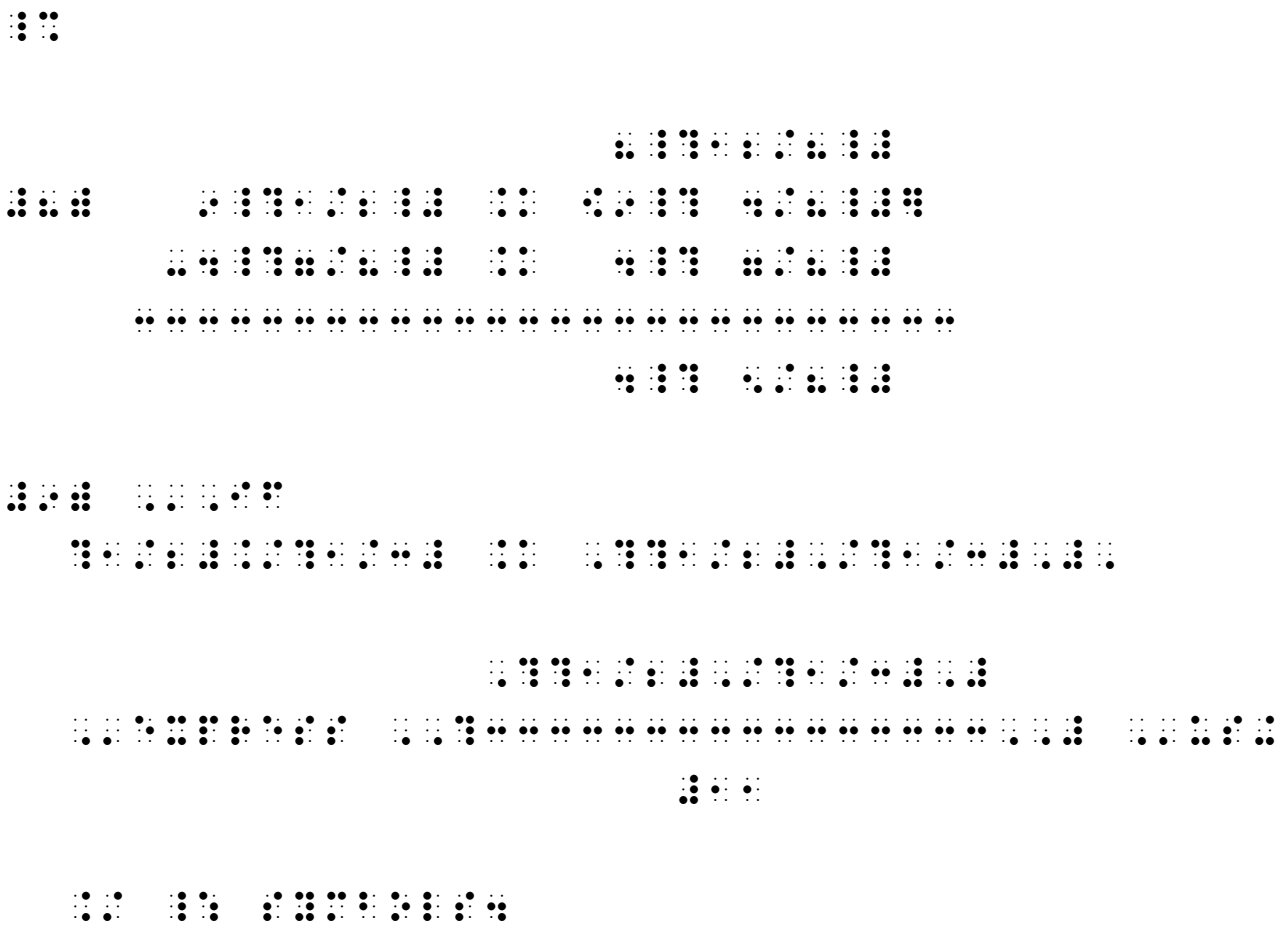




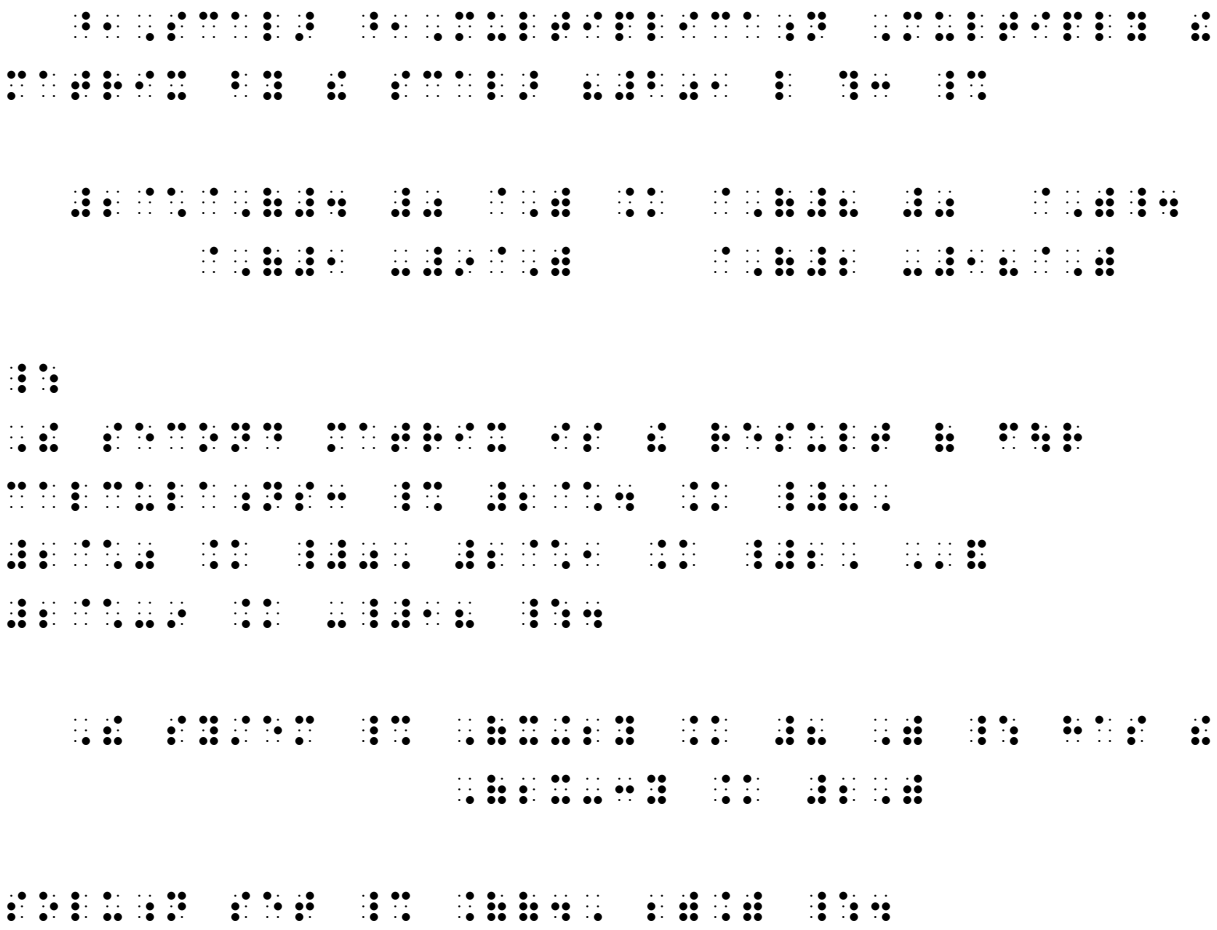
Lesson 14 Reading Practice



Lesson 15 Reading Practice



Lesson 16 Reading Practice



There are no reading practices for lessons 17 and 18.

ANSWERS

Lesson 1

$\$4.98,\ 27¢,\ \$0.11$

$6'8'' = 80''$

$27 - 31 < 31 - 27$

$6 : 2 :: 12 : 4$

$49 \div 7 > 1 \times 5$

$-.5 < .5$

$10 \cdot 10 \cdot 10 = 1{,}000$

Lesson 2

1. Show with cubes that $8 + 3 = 3 + 8$.
2. Fill in the missing numbers: $-7 \quad -5 \quad \underline{\ ?\ } \quad -1 \quad 1 \quad \underline{\ ?\ } \quad 5 \quad \underline{\ ?\ }$
3. I scored 100% on the "Counting By 8s" quiz!
4. Craig bought a bunch of bananas at \$.48 per pound. He spent \$1.68. How many pounds of bananas did he buy? *Answer:* 3.5 pounds
5. The test scores ranged from 26.5-98.9.

Lesson 3

1) On a number line, the distance from 0 to –3 is its absolute value—that is, $|-3| = 3$.
2) What is $|-13|$? $-(-13) = +13$ because two $-$'s make a $+$.
3) $[(3 + 2) \times (6 - 4) + 2] \times 4 = [(5 \times 2) + 2] \times 4 = 48$
4) Complete the number series: .25, .5, .75, ___, ___.

Lesson 4

Since 1 yr = 52 weeks, how many weeks are there in 2 yrs?

The set of children wearing red socks today: {Chloe, Oliver, Charlie}.

Ava counted 7 drops in 1.5 sec. How many drops will fall in 1.5 min?

Here are the commutative properties of addition and multiplication expressed using a and b: $a + b = b + a$ and $a \cdot b = b \cdot a$ or $ab = ba$.

The area of rectangle PQRS is 4.5 sq m. If side PQ is 3 cm, how long is side QR?

Lesson 5

i) The area of an ellipse, expressed as "A": $A = \pi ab$.

ii) Point (5, 7) is on ray ST.

iii) Power set notation may use the "Weierstrass P" as in P(S). If $S = \{\}$ then P(S)= {{ }} is returned.

iv) The hexadecimal system uses symbols 0-9 and A-F. For example, 45,997 in base 10 is B3AD in base 16.

TN Letters representing hexadecimal digits are capitalized in print. TN

Lesson 6

$\sim p \vee q$ spoken: "not p or q (or both)".

Integer division is sometimes denoted \, as illustrated here: $10/3 = 3 + 1/3$, so $10\backslash 3 = 3$. The remainder is not noted.

Graph this inequality: $y \leq x + 2$

Use a number line to explain why $-6 - -6 = 0$. Is this the same as $-6 - +6$?

1 # (2 & 3) = (1 # 2) & (1 # 3)

Lesson 7

1. Several parallel β-sheets form a left-handed β-helix.
2. In the study of logic, use of the boldface equality sign $\mathbf{=}$ avoids unintended mix-ups with the standard equals sign.

   TN In the next question, double-struck letters are indicated in braille with the script typeform indicator. TN

3. Which symbol denotes the set of natural numbers?
   a. $\mathbb{Z}$
   b. $\mathbb{N}$
   c. $\mathbb{R}$
4. Is there a vector $\mathbf{s}$ such that $\mathbf{r} + \mathbf{s} = \mathbf{t}$?

Lesson 8

Isotopes

Hydrogen		${}^{1}_{1}\mathrm{H}$
Uranium		${}^{238}_{92}\mathrm{U}$

Cations

Sodium	Na^{+}	${}^{23}_{11}\mathrm{Na}^{+}$
Aluminum	Al^{3+}	${}^{27}_{13}\mathrm{Al}^{3+}$

Anions

Iodine	I^{-}	${}^{127}_{53}\mathrm{I}^{-}$
Oxygen	O^{2-}	${}^{16}_{8}\mathrm{O}^{2-}$

Lesson 9

To simplify a radical expression, use the product and quotient properties of radicals,

$$\sqrt{xy} = \sqrt{x} \cdot \sqrt{y} \text{ and } \sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$$

as demonstrated below.

$$\sqrt{16x} = \sqrt{16} \cdot \sqrt{x} = \sqrt{4^2} \cdot \sqrt{x} = 4\sqrt{x}$$

$$\sqrt{\frac{25}{16}\,x^2} = \frac{\sqrt{25}}{\sqrt{16}} \cdot \sqrt{x^2} = \frac{5}{4}x$$

$$\sqrt{\frac{15}{16}} = \frac{\sqrt{15}}{\sqrt{16}} = \frac{\sqrt{15}}{4}$$

Lesson 10

Prove that S is a subspace of V if and only if:

(1) $v, w \in S \Rightarrow v + w \in S$

(2) $\lambda \in K, V \in S \rightarrow \lambda \cdot v \in S$

Find the mistake:

$$\begin{array}{r} {\scriptstyle 4\ 13} \\ \not{5}\ \not{3}\ 3\ 5 \\ -\ 4\ 9\ 2\ 5 \\ \hline 1\ 4\ 1\ 0 \end{array}$$

Lesson 11

1. Calculate the total cost of an item selling for $8.79 with 5.5% tax added.

   8 [.] 7 9 [+] 5 [.] 5 [%] [=]

2. $\angle ABD + \angle DBE = ?$

3. [a, b] ⨟ [c, d]

4. $\{\triangle, \diamond, \bigcirc\} \cup \{\square\}$

5. Add $\angle 30°$ and $\angle 20°$.

Lesson 12

Note: Your translation may show a different omission symbol in item (ii).

(i) Rewrite as a simplified fraction.

$0.\overline{3}$

$3.1\overline{6}$

$2.\overline{18}$

(ii)
$$\begin{array}{r} 197 \\ \times\ 76 \\ \hline 11\square 2 \\ 1\square 79\ \ \\ \hline 1\square 9\square 2 \end{array}$$

(iii) Versor $\hat{\mathbf{u}}$ of a non-zero vector $\mathbf{u}$ is $\hat{\mathbf{u}} = \frac{\mathbf{u}}{|\mathbf{u}|}$ where $|\mathbf{u}|$ is the length of $\mathbf{u}$.

Lesson 13

The division problem $4.2\overline{)3313.8}$ is demonstrated below.

$$\begin{array}{r} 789.0 \\ 4.2\hat{}\,\overline{)3313.8\hat{}0} \\ 294\ \ \ \ \ \ \\ \hline 373\ \ \ \ \\ 336\ \ \ \ \\ \hline 37\ 8\ \ \\ 37\ 8\ \ \\ \hline 0 \end{array}$$

Compute the numbers indicated by the factorial notation.

1. $\frac{6!}{3!2!}$
2. $7!$
3. $\frac{10!}{5!5!}$

Lesson 14

Find the square root of 484.

$$\begin{array}{r} 2\ 2 \\ \sqrt{484} \\ 4 \\ 42\ \big|\ 084 \\ \times 2\ \big|\ 84 \\ \hline 0 \end{array}$$

Since the remainder is 0, $\sqrt{484} = 22$.

Provide your answers to the following problems on a separate sheet of paper.

1. Find the value of $\sin\frac{5\pi}{12}$.
2. What is Arc csc $\left(-\sqrt{2}\right)$?
3. Simplify: $e^{\log_e e}$
4. Prove this reduction formula : $\tan(90° + \theta) = -\cot\theta$.
5. Is the following identity true? $\cot 2\theta = \frac{\cot^2\theta - 1}{2\cot\theta}$

Lesson 15

8) $$\begin{array}{r} 8\frac{12}{8} \\ 9\frac{1}{2} = \cancel{9}\frac{4}{8} \\ -\ 4\frac{7}{8} = 4\frac{7}{8} \\ \hline 4\frac{5}{8} \end{array}$$

9) If $\frac{1}{2} \div \frac{1}{3} = \frac{\frac{1}{2}}{\frac{1}{3}}$, express $\frac{\frac{\frac{1}{2}}{\frac{1}{3}}}{11}$ using ÷ symbols.

Lesson 16

Scalar Multiplication Multiply the matrix by the scalar "2", like this:

$$2 \times \begin{bmatrix} 4 & 0 \\ 1 & -9 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 2 & -18 \end{bmatrix}.$$

The second matrix is the result of four calculations: $2 \times 4 = \mathbf{8}$, $2 \times 0 = \mathbf{0}$, $2 \times 1 = \mathbf{2}$, and $2 \times -9 = \mathbf{-18}$.

The system $\begin{pmatrix} x+2y=8 \\ 2x-3y=2 \end{pmatrix}$ has the solution set $\{(4,\ 2)\}$.

APPENDIX B — GLOSSARY OF TERMS

This glossary contains a sampling of terminology encountered in K-12 mathematics which transcribers may find useful when communicating in workshops, forums, and blogs. There are many dictionaries of mathematical terms for those interested in a more comprehensive glossary.

Mathematical expression: A combination of mathematical symbols grouped together to designate the value of something.

Numerical expression: A mathematical expression containing only numbers and operation symbols. EXAMPLE $14 \div 2$

Algebraic expression: A mathematical expression containing variables, numbers, and operation symbols. EXAMPLE $3x + 2y - 7$

The Four Basic Operations

Addition

```
  2    addend
+3     addend
  5    sum
```

Subtraction

```
  7    minuend
-1     subtrahend
  6    difference
```

Multiplication

```
  2    multiplicand
×3     multiplier
  6    product
```

```
   5.009
  × .27
  35063     partial product
 10018      partial product
1,352.43    product
```

Division dividend ÷ divisor = quotient

```
    4              quotient
10 )40          divisor )dividend
```

```
   4 r4         the "r" number is the remainder
9 )40
  36            partial quotient
   4            difference
```

Many Ways to Denote Multiplication

Print has several ways of denoting multiplication. Nemeth Code provides symbols for all of them, which enables us to follow print. Print may space between items for clarity, but factors are not spaced in braille unless abbreviations, function names, or certain symbols are contained in the expression.

All of the following mean "two times y".

$2 \times y$		(a)
$2 \cdot y$		(b)
$2 * y$		(c)
$2y$		(d)
$(2)(y)$		(e)

(a) 2×3 The multiplication cross is used the most in the lower grades.

(b) $2 \cdot 3$ The multiplication dot is used when the "x" variable is introduced, since the x and the multiplication cross can be confused in print.

(c) $2 * 3$ The multiplication asterisk is used most often in calculator operations since, on calculator keys, the multiplication cross may be confused with the variable x and the multiplication dot may be confused with the decimal.

(d) $2y$ is the form you will see most often when one of the factors is a variable, a constant, or a radical expression. $2xy$ means "two time x times y". 2π means "two times pi". $2\sqrt{5}$ means "two times the square root of 5".

(e) Enclosure symbols are used to group factors when the multipliers require calcuations.

EXAMPLE $(x + y)(x - y)$ means "multiply $x + y$ times $x - y$".

If a factor uses parentheses, brackets group the outer factor.

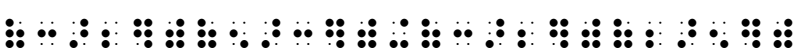
EXAMPLE $5[(x + y)(x - y)]$ means "five times the product of $x + y$ times $x - y$".

The expression $(3\sqrt{2})(5\sqrt{3}) + (3\sqrt{2})(2\sqrt{5})$ can be analyzed as follows.

- The overall expression is the sum (addition) of two terms. Addition is indicated with a plus sign. __ + __
- Each addend is the product (multiplication) of two factors, indicated with side-by-side parentheses. (__)(__)
- Each multiplier is the product of a numeral and a square root. No multiplication sign is shown but multiplication is implied. _$\sqrt{\ }$

Spoken: "(Three times the square root of two TIMES five times the square root of three) PLUS (three times the square root of two TIMES two times the square root of five)."

Transcription: The braille contains no spaces:



Fractions

A fraction consists of a *numerator* and a *denominator* separated by a *fraction line*.

Simple Fraction This fraction has a horizontal fraction line.

$\frac{3}{4}$ numerator
denominator

This fraction has a diagonal fraction line. $3/4$

Mixed Number A mixed number is one quantity made up of a whole number and a simple fraction.

$7\frac{1}{2}$

Complex Fraction A complex fraction's numerator and/or denominator is also a fraction.

$\frac{1/2}{4}$ numerator denominator $\frac{4}{1/2}$

Geometry

Irregular polygon: A closed figure with at least two unequal sides and two unequal angles

Regular polygon: A closed figure with equal sides and equal angles

Types of triangles:
Acute: All angles are less than 90°
Equilateral: Three equal sides; three equal angles (always 60°)
Isosceles: Two equal sides; two equal angles
Obtuse: Has one angle greater than 90°
Right: Contains one right angle (90°)
Scalene: No equal sides; no equal angles

Radical Expressions

$\sqrt{144}^{12}$ $\sqrt{\text{radicand}}^{\text{root}}$

Radical sign: √
Vinculum: The line which extends above the radicand.
Index: A small number that may appear next to the radical sign is the *index* of the radical.

$\sqrt[3]{9}$ 3 is the index of this radical.

Statements and Functions

Exponential statement:	$b^e = x$	*b* is the base; *e* is the exponent.
Logarithmic statement:	$\log_b x = y$	*x* is the *argument* of the log.
Trigonometric function:	$\sin\theta = 1$	θ (theta) is the *argument* of the function.

Algebra

The following algebraic expression is used as a sample in the definitions below. $3x + 2y - 7$

Coefficient: When a term is made up of a constant placed before a variable, the constant is called a coefficient. In the sample expression, the coefficient of *x* is 3. The coefficient of *y* is 2.

Constant: A constant is a number that is fixed and known, or a letter which stands for a fixed number, such as π. The constants in the sample expression are 3, 2, and 7.

Term: An algebraic expression is made up of terms. Each term is separated by a + or a – sign. The terms in the sample expression are 3*x*, 2*y*, and 7.

Variable: A variable is a symbol for an unknown number, usually a letter. The variables in the sample expression are *x* and *y*.

Other Terminology Used With Algebraic Expressions

Binomial: An algebraic expression consisting of the sum or the difference of two terms is a binomial. EXAMPLE 3x + 9

Degree: The degree of a term is the sum of the term's exponents. For example, the degree of the term $2y^3$ is 3. The degree of the term $16x^2y^3$ is 5.

Equality: An equality consists of two expressions which have the same value. EXAMPLE A = B

Equation: An equation is a mathematical statement of an equality which contains at least one variable. EXAMPLE $x + 1 = 4$
The process of finding out the variable value that makes the equation true is called *solving* the equation.

Equivalent equations: Two equations that have the same solution are called equivalent equations. EXAMPLE $x = 1$ and $2x = 2$.

Exponent: In x^2, the exponent is 2.

Factor: Factors are numbers which, when multiplied together, result in another number. A number can have many factors. For example, 3 and 4 are factors of 12 because $3 \times 4 = 12$. 2 and 6 are also factors of 12 because $2 \times 6 = 12$.

Inequality: An inequality consists of two expressions, one on each side of a comparison sign that is not an equals sign. EXAMPLES $3x < 10$ and $a \leq b$.

Like terms: Like terms are terms which have the same variable raised to the same exponent. For example, $3x^2$ and $9x^2$ are like terms.

Monomial: An algebraic expression consisting of one term is a monomial. Monomials include numbers, whole numbers and variables that are multiplied together, and variables that are multiplied together. EXAMPLE 3xy

Polynomial: An algebraic expression consisting of more than two terms is a polynomial. The sample expression 3x + 2y – 7 is a polynomial. A polynomial may also include a term with exponents. EXAMPLE $x^2 - 4x + 7$.

APPENDIX C

NEMETH CODE FORMAT SUMMARIES

Read about this PROVISIONAL EDITION in the front matter to this book.
Check the NFB website periodically for updates to this appendix.

- General Principles
- "Keep Together"
- Margins
  - Itemized Material
  - Displayed Mathematical Material
- Labeled Mathematical Statements
- Formal Proofs
- Division of Mathematical Expressions Between Braille Lines
- Placement of Code Switch Indicators
- Spatial Arrangements

"Formatting" refers to layout on the page, such as indentations (margins), line spacing (blank lines), centering, and pagination. *The Nemeth Braille Code for Mathematics and Science Notation* specifies certain formats which are summarized in this appendix. For illustrative examples, go to the sections in this lesson material which are cited as bold numbers in brackets.

General Principles

When an item in a UEB transcription requires the use of Nemeth Code symbols, format rules of *The Nemeth Braille Code for Mathematics and Science Notation* are to be applied to the entire transcription including those portions transcribed in UEB. When a format is not specifically addressed in the Nemeth Code, the principles provided in *Braille Formats Principles of Print-to-Braille Transcription* should be followed. [**1.7**]

Example: A ditto mark is centered beneath the material to which it applies and is separated from any expression which precedes or follows it by at least one space. In UEB context, the UEB ditto mark symbol is used but Nemeth Code format is applied: center the ditto under the word or item to which it applies. **[13.2.3]**

"Keep Together"

Abbreviations: An abbreviation and a preceding or following numeral or letter to which it applies must not be divided between braille lines. [**4.4.1**]

Enclosed List: Items in an enclosed list must not be divided between braille lines if the entire list will fit on a single braille line. If the enclosed list will not fit on a single braille line, use as much of the current line as possible and begin a runover line after a comma. [**5.15.3**]

Function Names: A function name or its abbreviation and the sign which follows it must not be divided between braille lines. [**14.7**]

Hyphenated Expressions: A hyphenated expression containing one or more mathematical components must not be divided between braille lines. [**3.19**]

Keystroke Constructions: A single keystroke construction must not be divided between braille lines. [**11.25**]

Mathematical Expressions: A mathematical expression that will fit on one braille line within the current margins must not be divided between lines. The entire expression is brought down to the next line. [**2.5**] If a page number on line 25 or line 1 does not allow the entire expression to fit on the line, the expression is brought down to the next line that has enough usable cells. [**4.7**]

Signs of Shape: A sign of shape and the letter, sequence of letters, or numeral which follows it is regarded as a single mathematical item and therefore should not be divided between braille lines. [**11.31.1**]

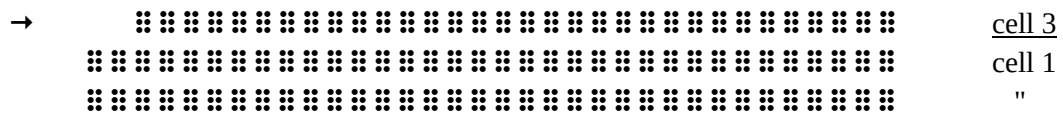
Symbols: The components of the following signs must not be divided between braille lines: plus or minus; minus or plus; plus followed by minus; minus followed by plus; minus followed by minus; signs of comparison compounded vertically or horizontally; superposed signs; tally marks belonging to the same group; signs of shape with structural or interior modification. [**6.2, 6.8, 6.9, 11.7-11.18, 13.4, 13.5-13.6**]

Margins

For margins with spatial material, see the section on Spatial Arrangements.

Nemeth Code formatting can be broken down into two major styles: indented and hanging. In the "indented" styles, the first line is indented two cells to the right of the runover margin. In the "hanging" styles, the first line begins two cells to the left of the runover margin, or four cells to the left of the runover margin.

Narrative Portions of Text (3-1): Narrative paragraphs in a document governed by Nemeth Code formatting begin in cell 3 with runovers in cell 1. Blocked paragraphing is not used in a technical transcription. [**2.14**]

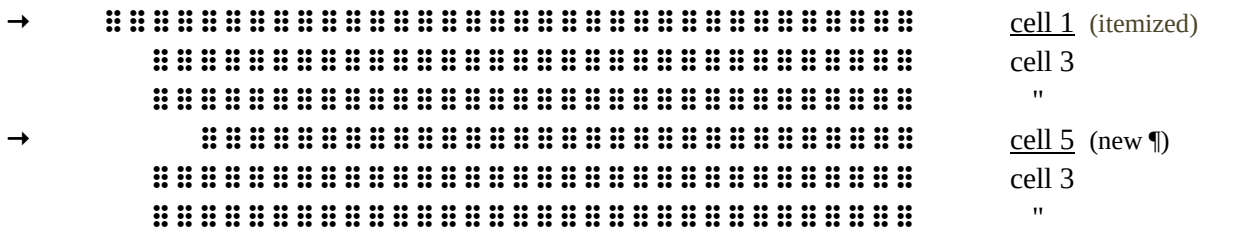


Instructions Preceding Unitemized Material (3-1): If instructional text is not followed by lettered or numbered exercise material, the text is considered to be a narrative paragraph and begins in cell 3 with runovers beginning in cell 1. The (5-5) style of "directions" given in *Braille Formats* does not apply in a transcription that contains Nemeth Code. [**6.12.1**]

Continuation of Narrative Following Displayed Material: Look for context clues in the print copy to determine if the narrative following a displayed mathematical expression is a new paragraph or if it is a continuation of the interrupted paragraph. Apply proper margins to the text accordingly. [**8.25.2**]

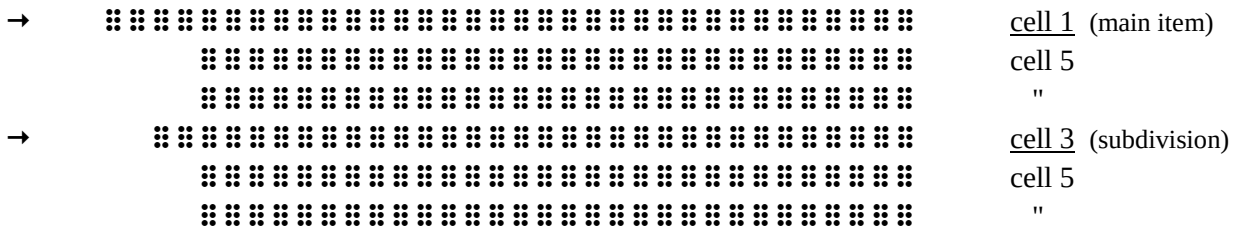
Margins: Itemized Material

Itemized Material with No Subdivisions (1-3): The identifier begins in cell 1; runovers begin in cell 3. If the material contains more than one paragraph, each subparagraph begins in cell 5 with runovers in cell 3. [**2.16**]

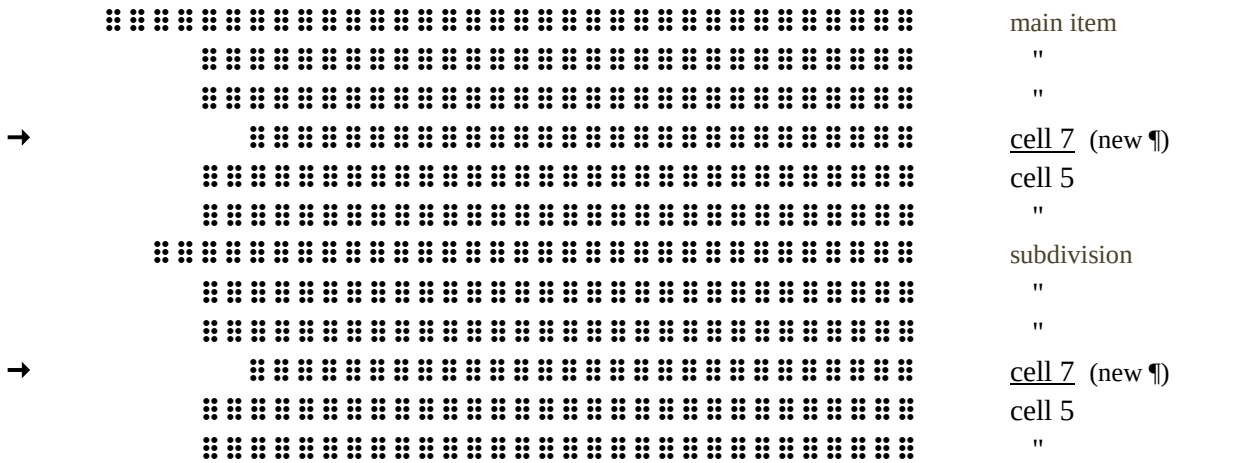


Itemized Material with No Subdivisions—Side-by-Side Print Layout: When the print copy arranges itemized material side-by-side across the page and there are no subdivisions, the braille format is changed so that all identifiers start in cell 1. [**3.20**] **EXCEPTION:** Spatially-arranged material may be arranged side-by-side across the width of the page as long as there are no subdivisions. [**10.21**]

Itemized Material with Subdivisions (1-5; 3-5): The main item designation begins in cell 1, and its runovers begin in cell 5. Each lettered or numbered subdivision—regardless of print indentation—begin in cell 3, with any runovers in cell 5. [**7.18**]



Itemized Material with Subdivisions with More Than One Paragraph (7-5): If a main item or a subdivision has more than one paragraph, each new paragraph begins in cell 7, and its runovers begin in cell 5. [**7.18.1**]

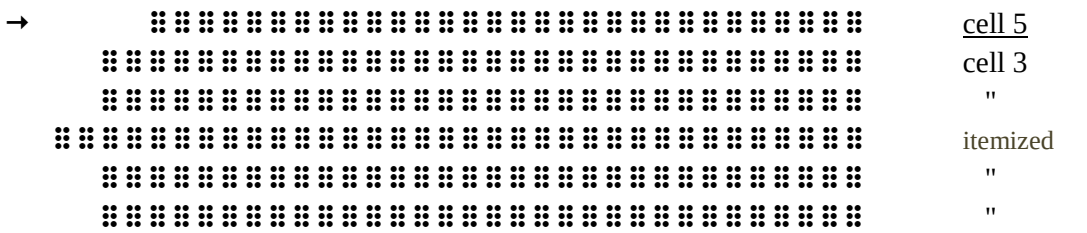


Itemized Material with Subdivisions—Side-by-Side Braille Layout Option: It is permissible to place the subdivisions side by side across a page if and only if all the subdivisions can be accommodated on one braille line. If all the subdivisions cannot be accommodated on one braille line, the (1-5; 3-5) format is used. This is an optional format style and applies only to subdivisions. (This restriction does not apply to spatial material, which will be studied later in this course.) [**7.18.2**]

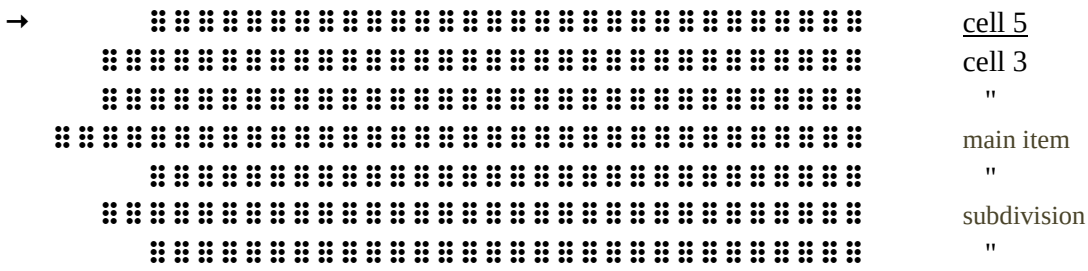
Tabular Form: When itemized material is arranged in tabular form so that items are numbered at the margin and subdivisions are aligned beneath lettered column headings, the material is transcribed in one of the following ways: [**7.18.3**]

- If all the columns can be accommodated across the braille page, the print columnar arrangement is followed. Each problem number begins in cell 1. The letter identifying each column is aligned with the first cell of the related column. A blank line is left above and below the lettered column headings. Two blank cells separate the columns.
- If all the columns cannot be accommodated across the braille page, each subdivision in each problem must be lettered individually, and the (1-5; 3-5) format is followed.

Instructions Preceding Itemized Material (5-3): When a group of numbered or lettered problems is preceded by instructions, the instructions begin in cell 5, with runovers in cell 3. One line is left blank above instructions unless the instructions follow a cell-5 or a cell-7 heading. Instructions may begin on line 1 of the braille page if no running head is in use. The related itemized material follows on the next line unless the material itself requires a blank line before it.At least one line of the instructions must be on the same braille page as the related problems which follow. [**6.11**]



OR

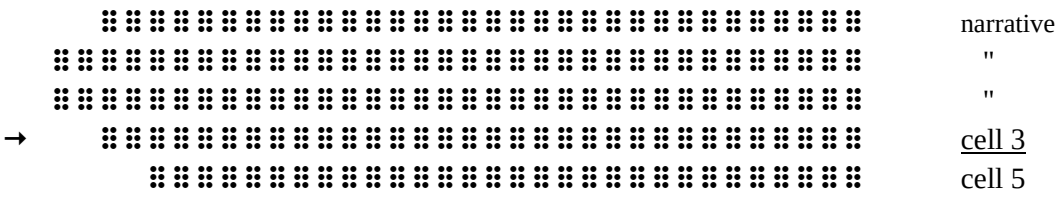


Exercise Set: Runover margins for itemized material are determined individually for each question. A problem with no subentries will be (1-3); the next problem may have subentries and so will be (1-5; 3-5), etc. [**7.19**]

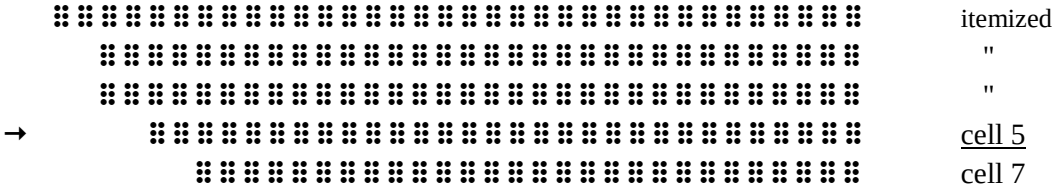
Margins: Displayed Mathematical Material

Displayed Mathematical Expressions: When mathematical material is set apart from the body of the text in the print copy, it is referred to as a displayed expression. In braille, margins for displayed mathematical material depend upon the margins of the surrounding text. The first cell of the displayed material is indented two cells to the right of the runover cell of the preceding material. These margins apply regardless of the presence of runover material in the preceding paragraph. [**8.25**]

— *In unitemized explanatory portions of the text*, displayed mathematical material begins in cell 3. Runovers begin in cell 5. [Text (3-1); displayed material (3-5)]



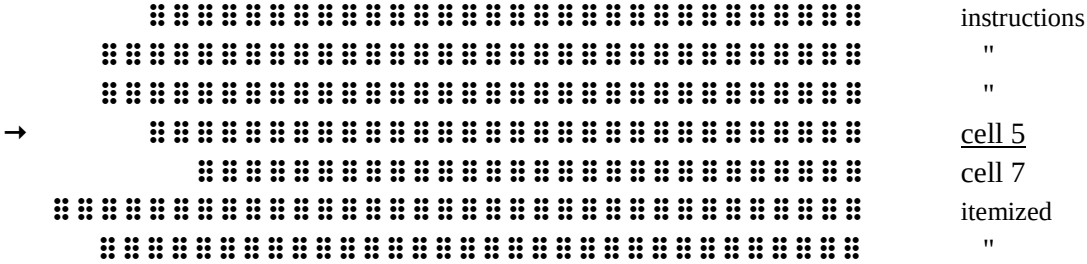
— *In itemized text without subdivisions*, displayed mathematical material begins in cell 5. Runovers begin in cell 7. [Text (1-3); displayed material (5-7)]



— *In itemized text with subdivisions*, displayed mathematical material begins in cell 7. Runovers begin in cell 9. [Main division text (1-5); displayed material (7-9). Subdivision text (3-5); displayed material (7-9)]



— *Within or following instructions*, displayed mathematical material begins in cell 5. Runovers begin in cell 7. [Instructions (5-3); displayed material (5-7)] [**8.25.3**]



Blank Lines with Displayed Material: A line is not skipped above or below displayed mathematical material unless the preceding or following material requires a blank line. [**8.25**]

Labeled Displayed Material: When a number or letter is used to label a displayed mathematical expression it is placed at the left of the expression in braille regardless of the location of the label in the print copy. The label begins in the appropriate cell for displayed material. [**11.36.1**]

Linked Expressions Requiring Special Margins: The print layout denoted as "special" (defined in **9.23**) can occur in one of the following three layouts. Note that the first cell of the displayed anchor is indented two cells to the right of the runover cell of the preceding material.

- **In Narrative:** When a linked expression requiring special margins is displayed to (3-1) unitemized explanatory portions of text, the *anchor* begins in cell 3. If the anchor has runovers, they begin in cell 7. *Each link* begins in cell 5. If a link has runovers, they also begin in cell 7. [**9.23.1**]
- **In Itemized Text Without Subdivisions:** When a linked expression requiring special margins is displayed to (1-3) itemized text containing no subdivisions, the *anchor* begins in cell 5. If the anchor has runovers, they begin in cell 9. *Each link* begins in cell 7. If a link has runovers, they also begin in cell 9. [**9.23.2**]
- **In Itemized Text With Subdivisions:** When a linked expression requiring special margins is displayed to itemized text containing subdivisions (1-5; 3-5), the *anchor* begins in cell 7. If the anchor has runovers, they begin in cell 11. *Each link* begins in cell 9. If a link has runovers, they also begin in cell 11. [**9.23.3**]

Labeled Mathematical Statements

Spacing and Margins: A line is left blank before the beginning and after the end of the entire labeled statement. Normal paragraphing (3-1) is applied, with the label beginning the paragraph. [**11.38.3**] The label is transcribed in full capitals regardless of print style and is placed as a paragraph heading regardless of its location in the print copy. Nonregular typeface in the label is ignored. [**11.38.1**] See **11.39** for details regarding typeface in the text of the statement.

Auxiliary Captions: If a proof or labeled statement contains auxiliary captions such as *Given*, *Prove*, or *Conclusion*, etc., such captions begin a new paragraph in cell 3 with runovers in cell 1. A line is not skipped above a caption. If a caption is printed in a nonregular typeface, UEB typeform indicators are applied in accordance with the print text. Print capitalization is maintained. [**12.17.2**]

Formal Proofs

Spacing and Margins: A line is left blank before the beginning and after the end of the entire formal proof. The narrative portion of the formal proof begins in cell 3 and its runovers begin in cell 1. Portions of a formal proof which satisfy the definition of a labeled statement follow the format directives outlined in **Lesson 11**. [**12.17.1**]

Step-Number Format: When a formal proof is presented in step-number form and divided into two columns, often headed *Statements* and *Reasons*, the following format is used in the braille transcription. [**12.17.3**]

- A line is left blank before the beginning and after the end of the step-numbered items.
- Each item from the *Reason* column is placed beneath its matching item from the *Statement* column.
- The letters "S" for *Statement* and "R" for *Reason* is placed immediately after the appropriate step number. Similarly, other column headings should be indicated by appropriate letters. The step numbers are brailled in UEB.
- Each step number begins in cell 1 and any runovers begin in cell 3.

Division of Mathematical Expressions Between Braille Lines

When a mathematical expression is too long to fit on one braille line within the current margins the expression is divided between braille lines according to the rules of the Nemeth Code. A new line need not be forced if there is room on the line to begin the expression, provided the division is made in accordance with the principles defined below. Runovers conform to the margin requirements currently in effect – be it itemized, instructional, explanatory, labeled, subdivided, or displayed material.

Long Numeral: A long numeral is divided after a comma if a comma is present, and a hyphen is inserted. If the numeral does not contain a comma, the hyphen may be inserted after any digit. When a numeral is divided between braille lines, the numeric indicator is used before the first digit of the numeral on the next line. [**2.7.1**]

Enclosed List: If an enclosed list will not fit on a single braille line, use as much of the current line as possible and begin a runover line after a comma. [**5.15.3.a**]

Linked Expressions: If a linked expression is too long to fit on one line, the expression continues on the next line, beginning with a sign of comparison. If the expression contains more than one link, it is not necessary to divide at every comparison sign – use as much as the line as possible before dividing the expression.. The new line begins in the runover cell of the current format. [**9.22**] The comparison sign at which the new line begins must be on the baseline of writing. An expression should not be divided before a comparison sign that is part of an item enclosed in grouping symbols, between fraction indicators, or within radical signs. [**9.22.1**] A transition to a new braille line made before a sign of comparison terminates the effect of any level indicator used on the line above. [**9.22.2**]

Mathematical expressions: Formulas, equations, etc., which cannot fit on one braille line within the boundaries of the margins used should be divided between lines in the following order of preference. After making divisions according to an item higher up on the priority list, if the expression requires further division a division may be made at an item lower on the priority list.

- Before a sign of comparison, providing that the sign is on the baseline of writing. Additionally, the sign cannot be part of an item enclosed in grouping symbols, fraction indicators, radical signs, or be enclosed between the symbols of a modified expression. A transition to a new braille line made before a sign of comparison terminates the effect of any level indicator used on the line above, just as it would if it were not divided between lines. [**14.13**]
- Before a sign of operation, providing that the sign is on the baseline of writing. If the transition to a new line is made before a minus sign, the numeric indicator is used after the minus sign when followed by a numeral or a decimal point and a numeral. [**14.14**]
- Before a fraction line, providing that the fraction line belongs to a fraction on the baseline of writing. Avoid division within a numerator or within a denominator. If the numerator or denominator requires a division, division at the fraction line is also applied.. [**14.15**]
- Before a baseline indicator [**14.16**]
- Before a level indicator [**14.17**]
- Between factors that are enclosed within grouping signs. Division should not be made within a factor unless unavoidable. [**14.18**]
- After a termination indicator of a modified expression or a radical. [**14.19**]

Placement of Code Switch Indicators

About Code Switching: The base code used in a complete transcription is Unified English Braille ("UEB"). When mathematical content occurs anywhere in the transcription, the non-technical notation follows UEB rules while the technical notation follows the rules of the Nemeth Code. The reader will assume he is reading Unified English Braille unless signaled otherwise by the use of an opening code switch indicator. The material between the opening Nemeth Code indicator and the Nemeth Code terminator follows Nemeth Code rules. UEB symbols are not used within the Nemeth Code switch indicators. [**2.1**]

The main objective is to keep the switch indicators on the same line as the mathematics to which they apply, with some exceptions listed below. There are also guidelines to follow when the switch indicators do not fall neatly on a line or on a page.

General Principle in Narrative Context: The opening Nemeth Code indicator is followed by a space. The Nemeth Code terminator is preceded by a space (unless it begins a new line). These spaces do not represent spaces in print. [**2.1**] Within a paragraph, the switch indicators should appear on the same line as the expression if the mathematical expression and the two indicators will fit on one braille line within the current margins. [**2.5.1**] If two or more math expressions are brailled between the same code switch indicators, the line may wrap at the space between the expressions even if the entire Nemeth Code portion could fit on one line. [**2.5.1.a**]

Punctuation that relates to the main text is generally placed outside of the switch indicators when the surrounding text is in UEB. There is no space between the terminator and the following punctuation. [**2.4]** When punctuation occurs within mathematical material, Nemeth Code punctuation is used. To avoid excessive code switching between mathematical items, Nemeth Code punctuation may be used for punctuation which belongs to the sentence structure **[3.3**]; embedded identifiers may be brailled in Nemeth Code. [**3.18**]

Switch Indicators May Stand Alone on a Line: If a math expression will fit on one line but there is not room for one or both of the switch indicators, one or both switch indicators may stand alone on a line. Keeping the mathematical expression intact on one line is the priority. [**2.5.2**]

Switch Indicators ...

> **... after Headings:** When itemized material immediately follows a centered heading, place the opening Nemeth Code indicator alone on the line immediately before the first identifier. [**5.24.1**] When itemized material immediately follows a cell-5 or a cell-7 heading, place the opening Nemeth Code indicator after the last word in the heading. If there is no room on the line, the indicator will fall in the runover cell of the heading. [**5.24.2**]
>
> **... at Page Turns:** Keep the opening switch indicator on the same braille page as the mathematical material to which it applies. Unless it is impossible to arrange, both the opening Nemeth Code indicator and the Nemeth Code terminator must appear on the same page as the expression to which they apply. The opening Nemeth Code indicator should not stand alone at the bottom of a braille page, nor should the Nemeth Code terminator stand alone at the top of a braille page. Part of the expression to which it applies should appear on the same braille page as the indicator. [**2.5.3, 4.7, 4.8**]
>
> **... with Boxed Material:** When literary content is followed by boxed mathematical material, if all of the material in the box is in Nemeth Code place the opening Nemeth Code indicator at the beginning of the top box line, followed by a blank space. Place the Nemeth Code terminator at the end of the bottom box line, preceded by a space. See **17.6** for further details regarding code switching and box lines.

... with Comments: When short narrative comments alternate with math problems, switch out of Nemeth Code to transcribe the comments in contracted braille. [**15.11**]

... with Instructions: If exercise directions ("instructions") end with an expression in Nemeth Code and the subsequent math problem starts with Nemeth Code, Nemeth Code may be left in effect between the end of the directions and the start of the problem. [**6.11.1**]

... with Itemized Material: Identifiers are transcribed according to the rules for the code in use at the time. All identifiers in a section do not need to be transcribed in the same code (code switching may be required within the itemized portion). [**3.18**] To ensure that all identifiers begin in the same cell, the opening Nemeth Code indicator is placed at the end of the line of text that precedes the itemized material, or on the next line in the appropriate runover cell. The code switch indicator does not take the place of the blank line that must precede the itemized material. [**3.17**] If space permits, the terminator should be placed on the same line where Nemeth Code ends.

... with Displayed Mathematical Material: When displayed mathematical material is both preceded and followed by UEB text, the expression and the two switch indicators may be placed all together on one line if they will fit within current margins. If more than one line is required for the expression, the opening Nemeth Code indicator is placed at the end of the text line preceding the displayed material. The Nemeth Code terminator is placed at the completion of the displayed expression. In either case, if the indicator will not fit on the current line, it is placed on the following line in the runover position. [**8.25.1**]

... with Spatial Arrangements: The opening Nemeth Code indicator and the Nemeth Code terminator do not take the place of the required blank line before and after a spatial arrangement. The switch indicators must not interfere with the layout of the problem. An opening Nemeth Code indicator that precedes a spatial problem may be placed on the same line with the end of the text above the problem, if space permits. The required blank line follows the opening Nemeth Code indicator. If there is not room on the line with the preceding text, the opening Nemeth Code indicator is placed in the runover position if it precedes itemized material, or in cell 1 on a line by itself if it precedes unitemized material. The required blank line follows. To close Nemeth Code after a spatial problem, first insert the required blank line that follows the arrangement. Then place the Nemeth Code terminator in cell 1 by itself on the following braille line. [**10.35**] Keep in mind that the blank lines required by the Nemeth Code are part of the spatial problem and so must be inside the Nemeth switches.

... with Tables: When table entries consist of technical material, the entire body of the table is brailled in Nemeth Code, including any words. One opening Nemeth Code indicator precedes row 1 and one Nemeth Code terminator is placed at the end of the table. Words within the table are brailled without contractions, and the single-word switch indicator is not used. **[6.5, 17.5]** When a mixture of narrative entries and mathematical data occur in the table, a switch to Nemeth Code may be applied only where needed. A table may be more clearly presented by brailling it entirely in Nemeth Code even when some entries do not require a switch. [**17.4.3**] See **17.6** for details regarding code switching and box lines.

... with Tactile Graphics: Nemeth Code remains in effect for a tactile graphic if the graphic intervenes between two items in Nemeth Code. [**17.9**]

For further code-switching guidelines and illustrative examples see the following BANA publication: "Guidance for Transcription Using the Nemeth Code within UEB Context."

Spatial Arrangements

Details regarding various spatial arrangements are in the following lesson sections.

Addition **Lesson 10**

Arrays **16.12-16.18**

Cancellation

in Long Division Problems **13.16**

in Subtraction Problems **10.29**

with Fractions **15.5**

Carried Numbers

with Addition **10.28**

with Division **13.15**

with Multiplication **12.26**

Code Switch Indicators, Placement of **10.34-10.36, 10.38**

Determinants **16.12-16.18**

Displayed Spatial Arrangements **10.38**

Division **Lesson 13**

Partial Quotients **14.25**

Square Roots **14.22-14.24**

Synthetic Division **14.26**

Fractions and Mixed Numbers **10.26, 12.23, 15.4, 15.5**

Hypercomplex Fractions **15.7-15.8**

Instructions Preceding Itemized Spatial Arrangements **10.37**

Itemized Spatial Arrangements **10.27-10.28, 10.33, 12.27, 13.17**

Matrices **16.12-16.18**

Multiplication **Lesson 12**

Omissions **10.23, 11.30, 12.22, 13.14**

Polynomials **12.24**

Remarks **15.11, 16.10**

Square Roots **14.22-14.24**

Stem-and-Leaf Plots **15.11-15.15**

Subscripts Denoting Nondecimal Bases **12.25**

Subtraction **Lesson 10**